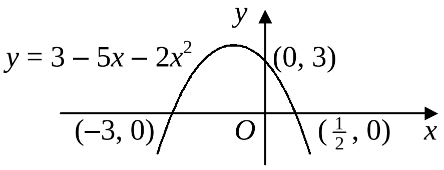
ALGEBRA — Answers

1 **a** $= 2x(10 - x - 3x^2)$
$= 2x(2 + x)(5 - 3x)$
b $2x(2 + x)(5 - 3x) = 0$
$x = -2, 0 \text{ or } \frac{5}{3}$

2 **a** $AB^2 = (6 + 2)^2 + (k - 1)^2 = 64 + k^2 - 2k + 1$
$= k^2 - 2k + 65$
b $k^2 - 2k + 65 = 10^2 = 100$
$k^2 - 2k - 35 = 0$
$(k + 5)(k - 7) = 0$
$k = -5 \text{ or } 7$

3 **a** $x^2 - 5 = 4x$
$x^2 - 4x - 5 = 0$
$(x + 1)(x - 5) = 0$
$x = -1 \text{ or } 5$
b $9 - (5 - x) = 2x(5 - x)$
$2x^2 - 9x + 4 = 0$
$(2x - 1)(x - 4) = 0$
$x = \frac{1}{2} \text{ or } 4$

4 **a** $y = -2[x^2 + \frac{5}{2}x] + 3$
$= -2[(x + \frac{5}{4})^2 - \frac{25}{16}] + 3$
$= -2(x + \frac{5}{4})^2 + \frac{49}{8}$
$\therefore$ turning point is $(-\frac{5}{4}, \frac{49}{8})$
b $3 - 5x - 2x^2 = 0$
$2x^2 + 5x - 3 = 0$
$(2x - 1)(x + 3) = 0, \ x = -3 \text{ or } \frac{1}{2}$



5 $x = \dfrac{-5\sqrt{2} \pm \sqrt{50 + 48}}{4}$
$= \dfrac{-5\sqrt{2} \pm \sqrt{98}}{4}$
$= \dfrac{-5\sqrt{2} \pm 7\sqrt{2}}{4}$
$= -3\sqrt{2} \text{ or } \frac{1}{2}\sqrt{2}$

6 **a** $y = 3[x^2 - 3x] + k = 3[(x - \frac{3}{2})^2 - \frac{9}{4}] + k$
$= 3(x - \frac{3}{2})^2 - \frac{27}{4} + k$
$\therefore$ x-coordinate of $P = \frac{3}{2}$
b y-coord of $P = k - \frac{27}{4} = \frac{17}{4}$ $\therefore k = 11$
$\therefore$ curve is $y = 3x^2 - 9x + 11$
$\therefore$ coordinates of Q are $(0, 11)$

7 $y^2 - 10y + 16 = 0$
$(y - 2)(y - 8) = 0$
$y = 2^x = 2 \text{ or } 8$
$x = 1 \text{ or } 3$

8 equal roots $\therefore b^2 - 4ac = 0$
$4 - 4k(3 - 2k) = 0$
$2k^2 - 3k + 1 = 0$
$(2k - 1)(k - 1) = 0$
$k = \frac{1}{2} \text{ or } 1$

9 **a** $f(x) = -[x^2 - 4x] + 3$
$= -[(x - 2)^2 - 4] + 3$
$= -(x - 2)^2 + 7$
b turning point is $(2, 7)$
c $-(x - 2)^2 + 7 = 2$
$(x - 2)^2 = 5$
$x = 2 \pm \sqrt{5}$

10 **a** $x = \dfrac{5 \pm \sqrt{25 - 12}}{6}$
$= \frac{1}{6}(5 \pm \sqrt{13})$
b $x(x - 1) = 3(x + 2)$
$x^2 - 4x - 6 = 0$
$x = \dfrac{4 \pm \sqrt{16 + 24}}{2} = \dfrac{4 \pm 2\sqrt{10}}{2}$
$= 2 \pm \sqrt{10}$

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11 **a** $(x-2k)^2-4k^2+6=0$

$(x-2k)^2=4k^2-6$

$x-2k=\pm\sqrt{4k^2-6}$

$x=2k\pm\sqrt{4k^2-6}$

b $k=3$

$\therefore\ x=6\pm\sqrt{36-6}$

$=6\pm\sqrt{30}$

12 **a** $x^2-6x-3=0$

$x=\frac{6\pm\sqrt{36+12}}{2}=\frac{6\pm4\sqrt{3}}{2}$

$=3\pm2\sqrt{3}$

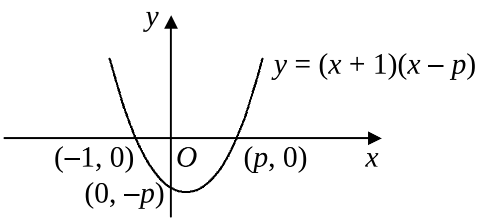
b $y(2y^2+y-15)=0$

$y(2y-5)(y+3)=0$

$y=-3,\ 0$ or $\frac{5}{2}$

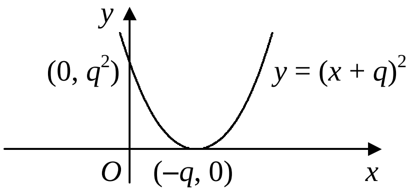
13 **a** $x=0\ \Rightarrow\ y=-p$

$y=0\ \Rightarrow\ x=-1$ or p



b $x=0\ \Rightarrow\ y=q^2$

$y=0\ \Rightarrow\ x=-q\quad[-q>0]$



14 **a** $\mathrm{f}(x)=2[x^2-3x]+5$

$=2[(x-\frac{3}{2})^2-\frac{9}{4}]+5$

$=2(x-\frac{3}{2})^2+\frac{1}{2}$

$\therefore\ A=2,\ B=-\frac{3}{2},\ C=\frac{1}{2}$

b minimum value of $\mathrm{f}(x)=\frac{1}{2}$

15 **a** $x^{\frac{2}{3}}=(x^{\frac{1}{3}})^2=t^2$

b let $t=x^{\frac{1}{3}}\ \Rightarrow\ 2t^2+t-6=0$

$(2t-3)(t+2)=0$

$t=-2$ or $\frac{3}{2}$

but $x=t^3\ \therefore\ x=-8$ or $\frac{27}{8}$

16 **a** $=(k-4)^2-16+20$

$=(k-4)^2+4$

b $x^2-kx+2k-5=0$

discriminant $=b^2-4ac$

$=k^2-4(2k-5)$

$=k^2-8k+20$

using **a** $=(k-4)^2+4$

for all real k, $(k-4)^2\geq0$

$\therefore$ discriminant >0

$\therefore$ real and distinct roots for all real k

17 **a** $(x^2+2x-3)(x^2-3x-4)\equiv x^2(x^2-3x-4)+2x(x^2-3x-4)-3(x^2-3x-4)$

$\equiv x^4-3x^3-4x^2+2x^3-6x^2-8x-3x^2+9x+12$

$\equiv x^4-x^3-13x^2+x+12$

b $(x^2+2x-3)(x^2-3x-4)=0$

$(x+3)(x-1)(x+1)(x-4)=0$

$x=-3,\ -1,\ 1$ or 4

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