ALGEBRA — Answers

1 **a** $f(-2) = 0 \Rightarrow -8 - 20 - 2a + b = 0$

$\Rightarrow -2a + b = 28 \quad (1)$

$f(3) = 0 \Rightarrow 27 - 45 + 3a + b = 0$

$\Rightarrow 3a + b = 18 \quad (2)$

$(2) - (1) \;\; 5a = -10 = 0 \Rightarrow a = -2$

sub. (1) $\Rightarrow b = 24$

b $f(x) \equiv x^3 - 5x^2 - 2x + 24$

$(x + 2)(x - 3)(ax + b) \equiv x^3 - 5x^2 - 2x + 24$

by inspection

$f(x) \equiv (x + 2)(x - 3)(x - 4)$

2 $f(k) = 8f(\tfrac{1}{2}k)$

$8k^3 - k^2 + 7 = 8(k^3 - \tfrac{1}{4}k^2 + 7)$

$8k^3 - k^2 + 7 = 8k^3 - 2k^2 + 56$

$k^2 = 49$

$k = \pm 7$

3 **a** $f(2) = 24 - 4 - 24 + 4 = 0$

$\therefore (x - 2)$ is a factor of $f(x)$

b

$$\begin{array}{r|l} & 3x^2 + 5x - 2 \\ x - 2 & 3x^3 - x^2 - 12x + 4 \\ & 3x^3 - 6x^2 \\ & \quad\quad 5x^2 - 12x \\ & \quad\quad 5x^2 - 10x \\ & \quad\quad\quad\quad -2x + 4 \\ & \quad\quad\quad\quad -2x + 4 \end{array}$$

$\therefore f(x) = (x - 2)(3x^2 + 5x - 2)$

$= (x - 2)(3x - 1)(x + 2)$

$f(x) = 0 \Rightarrow (x - 2)(3x - 1)(x + 2) = 0$

$x = -2, \tfrac{1}{3}$ or 2

4 $6 + 7x - x^3 = 0$

let $f(x) = 6 + 7x - x^3$

$f(1) = 12,\; f(2) = 12,\; f(-1) = 0$

$\therefore (x + 1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & -x^2 + x + 6 \\ x + 1 & -x^3 + 0x^2 + 7x + 6 \\ & -x^3 - x^2 \\ & \quad\quad x^2 + 7x \\ & \quad\quad x^2 + x \\ & \quad\quad\quad\quad 6x + 6 \\ & \quad\quad\quad\quad 6x + 6 \end{array}$$

$\therefore (x + 1)(-x^2 + x + 6) = 0$

$-(x + 1)(x - 3)(x + 2) = 0$

$x = -2, -1, 3$

$\therefore (-2, 0), (-1, 0)$ and $(3, 0)$

5 **a** $f(-1) = -4$

$\therefore\ -3 + p - 8 + q = -4$

$p + q = 7 \qquad (1)$

$f(2) = 80$

$\therefore\ 24 + 4p + 16 + q = 80$

$4p + q = 40 \qquad (2)$

$(2) - (1) \Rightarrow 3p = 33$

$\therefore\ p = 11,\ q = -4$

b $f(x) \equiv 3x^3 + 11x^2 + 8x - 4$

$f(-2) = -24 + 44 - 16 - 4 = 0$

$\therefore\ (x + 2)$ is a factor

c

$$
\begin{array}{r|l}
 & 3x^2 + 5x - 2 \\
\hline
x + 2 & 3x^3 + 11x^2 + 8x - 4 \\
 & \underline{3x^3 + 6x^2} \\
 & 5x^2 + 8x \\
 & \underline{5x^2 + 10x} \\
 & -2x - 4 \\
 & \underline{-2x - 4}
\end{array}
$$

$\therefore\ f(x) = (x + 2)(3x^2 + 5x - 2)$

$= (3x - 1)(x + 2)^2$

$\therefore\ f(x) = 0 \Rightarrow x = -2$ or $\frac{1}{3}$

6 **a** let $f(x) = x^3 - 4x^2 - 7x + 10$

$f(1) = 1 - 4 - 7 + 10 = 0$

$\therefore\ (x - 1)$ is a factor

$$
\begin{array}{r|l}
 & x^2 - 3x - 10 \\
\hline
x - 1 & x^3 - 4x^2 - 7x + 10 \\
 & \underline{x^3 - x^2} \\
 & -3x^2 - 7x \\
 & \underline{-3x^2 + 3x} \\
 & -10x + 10 \\
 & \underline{-10x + 10}
\end{array}
$$

$\therefore\ (x - 1)(x^2 - 3x - 10) = 0$

$(x - 1)(x + 2)(x - 5) = 0$

$x = -2, 1, 5$

b $y^2 = x$ in part **a**

$y^2 = 1, 5$ or -2 [no solutions]

$y = \pm 1, \pm\sqrt{5}$

7 **a** $= f(-1) = -1 + 7 - 14 + 3 = -5$

b

$$
\begin{array}{r|l}
 & n^2 + 6n + 8 \\
\hline
n + 1 & n^3 + 7n^2 + 14n + 3 \\
 & \underline{n^3 + n^2} \\
 & 6n^2 + 14n \\
 & \underline{6n^2 + 6n} \\
 & 8n + 3 \\
 & \underline{8n + 8} \\
 & -5
\end{array}
$$

$\therefore\ f(n) = (n + 1)(n^2 + 6n + 8) - 5$

$f(n) = (n + 1)(n + 2)(n + 4) - 5$

c $(n + 1)$ and $(n + 2)$ are consecutive integers

$\therefore$ either $(n + 1)$ or $(n + 2)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4) - 5$ is odd