ALGEBRA

1 **a** Factorise fully the expression

$$20x - 2x^2 - 6x^3.$$

b Hence, find all solutions to the equation

$$20x - 2x^2 - 6x^3 = 0.$$

2 A is the point $(-2, 1)$ and B is the point $(6, k)$.

a Show that $AB^2 = k^2 - 2k + 65$.

Given also that $AB = 10$,

b find the possible values of k.

3 Solve the equations

a $x - \dfrac{5}{x} = 4$

b $\dfrac{9}{5-x} - 1 = 2x$

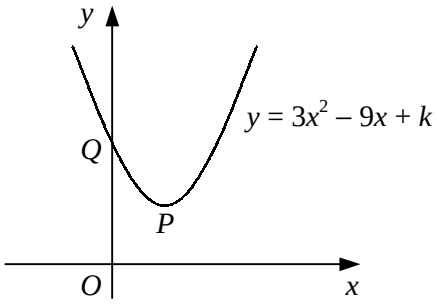
4 **a** Find the coordinates of the turning point of the curve with equation $y = 3 - 5x - 2x^2$.

b Sketch the curve $y = 3 - 5x - 2x^2$, showing the coordinates of any points of intersection with the coordinate axes.

5 Find in the form $k\sqrt{2}$ the solutions of the equation

$$2x^2 + 5\sqrt{2}\,x - 6 = 0.$$

6



The diagram shows the curve with equation $y = 3x^2 - 9x + k$ where k is a constant.

a Find the x-coordinate of the turning point of the curve, P.

Given that the y-coordinate of P is $\frac{17}{4}$,

b find the coordinates of the point Q where the curve crosses the y-axis.

7 By letting $y = 2^x$, or otherwise, solve the equation

$$2^{2x} - 10(2^x) + 16 = 0.$$

8 Given that the equation

$$kx^2 - 2x + 3 - 2k = 0$$

has equal roots, find the possible values of the constant k.

9 $f(x) \equiv 3 + 4x - x^2$.

a Express $f(x)$ in the form $a(x + b)^2 + c$.

b State the coordinates of the turning point of the curve $y = f(x)$.

c Solve the equation $f(x) = 2$, giving your answers in the form $d + e\sqrt{5}$.

10 Giving your answers in terms of surds, solve the equations

a $3x^2 - 5x + 1 = 0$

b $\dfrac{x}{x+2} = \dfrac{3}{x-1}$

11 **a** By completing the square, find, in terms of k, the solutions of the equation

$$x^2 - 4kx + 6 = 0.$$

b Using your answers to part **a**, solve the equation

$$x^2 - 12x + 6 = 0.$$

12 **a** Find in the form $a + b\sqrt{3}$, where a and b are integers, the values of x such that

$$2x^2 - 12x = 6.$$

b Solve the equation

$$2y^3 + y^2 - 15y = 0.$$

13 Labelling the coordinates of any points of intersection with the coordinate axes, sketch the curves

a $y = (x + 1)(x - p)$ where $p > 0$,

b $y = (x + q)^2$ where $q < 0$.

14 $f(x) \equiv 2x^2 - 6x + 5$.

a Find the values of A, B and C such that

$$f(x) \equiv A(x + B)^2 + C.$$

b Hence deduce the minimum value of $f(x)$.

15 **a** Given that $t = x^{\frac{1}{3}}$ express $x^{\frac{2}{3}}$ in terms of t.

b Hence, or otherwise, solve the equation

$$2x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0.$$

16 **a** Express $k^2 - 8k + 20$ in the form $a(k + b)^2 + c$, where a, b and c are constants.

b Hence prove that the equation

$$x^2 - kx + 2k = 5$$

has real and distinct roots for all real values of k.

17 **a** Show that

$$(x^2 + 2x - 3)(x^2 - 3x - 4) \equiv x^4 - x^3 - 13x^2 + x + 12.$$

b Hence solve the equation

$$x^4 - x^3 - 13x^2 + x + 12 = 0.$$