ALGEBRA

1 $$f(x) \equiv x^3 - 5x^2 + ax + b.$$

Given that $(x + 2)$ and $(x - 3)$ are factors of $f(x)$,

a show that $a = -2$ and find the value of b.

b Hence, express $f(x)$ as the product of three linear factors.

2 $$f(x) \equiv 8x^3 - x^2 + 7.$$

The remainder when $f(x)$ is divided by $(x - k)$ is eight times the remainder when $f(x)$ is divided by $(2x - k)$.

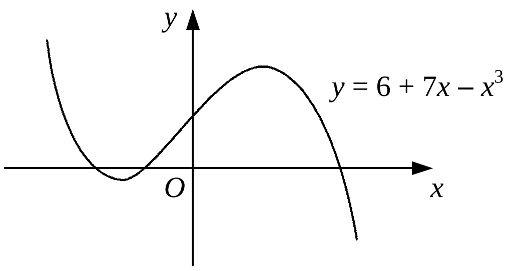
Find the two possible values of the constant k.

3 $$f(x) \equiv 3x^3 - x^2 - 12x + 4.$$

a Show that $(x - 2)$ is a factor of $f(x)$.

b Solve the equation $f(x) = 0$.

4



The diagram shows the curve with the equation $y = 6 + 7x - x^3$.

Find the coordinates of the points where the curve crosses the x-axis.

5 $$f(x) \equiv 3x^3 + px^2 + 8x + q.$$

When $f(x)$ is divided by $(x + 1)$ there is a remainder of -4.

When $f(x)$ is divided by $(x - 2)$ there is a remainder of 80.

a Find the values of the constants p and q.

b Show that $(x + 2)$ is a factor of $f(x)$.

c Solve the equation $f(x) = 0$.

6 **a** Solve the equation

$$x^3 - 4x^2 - 7x + 10 = 0.$$

b Hence, solve the equation

$$y^6 - 4y^4 - 7y^2 + 10 = 0.$$

7 $$f(n) \equiv n^3 + 7n^2 + 14n + 3.$$

a Find the remainder when $f(n)$ is divided by $(n + 1)$.

b Express $f(n)$ in the form

$$f(n) \equiv (n + 1)(n + a)(n + b) + c,$$

where a, b and c are integers.

c Hence, show that $f(n)$ is odd for all positive integer values of n.