

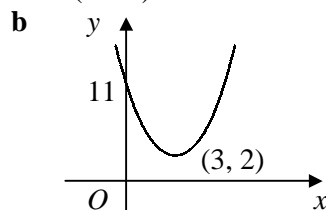
ALGEBRA

Answers

1 a $2^{x-1} = 2^4$
 $x - 1 = 4$
 $x = 5$

b $3^y - 10 = 17$
 $3^y = 27$
 $y = 3$

2 a $= (x - 3)^2 - 9 + 11$
 $= (x - 3)^2 + 2$



3 a $= \left(\frac{49}{4}\right)^{-\frac{1}{2}} = \sqrt{\frac{4}{49}} = \frac{2}{7}$

b $3x^{-3} = \frac{64}{9}$

$$x^3 = \frac{27}{64}$$

$$x = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

4 $2x\sqrt{3} + 9 = x\sqrt{3}$

$$x\sqrt{3} = -9$$

$$x = \frac{-9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -3\sqrt{3}$$

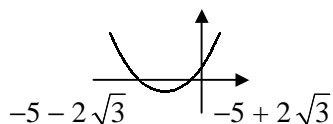
5 a $x = \frac{-10 \pm \sqrt{100 - 52}}{2}$

$$= \frac{-10 \pm \sqrt{48}}{2}$$

$$= \frac{-10 \pm 4\sqrt{3}}{2}$$

$$= -5 \pm 2\sqrt{3}$$

b



$$x < -5 - 2\sqrt{3} \text{ or } x > -5 + 2\sqrt{3}$$

6 a $42x - 49 = 9x^2$

$$9x^2 - 42x + 49 = 0$$

$$(3x - 7)^2 = 0$$

$$x = \frac{7}{3}$$

b $2 + (y + 1) = 2y(y + 1)$

$$2y^2 + y - 3 = 0$$

$$(2y + 3)(y - 1) = 0$$

$$y = -\frac{3}{2} \text{ or } 1$$

7 $y = x + 3$

sub.

$$3x^2 - 2x(x + 3) + (x + 3)^2 - 17 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\therefore x = -2, y = 1 \text{ or } x = 2, y = 5$$

8 a $x^{\frac{1}{3}} = \sqrt[3]{64} = 4$

$$x = 4^2 = 16$$

b $\frac{\sqrt{3}+1}{2\sqrt{3}-3} = \frac{\sqrt{3}+1}{2\sqrt{3}-3} \times \frac{2\sqrt{3}+3}{2\sqrt{3}+3} = \frac{(\sqrt{3}+1)(2\sqrt{3}+3)}{12-9}$

$$= \frac{1}{3}(6 + 3\sqrt{3} + 2\sqrt{3} + 3)$$

$$= 3 + \frac{5}{3}\sqrt{3}$$

$$\therefore a = 3, b = \frac{5}{3}$$

9 a let A be (2, 4)

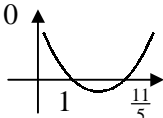
$$\therefore AP^2 = (2k - 2)^2 + (k - 4)^2$$

$$AP < 3 \therefore (2k - 2)^2 + (k - 4)^2 < 9$$

$$5k^2 - 16k + 11 < 0$$

b $(5k - 11)(k - 1) < 0$

$$1 < k < \frac{11}{5}$$



10 a $2x \leq 7$

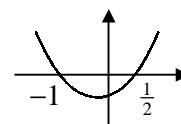
$$x \leq \frac{7}{2}$$

b $2x^2 + x < 1$

$$2x^2 + x - 1 < 0$$

$$(2x - 1)(x + 1) < 0$$

$$-1 < x < \frac{1}{2}$$



$$\begin{aligned}
 \mathbf{11} \quad \mathbf{a} \quad f(x) &= 2[x^2 - 4x] + 5 \\
 &= 2[(x-2)^2 - 4] + 5 \\
 &= 2(x-2)^2 - 3
 \end{aligned}$$

$$\mathbf{b} \quad (2, -3)$$

$$\begin{aligned}
 \mathbf{c} \quad 2(x-2)^2 - 3 &= 0 \\
 x-2 &= \pm \sqrt{\frac{3}{2}} \\
 x &= 2 \pm \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
 x &= 2 \pm \frac{1}{2}\sqrt{6}
 \end{aligned}$$

13 no real roots

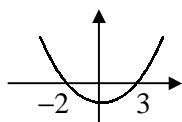
$$\therefore b^2 - 4ac < 0$$

$$4k^2 - 4(k+6) < 0$$

$$k^2 - k - 6 < 0$$

$$(k+2)(k-3) < 0$$

$$-2 < k < 3$$



$$\begin{aligned}
 \mathbf{15} \quad (2^2)^{2y+7} &= (2^3)^{y+3} \\
 4y + 14 &= 3y + 9 \\
 y &= -5
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{17} \quad \mathbf{a} \quad x^2 + 4x + k &= 0 \\
 (x+2)^2 - 4 + k &= 0 \\
 x+2 &= \pm \sqrt{4-k} \\
 x &= -2 \pm \sqrt{4-k}
 \end{aligned}$$

b real roots only if $4 - k \geq 0$

$$\therefore k \leq 4$$

$$\mathbf{c} \quad k = -4$$

$$\begin{aligned}
 \therefore x &= -2 \pm \sqrt{8} \\
 x &= -2 \pm 2\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{12} \quad \mathbf{a} &= 2\sqrt{3} - \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= 2\sqrt{3} - \frac{5}{3}\sqrt{3} \\
 &= \frac{1}{3}\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= \frac{64x\sqrt{x}}{16x} \\
 &= 4\sqrt{x}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{14} \quad \mathbf{a} \quad AM &= \frac{1}{2}AC = 2 + 2\sqrt{3} \\
 BM^2 &= AB^2 - AM^2 \\
 &= (4 + \sqrt{3})^2 - (2 + 2\sqrt{3})^2 \\
 &= 16 + 8\sqrt{3} + 3 - (4 + 8\sqrt{3} + 12) = 3 \\
 \therefore BM &= \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= \frac{1}{2} \times AC \times BM \\
 &= \frac{1}{2} \times (4 + 4\sqrt{3}) \times \sqrt{3} \\
 &= \frac{1}{2}(4\sqrt{3} + 12) = 6 + 2\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{16} \quad \text{LHS} &= x^2(2x^2 - 3x - 9) - x(2x^2 - 3x - 9) \\
 &\quad + 3(2x^2 - 3x - 9) \\
 &= 2x^4 - 3x^3 - 9x^2 - 2x^3 + 3x^2 + 9x \\
 &\quad + 6x^2 - 9x - 27 \\
 &= 2x^4 - 5x^3 - 27 \\
 \therefore A &= 2, B = -5 \text{ and } C = -27
 \end{aligned}$$