

## ALGEBRA

## Answers

- 1**
- a**  $3x = 2x + 1$   
 $x = 1$   
 $\therefore x = 1, y = 3$
- b**  $x - 6 = \frac{1}{2}x - 4$   
 $x = 4$   
 $\therefore x = 4, y = -2$
- c**  $2x + 6 = 3 - 4x$   
 $x = -\frac{1}{2}$   
 $\therefore x = -\frac{1}{2}, y = 5$
- d** subtracting  
 $y + 4 = 0$   
 $y = -4$   
 $\therefore x = 7, y = -4$
- e**  $2x + 4y + 22 = 0$   
 $2x - 3y + 1 = 0$   
 subtracting  
 $7y + 21 = 0$   
 $y = -3$   
 $\therefore x = -5, y = -3$
- f**  $6x + 6y + 8 = 0$   
 $15x - 6y - 15 = 0$   
 adding  
 $21x - 7 = 0$   
 $x = \frac{1}{3}$   
 $\therefore x = \frac{1}{3}, y = -\frac{5}{3}$
- 2**
- a**  $x + 2 = x^2 - 4$   
 $x^2 - x - 6 = 0$   
 $(x + 2)(x - 3) = 0$   
 $x = -2$  or  $3$   
 $\therefore (-2, 0)$  and  $(3, 5)$
- b**  $4x + 11 = x^2 + 3x - 1$   
 $x^2 - x - 12 = 0$   
 $(x + 3)(x - 4) = 0$   
 $x = -3$  or  $4$   
 $\therefore (-3, -1)$  and  $(4, 27)$
- c**  $2x - 1 = 2x^2 + 3x - 7$   
 $2x^2 + x - 6 = 0$   
 $(2x - 3)(x + 2) = 0$   
 $x = -2$  or  $\frac{3}{2}$   
 $\therefore (-2, -5)$  and  $(\frac{3}{2}, 2)$
- 3**
- a** subtracting  
 $x^2 - x - 2 = 0$   
 $(x + 1)(x - 2) = 0$   
 $x = -1$  or  $2$   
 $\therefore x = -1, y = 4$   
 or  $x = 2, y = 7$
- b** adding  
 $2x^2 - 7x + 3 = 0$   
 $(2x - 1)(x - 3) = 0$   
 $x = \frac{1}{2}$  or  $3$   
 $\therefore x = \frac{1}{2}, y = -\frac{7}{2}$   
 or  $x = 3, y = -6$
- c**  $y = 2x - 5$   
 sub  
 $x^2 + (2x - 5)^2 = 25$   
 $x^2 - 4x = 0$   
 $x(x - 4) = 0$   
 $x = 0$  or  $4$   
 $\therefore x = 0, y = -5$   
 or  $x = 4, y = 3$
- d**  $y = 2x + 10$   
 sub.  
 $x^2 + 2x(2x + 10) + 15 = 0$   
 $x^2 + 4x + 3 = 0$   
 $(x + 3)(x + 1) = 0$   
 $x = -3$  or  $-1$   
 $\therefore x = -3, y = 4$   
 or  $x = -1, y = 8$
- e**  $y = 1 - x$   
 sub.  
 $x^2 - 2x(1 - x) - (1 - x)^2 = 7$   
 $x^2 = 4$   
 $x = \pm 2$   
 $\therefore x = -2, y = 3$   
 or  $x = 2, y = -1$
- f**  $y = 1 - x$   
 sub.  
 $3x^2 - x - (1 - x)^2 = 0$   
 $2x^2 + x - 1 = 0$   
 $(2x - 1)(x + 1) = 0$   
 $x = -1$  or  $\frac{1}{2}$   
 $\therefore x = -1, y = 2$   
 or  $x = \frac{1}{2}, y = \frac{1}{2}$
- g**  $y = 4 - x$   
 sub.  
 $2x^2 + x(4 - x) + (4 - x)^2 = 22$   
 $x^2 - 2x - 3 = 0$   
 $(x + 1)(x - 3) = 0$   
 $x = -1$  or  $3$   
 $\therefore x = -1, y = 5$   
 or  $x = 3, y = 1$
- h**  $x = 2y$   
 sub.  
 $(2y)^2 - 4y - y^2 = 0$   
 $3y^2 - 4y = 0$   
 $y(3y - 4) = 0$   
 $y = 0$  or  $\frac{4}{3}$   
 $\therefore x = 0, y = 0$   
 or  $x = \frac{8}{3}, y = \frac{4}{3}$
- i**  $y = 3 - \frac{3}{2}x$   
 sub.  
 $x^2 + x(3 - \frac{3}{2}x) = 4$   
 $x^2 - 6x + 8 = 0$   
 $(x - 2)(x - 4) = 0$   
 $x = 2$  or  $4$   
 $\therefore x = 2, y = 0$   
 or  $x = 4, y = -3$

- j**  $y = 2x - 3$   
sub.  
 $2x^2 + (2x - 3) - (2x - 3)^2 = 8$   
 $x^2 - 7x + 10 = 0$   
 $(x - 2)(x - 5) = 0$   
 $x = 2$  or  $5$   
 $\therefore x = 2, y = 1$   
or  $x = 5, y = 7$
- k**  $y = 2x - 7$   
sub.  
 $x^2 - x(2x - 7) + (2x - 7)^2 = 13$   
 $x^2 - 7x + 12 = 0$   
 $(x - 3)(x - 4) = 0$   
 $x = 3$  or  $4$   
 $\therefore x = 3, y = -1$   
or  $x = 4, y = 1$
- l**  $y = 5 - 3x$   
sub.  
 $x^2 - 5x + (5 - 3x)^2 = 0$   
 $2x^2 - 7x + 5 = 0$   
 $(2x - 5)(x - 1) = 0$   
 $x = 1$  or  $\frac{5}{2}$   
 $\therefore x = 1, y = 2$   
or  $x = \frac{5}{2}, y = -\frac{5}{2}$
- m**  $x = 2y + 10$   
sub.  
 $3(2y + 10)^2 - y(2y + 10) + y^2 = 36$   
 $y^2 + 10y + 24 = 0$   
 $(y + 6)(y + 4) = 0$   
 $y = -6$  or  $-4$   
 $\therefore x = -2, y = -6$   
or  $x = 2, y = -4$
- n**  $y = \frac{3}{2}x - 2$   
sub.  
 $2x^2 + x - 4(\frac{3}{2}x - 2) = 6$   
 $2x^2 - 5x + 2 = 0$   
 $(2x - 1)(x - 2) = 0$   
 $x = \frac{1}{2}$  or  $2$   
 $\therefore x = \frac{1}{2}, y = -\frac{5}{4}$   
or  $x = 2, y = 1$
- o**  $x = 3y - 17$   
sub.  
 $(3y - 17)^2 + (3y - 17) + 2y^2 - 52 = 0$   
 $y^2 - 9y + 20 = 0$   
 $(y - 4)(y - 5) = 0$   
 $y = 4$  or  $5$   
 $\therefore x = -5, y = 4$   
or  $x = -2, y = 5$
- 4 a** subtracting  
 $-\frac{1}{y} + 2y + 1 = 0$   
 $-1 + 2y^2 + y = 0$   
 $2y^2 + y - 1 = 0$   
 $(2y - 1)(y + 1) = 0$   
 $y = -1$  or  $\frac{1}{2}$   
 $\therefore x = -5, y = -1$   
or  $x = 4, y = \frac{1}{2}$
- b**  $y = x - 5$   
sub.  
 $x(x - 5) = 6$   
 $x^2 - 5x - 6 = 0$   
 $(x + 1)(x - 6) = 0$   
 $x = -1$  or  $6$   
 $\therefore x = -1, y = -6$   
or  $x = 6, y = 1$
- c**  $y = 7 - 4x$   
sub.  
 $\frac{3}{x} - 2(7 - 4x) + 4 = 0$   
 $3 - 2x(7 - 4x) + 4x = 0$   
 $8x^2 - 10x + 3 = 0$   
 $(4x - 3)(2x - 1) = 0$   
 $x = \frac{1}{2}$  or  $\frac{3}{4}$   
 $\therefore x = \frac{1}{2}, y = 5$   
or  $x = \frac{3}{4}, y = 4$
- 5**  $5 - x = x^2 - 3x + 2$   
 $x^2 - 2x - 3 = 0$   
 $(x + 1)(x - 3) = 0$   
 $x = -1$  or  $3$   
 $P$  and  $Q$  are the points  $(-1, 6)$  and  $(3, 2)$   
 $PQ^2 = (3 + 1)^2 + (2 - 6)^2$   
 $PQ = \sqrt{32} = 4\sqrt{2}$
- 6**  $3^{x-1} = (3^2)^{2y}$   $\therefore x - 1 = 4y$   
 $(2^3)^{x-2} = (2^2)^{1+y}$   $\therefore 3x - 6 = 2 + 2y$   
 $6x - 16 = 4y$   
 $\Rightarrow 6x - 16 = x - 1$   
 $x = 3$   
 $\therefore x = 3, y = \frac{1}{2}$
- 7**  $AB - A\sqrt{3} + 2B\sqrt{3} - 6 \equiv 9\sqrt{3} - 1$   
 $A$  and  $B$  integers  $\therefore AB - 6 = -1$  (1) and  $-A + 2B = 9$  (2)  
(2)  $\Rightarrow A = 2B - 9$   
sub. (1)  $B(2B - 9) - 6 = -1 \Rightarrow 2B^2 - 9B - 5 = 0$   
 $(2B + 1)(B - 5) = 0$   
 $B = -\frac{1}{2}$  or  $5$   
 $B$  integer  $\therefore B = 5 \Rightarrow A = 1, B = 5$