ALGEBRA

1 Find the quotient obtained in dividing

a $(x^3 + 2x^2 - x - 2)$ by $(x + 1)$

b $(x^3 + 2x^2 - 9x + 2)$ by $(x - 2)$

c $(20 + x + 3x^2 + x^3)$ by $(x + 4)$

d $(2x^3 - x^2 - 4x + 3)$ by $(x - 1)$

e $(6x^3 - 19x^2 - 73x + 90)$ by $(x - 5)$

f $(-x^3 + 5x^2 + 10x - 8)$ by $(x + 2)$

g $(x^3 - 2x + 21)$ by $(x + 3)$

h $(3x^3 + 16x^2 + 72)$ by $(x + 6)$

2 Find the quotient and remainder obtained in dividing

a $(x^3 + 8x^2 + 17x + 16)$ by $(x + 5)$

b $(x^3 - 15x^2 + 61x - 48)$ by $(x - 7)$

c $(3x^3 + 4x^2 + 7)$ by $(2 + x)$

d $(-x^3 - 5x^2 + 15x - 50)$ by $(x + 8)$

e $(4x^3 + 2x^2 - 16x + 3)$ by $(x - 3)$

f $(1 - 22x^2 - 6x^3)$ by $(x + 2)$

3 Use the factor theorem to determine whether or not

a $(x - 1)$ is a factor of $(x^3 + 2x^2 - 2x - 1)$

b $(x + 2)$ is a factor of $(x^3 - 5x^2 - 9x + 2)$

c $(x - 3)$ is a factor of $(x^3 - x^2 - 14x + 27)$

d $(x + 6)$ is a factor of $(2x^3 + 13x^2 + 2x - 24)$

e $(2x + 1)$ is a factor of $(2x^3 - 5x^2 + 7x - 14)$

f $(3x - 2)$ is a factor of $(2 - 17x + 25x^2 - 6x^3)$

4 $$f(x) \equiv x^3 - 2x^2 - 11x + 12.$$

a Show that $(x - 1)$ is a factor of $f(x)$.

b Hence, express $f(x)$ as the product of three linear factors.

5 $$g(x) \equiv 2x^3 + x^2 - 13x + 6.$$

Show that $(x + 3)$ is a factor of $g(x)$ and solve the equation $g(x) = 0$.

6 $$f(x) \equiv 6x^3 - 7x^2 - 71x + 12.$$

Given that $f(4) = 0$, find all solutions to the equation $f(x) = 0$.

7 $$g(x) \equiv x^3 + 7x^2 + 7x - 6.$$

Given that $x = -2$ is a solution to the equation $g(x) = 0$,

a express $g(x)$ as the product of a linear factor and a quadratic factor,

b find, to 2 decimal places, the other two solutions to the equation $g(x) = 0$.

8 $$f(x) \equiv x^3 + 2x^2 - 11x - 12.$$

a Evaluate $f(1)$, $f(2)$, $f(-1)$ and $f(-2)$.

b Hence, state a linear factor of $f(x)$ and fully factorise $f(x)$.

9 By first finding a linear factor, fully factorise

a $x^3 - 2x^2 - 5x + 6$

b $x^3 + x^2 - 5x - 2$

c $20 + 11x - 8x^2 + x^3$

d $3x^3 - 4x^2 - 35x + 12$

e $x^3 + 8$

f $12 + 29x + 8x^2 - 4x^3$

10 Solve each equation, giving your answers in exact form.

a $x^3 - x^2 - 10x - 8 = 0$

b $x^3 + 2x^2 - 9x - 18 = 0$

c $4x^3 - 12x^2 + 9x = 2$

d $x^3 - 5x^2 + 3x + 1 = 0$

e $x^2(x + 4) = 3(3x + 2)$

f $x^3 - 14x + 15 = 0$

11 $$\mathrm{f}(x) \equiv 2x^3 - x^2 - 15x + c.$$

Given that $(x - 2)$ is a factor of $\mathrm{f}(x)$,

a find the value of the constant c,

b fully factorise $\mathrm{f}(x)$.

12 $$\mathrm{g}(x) \equiv x^3 + px^2 - 13x + q.$$

Given that $(x + 1)$ and $(x - 3)$ are factors of $\mathrm{g}(x)$,

a show that $p = 3$ and find the value of q,

b solve the equation $\mathrm{g}(x) = 0$.

13 Use the remainder theorem to find the remainder obtained in dividing

a $(x^3 + 4x^2 - x + 6)$ by $(x - 2)$

b $(x^3 - 2x^2 + 7x + 1)$ by $(x + 1)$

c $(2x^3 + x^2 - 9x + 17)$ by $(x + 5)$

d $(8x^3 + 4x^2 - 6x - 3)$ by $(2x - 1)$

e $(2x^3 - 3x^2 - 20x - 7)$ by $(2x + 1)$

f $(3x^3 - 6x^2 + 2x - 7)$ by $(3x - 2)$

14 Given that when $(x^3 - 4x^2 + 5x + c)$ is divided by $(x - 2)$ the remainder is 5, find the value of the constant c.

15 Given that when $(2x^3 - 9x^2 + kx + 5)$ is divided by $(2x - 1)$ the remainder is -2, find the value of the constant k.

16 Given that when $(2x^3 + ax^2 + 13)$ is divided by $(x + 3)$ the remainder is 22,

a find the value of the constant a,

b find the remainder when $(2x^3 + ax^2 + 13)$ is divided by $(x - 4)$.

17 $$\mathrm{f}(x) \equiv px^3 + qx^2 + qx + 3.$$

Given that $(x + 1)$ is a factor of $\mathrm{f}(x)$,

a find the value of the constant p.

Given also that when $\mathrm{f}(x)$ is divided by $(x - 2)$ the remainder is 15,

b find the value of the constant q.

18 $$\mathrm{p}(x) \equiv x^3 + ax^2 + 9x + b.$$

Given that $(x - 3)$ is a factor of $\mathrm{p}(x)$,

a find a linear relationship between the constants a and b.

Given also that when $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is -30,

b find the values of the constants a and b.

19 $$\mathrm{f}(x) \equiv 4x^3 - 6x^2 + mx + n.$$

Given that when $\mathrm{f}(x)$ is divided by $(x + 1)$ the remainder is 3 and that when $\mathrm{f}(x)$ is divided by $(2x - 1)$ the remainder is 15, find the values of the constants m and n.

20 $$\mathrm{g}(x) \equiv x^3 + cx + 3.$$

Given that when $\mathrm{g}(x)$ is divided by $(x - 4)$ the remainder is 39,

a find the value of the constant c,

b find the quotient and remainder when $\mathrm{g}(x)$ is divided by $(x + 2)$.