ALGEBRA

1 **a** Find the value of x such that

$$2^{x-1} = 16.$$ **(3)**

b Find the value of y such that

$$2(3^y - 10) = 34.$$ **(2)**

2 **a** Express $x^2 - 6x + 11$ in the form $(x + a)^2 + b$. **(2)**

b Sketch the curve $y = x^2 - 6x + 11$, and show the coordinates of the turning point of the curve. **(3)**

3 **a** Express $(12\frac{1}{4})^{-\frac{1}{2}}$ as an exact fraction in its simplest form. **(2)**

b Solve the equation

$$3x^{-3} = 7\tfrac{1}{9}.$$ **(3)**

4 Solve the equation

$$x\sqrt{12} + 9 = x\sqrt{3},$$

giving your answer in the form $k\sqrt{3}$, where k is an integer. **(4)**

5 **a** Solve the equation

$$x^2 + 10x + 13 = 0,$$

giving your answers in the form $a + b\sqrt{3}$, where a and b are integers. **(4)**

b Hence find the set of values of x for which

$$x^2 + 10x + 13 > 0.$$ **(2)**

6 Solve the equations

a $7(6x - 7) = 9x^2$ **(3)**

b $\dfrac{2}{y+1} + 1 = 2y$ **(4)**

7 Solve the simultaneous equations

$$x - y + 3 = 0$$
$$3x^2 - 2xy + y^2 - 17 = 0$$ **(6)**

8 **a** Find the value of x such that

$$x^{\frac{3}{2}} = 64.$$ **(2)**

b Given that

$$\frac{\sqrt{3}+1}{2\sqrt{3}-3} \equiv a + b\sqrt{3},$$

find the values of the rational constants a and b. **(4)**

9 The point P $(2k, k)$ lies within a circle of radius 3, centre (2, 4).

a Show that $5k^2 - 16k + 11 < 0$. **(4)**

b Hence find the set of possible values of k. **(3)**

10 Solve each of the following inequalities.

a $4x - 1 \leq 2x + 6$ **(2)**

b $x(2x + 1) < 1$ **(4)**

11 $f(x) = 2x^2 - 8x + 5.$

a Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers. **(3)**

b Write down the coordinates of the turning point of the curve $y = f(x)$. **(1)**

c Solve the equation $f(x) = 0$, giving your answers in the form $p + q\sqrt{6}$, where p and q are rational. **(3)**

12 Simplify

a $\sqrt{12} - \frac{5}{\sqrt{3}}$ **(3)**

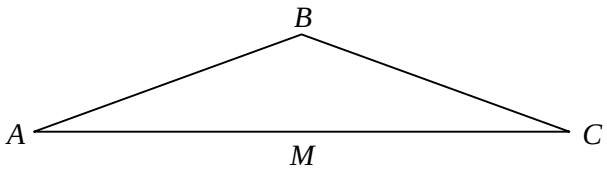
b $\frac{(4\sqrt{x})^3}{16x}$ **(2)**

13 Given that the equation

$$x^2 - 2kx + k + 6 = 0$$

has no real roots, find the set of possible values of the constant k. **(6)**

14



The diagram shows triangle ABC in which $AB = BC = 4 + \sqrt{3}$ and $AC = 4 + 4\sqrt{3}$.

Given that M is the mid-point of AC,

a find the exact length BM, **(4)**

b show that the area of triangle ABC is $6 + 2\sqrt{3}$. **(2)**

15 Solve the equation

$$4^{2y+7} = 8^{y+3}.$$ **(4)**

16 Show that

$$(x^2 - x + 3)(2x^2 - 3x - 9) \equiv Ax^4 + Bx^3 + C,$$

where A, B and C are constants to be found. **(4)**

17 $f(x) = x^2 + 4x + k.$

a By completing the square, find in terms of the constant k the roots of the equation $f(x) = 0$. **(4)**

b State the set of values of k for which the equation $f(x) = 0$ has real roots. **(1)**

c Use your answers to part **a** to solve the equation

$$x^2 + 4x - 4 = 0,$$

giving your answers in the form $a + b\sqrt{2}$, where a and b are integers. **(2)**