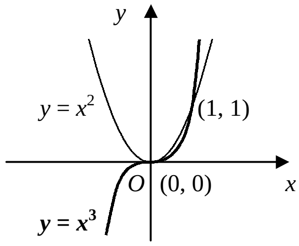
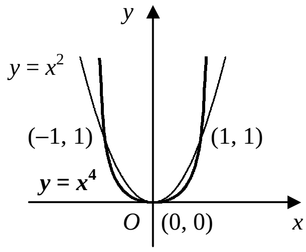
GRAPHS OF FUNCTIONS — *Answers*

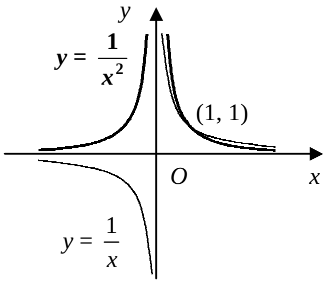
1 **a**



b

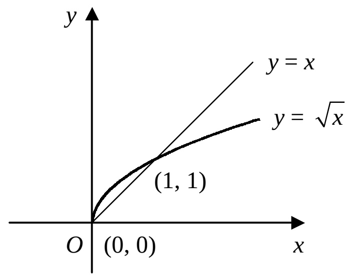


c



asymptotes: $y = 0$ and $x = 0$

d



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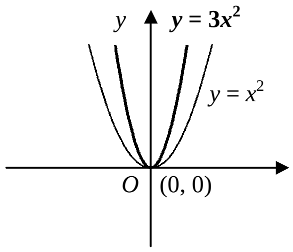

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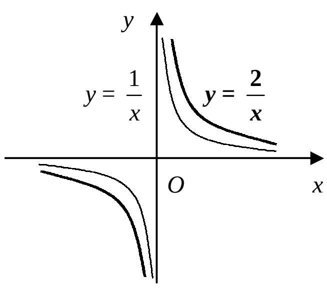
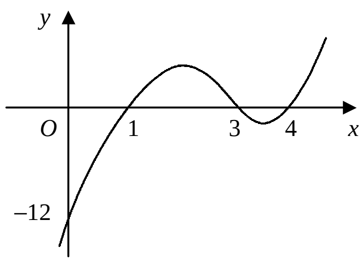


asymptotes: $y = 0$ and $x = 0$

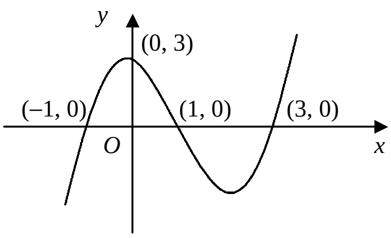
2 **a** $= (-1) \times (-3) \times (-4) = -12$

b $x = 1, 3, 4$

c



3 **a**



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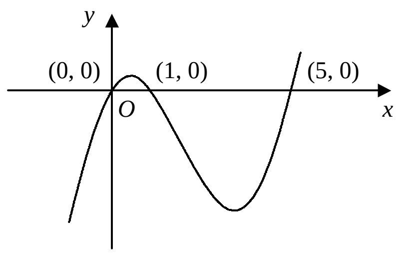

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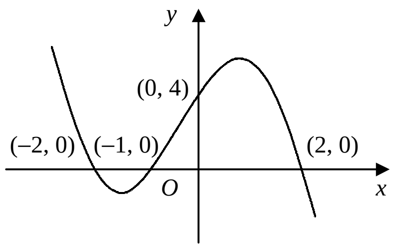

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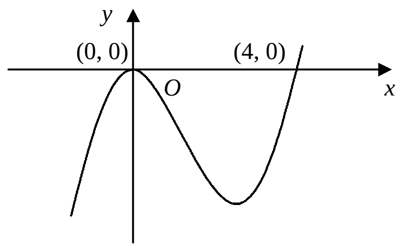

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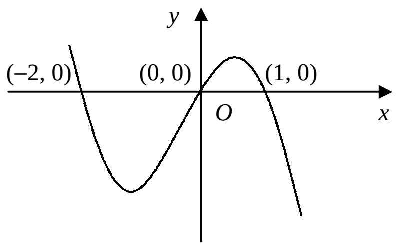

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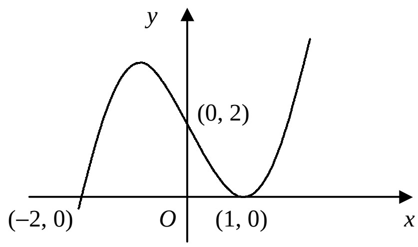
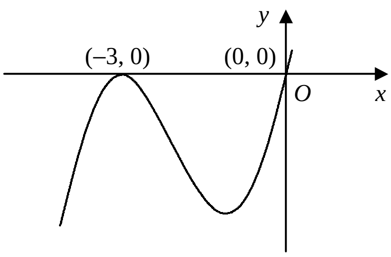
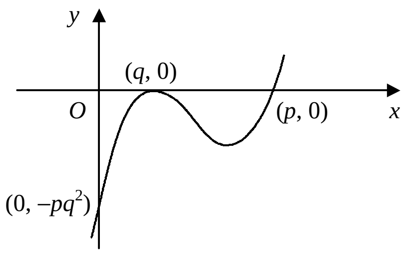


4 **a** $= x(x^2 + 6x + 9) = x(x + 3)^2$

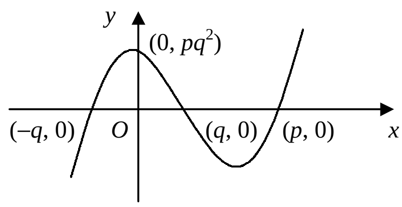
b



5 **a**



b $y = (x - p)(x + q)(x - q)$



6 TP at $(1, -2)$

$\therefore\ f(x) = k(x - 1)^2 - 2$

crosses y-axis at $(0, -5)$

$\therefore\ -5 = k - 2$

$k = -3$

$\therefore\ f(x) = -3(x - 1)^2 - 2$

$[\ f(x) = -3x^2 + 6x - 5\ ]$

7 crosses x-axis at $(-2, 0)$, $(1, 0)$ and $(2, 0)$

$\therefore\ y = k(x + 2)(x - 1)(x - 2)$

crosses y-axis at $(0, -8)$

$\therefore\ -8 = 4k$

$k = -2$

$\therefore\ y = -2(x + 2)(x - 1)(x - 2)$

$= -2(x + 2)(x^2 - 3x + 2)$

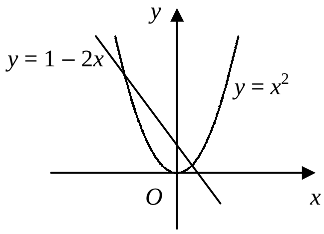
$= -2(x^3 - 3x^2 + 2x + 2x^2 - 6x + 4)$

$= -2x^3 + 2x^2 + 8x - 8$

$\therefore\ a = -2, b = 2, c = 8, d = -8$

8 **a** 4 **b** 0 **c** 2 **d** 3

9 **a**



b 2 roots as $x^2 + 2x - 1 = 0 \Rightarrow x^2 = 1 - 2x$ and the graphs of $y = x^2$ and $y = 1 - 2x$ intersect at 2 points

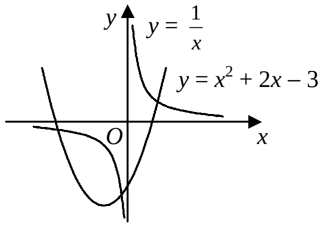
10 **a** $x^2 + 2x - 3 = (x + 1)^2 - 1 - 3 = (x + 1)^2 - 4$ $\therefore$ turning point is $(-1, -4)$

b $x^2 + 2x - 3 - \frac{1}{x} = 0 \Rightarrow x^2 + 2x - 3 = \frac{1}{x}$

$\therefore$ roots where $y = x^2 + 2x - 3$ and $y = \frac{1}{x}$ intersect

graphs intersect at 1 point for $x > 0$ and 2 points for $x < 0$

$\therefore$ one positive and two negative real roots



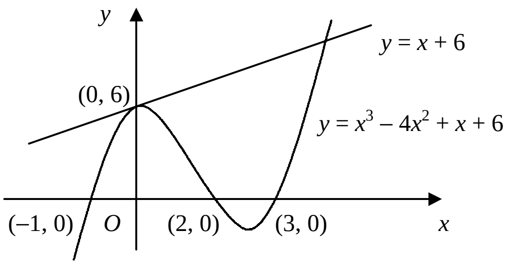
11 $x - 3 = x^2 - 5x + 6$
$x^2 - 6x + 9 = 0$
$(x - 3)^2 = 0$
repeated root
$\therefore$ $y = x - 3$ is tangent to $y = x^2 - 5x + 6$

12 **a** $x^2 + 5x + 8 = 3x + 7$
$x^2 + 2x + 1 = 0$
$(x + 1)^2 = 0$
$x = -1$ $\therefore$ $x = -1, y = 4$

b repeated root
$\therefore$ $y = 3x + 7$ is tangent to $y = x^2 + 5x + 8$ at the point $(-1, 4)$

13 **a** $x^3 - 4x^2 + x + 6 = x + 6$
$x^3 - 4x^2 = 0$
$x^2(x - 4) = 0$
$x = 0, 4$ $\therefore$ $(0, 6)$ and $(4, 10)$

b



14 $2x^2 - 5x + 1 = 3x + k$
$2x^2 - 8x + 1 - k = 0$
for tangent, repeated root $\therefore$ $b^2 - 4ac = 0$
$\therefore$ $64 - 8(1 - k) = 0$
$k = -7$

15 $x^2 + ax + 18 = 2 - 5x$
$x^2 + (a + 5)x + 16 = 0$
intersect at 2 points $\therefore$ $b^2 - 4ac > 0$
$\therefore$ $(a + 5)^2 - 64 > 0$
$a^2 + 10a - 39 > 0$
$(a + 13)(a - 3) > 0$
$a < -13$ or $a > 3$

16 **a** $x^2 - 2x + 6 = px + p$
$x^2 - (p + 2)x + 6 - p = 0$
for tangent, repeated root $\therefore$ $b^2 - 4ac = 0$
$\therefore$ $(p + 2)^2 - 4(6 - p) = 0$
$p^2 + 8p - 20 = 0$
$(p + 10)(p - 2) = 0$
$p = -10, 2$

b $x^2 - 2x + 6 = qx + 7$
$x^2 - (q + 2)x - 1 = 0$
for tangent, repeated root $\therefore$ $b^2 - 4ac = 0$
$\Rightarrow$ $(q + 2)^2 + 4 = 0$
but for real q, $(q + 2)^2 \geq 0$ $\therefore$ no solutions