

ALGEBRA

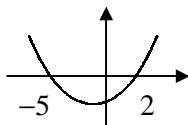
Answers

1 $x^2 + 3x + 2 \leq 12$

$$x^2 + 3x - 10 \leq 0$$

$$(x + 5)(x - 2) \leq 0$$

$$-5 \leq x \leq 2$$



2 **a** $= 8\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$

b $= x + 12\sqrt{x} + 36 + 4x - 12\sqrt{x} + 9$
 $= 5x + 45$

3 **a** $(-2, 0) \Rightarrow 0 = 8 - 2p + q$ (1)

$$(3, 0) \Rightarrow 0 = 18 + 3p + q$$
 (2)

$$(2) - (1) \quad 0 = 10 + 5p \Rightarrow p = -2$$

$$\text{sub.} \quad \Rightarrow q = -12$$

b $x\text{-coord} = \frac{-2+3}{2} = \frac{1}{2}$

$$\therefore y = -\frac{25}{2} \Rightarrow \left(\frac{1}{2}, -\frac{25}{2}\right)$$

4 $2x - 2\sqrt{32} = \sqrt{98} - x$

$$3x = 2\sqrt{32} + \sqrt{98}$$

$$3x = 8\sqrt{2} + 7\sqrt{2}$$

$$3x = 15\sqrt{2}$$

$$x = 5\sqrt{2}$$

5 **a** real and distinct roots

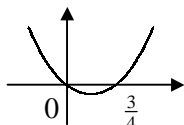
$$\therefore b^2 - 4ac > 0$$

$$16k^2 - 12k > 0$$

$$4k^2 - 3k > 0$$

$$k(4k - 3) > 0$$

b



$$k < 0 \text{ or } k > \frac{3}{4}$$

6 $(2^2)^{2x} = 2^{y-1}$

$$4x = y - 1$$
 (1)

$$(3^2)^{4x} = 3^{y+1}$$

$$8x = y + 1$$
 (2)

$$(1) \text{ and } (2) \Rightarrow y = 4x + 1 = 8x - 1$$

$$4x = 2$$

$$x = \frac{1}{2}, y = 3$$

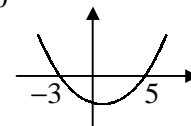
7 **a** $\text{LHS} = \left(x - \frac{7}{2}\right)^2 - \frac{49}{4} + 9$

$$= \left(x - \frac{7}{2}\right)^2 - \frac{13}{4}$$

$$\therefore a = -\frac{7}{2}, b = -\frac{13}{4}$$

b $x = \frac{7}{2}$

8 **a** $(y + 3)(y - 5) < 0$



$$-3 < y < 5$$

b $x(2 - x) = 4(x - 3)$

$$x^2 + 2x - 12 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 48}}{2} = \frac{-2 \pm 2\sqrt{13}}{2}$$

$$x = -1 \pm \sqrt{13}$$

9 $2^{x^2+2} = (2^3)^x$

$$x^2 + 2 = 3x$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$x = 1 \text{ or } 2$$

10 **a** $t - 2t^2 = 3t - 15$

$$2t^2 + 2t - 15 = 0$$

$$t = \frac{-2 \pm \sqrt{4 + 120}}{4} = \frac{-2 \pm \sqrt{124}}{4} = \frac{-2 \pm 2\sqrt{31}}{4}$$

$$t = \frac{1}{2}(-1 \pm \sqrt{31})$$

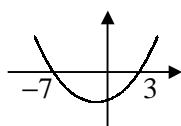
b $(x^2 + 2)(x^2 - 3) = 0$

$$x^2 = -2 \text{ [no solutions] or } 3$$

$$x = \pm\sqrt{3}$$

11 $x^2 + 4x - 21 \geq 0$
 $(x + 7)(x - 3) \geq 0$

$$x \leq -7 \text{ or } x \geq 3$$



12 **a** $3^{2x+2} = 3^2(3^x)^2 = 9y^2$

b $9y^2 - 10y + 1 = 0$
 $(9y - 1)(y - 1) = 0$
 $y = 3^x = \frac{1}{9}, 1$

$$\therefore x = -2, 0$$

13 **a** $= \sqrt{25 \times 3} = \sqrt{75}$

b $\sqrt{64} < \sqrt{75} < \sqrt{81}$

$$\therefore 8 < 5\sqrt{3} < 9$$

$$\therefore n = 8$$

14 $y = \frac{2x+7}{3}$

sub. $2x^2 - \left(\frac{2x+7}{3}\right)^2 - 7 = 0$

$$18x^2 - (2x+7)^2 - 63 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x+2)(x-4) = 0$$

$$x = -2 \text{ or } 4$$

$$\therefore x = -2, y = 1 \text{ or } x = 4, y = 5$$

15 **a** $= \sqrt{\frac{48}{12}} - \sqrt{\frac{600}{12}}$

$$= \sqrt{4} - \sqrt{50}$$

$$= 2 - 5\sqrt{2}$$

b $= \frac{\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = \frac{\sqrt{2}(4-3\sqrt{2})}{16-18}$

$$= -\frac{1}{2}(4\sqrt{2} - 6)$$

$$= 3 - 2\sqrt{2}$$

16 **a** $5^{x+1} = (5^2)^{y-3}$

$$x+1 = 2y-6$$

$$y = \frac{x+7}{2}$$

b $(4^2)^{x-1} = 4^z$

$$2x-2 = z$$

$$x = 2y-7 \quad \therefore z = 2(2y-7) - 2$$

$$z = 4y - 16$$

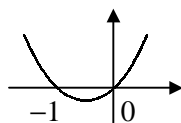
17 **a** $(x-k)^2 - k^2 - k = 0$

$$x-k = \pm \sqrt{k^2+k}$$

$$x = k \pm \sqrt{k^2+k}$$

b real roots $\therefore k^2+k \geq 0$

$$k(k+1) \geq 0$$



$$k \leq -1 \text{ or } k \geq 0$$

18 **a** $\frac{1}{y} - y = \frac{3}{2}$

$$2 - 2y^2 = 3y$$

$$2y^2 + 3y - 2 = 0$$

b $(2y-1)(y+2) = 0$

$$y = -2, \frac{1}{2}$$

$$x = y^5 = -32, \frac{1}{32}$$