ALGEBRA — *Answers*

1 **a** $= (x+1)^2 - 1 + 4$
$= (x+1)^2 + 3$

b $= (x-1)^2 - 1 + 4$
$= (x-1)^2 + 3$

c $= (x-2)^2 - 4 + 1$
$= (x-2)^2 - 3$

d $= (x+3)^2 - 9$

e $= (x+2)^2 - 4 + 8$
$= (x+2)^2 + 4$

f $= (x-4)^2 - 16 - 5$
$= (x-4)^2 - 21$

g $= (x+6)^2 - 36 + 30$
$= (x+6)^2 - 6$

h $= (x-5)^2 - 25 + 25$
$= (x-5)^2$

i $= (x+3)^2 - 9 - 9$
$= (x+3)^2 - 18$

j $= (x-2)^2 - 4 + 18$
$= (x-2)^2 + 14$

k $= (x+\frac{3}{2})^2 - \frac{9}{4} + 3$
$= (x+\frac{3}{2})^2 + \frac{3}{4}$

l $= (x+\frac{1}{2})^2 - \frac{1}{4} - 1$
$= (x+\frac{1}{2})^2 - \frac{5}{4}$

m $= (x-9)^2 - 81 + 100$
$= (x-9)^2 + 19$

n $= (x-\frac{1}{2})^2 - \frac{1}{4} - \frac{1}{2}$
$= (x-\frac{1}{2})^2 - \frac{3}{4}$

o $= (x+\frac{9}{2})^2 - \frac{81}{4} + 20$
$= (x+\frac{9}{2})^2 - \frac{1}{4}$

p $= (x-\frac{7}{2})^2 - \frac{49}{4} - 2$
$= (x-\frac{7}{2})^2 - \frac{57}{4}$

q $= (x-\frac{3}{2})^2 - \frac{9}{4} + 5$
$= (x-\frac{3}{2})^2 + \frac{11}{4}$

r $= (x-\frac{11}{2})^2 - \frac{121}{4} + 37$
$= (x-\frac{11}{2})^2 + \frac{27}{4}$

s $= (x+\frac{1}{3})^2 - \frac{1}{9} + 1$
$= (x+\frac{1}{3})^2 + \frac{8}{9}$

t $= (x-\frac{1}{4})^2 - \frac{1}{16} - \frac{1}{4}$
$= (x-\frac{1}{4})^2 - \frac{5}{16}$

2 **a** $= 2[x^2 + 2x] + 3$
$= 2[(x+1)^2 - 1] + 3$
$= 2(x+1)^2 + 1$

b $= 2[x^2 - 4x] - 7$
$= 2[(x-2)^2 - 4] - 7$
$= 2(x-2)^2 - 15$

c $= 3[x^2 - 2x] + 3$
$= 3[(x-1)^2 - 1] + 3$
$= 3(x-1)^2$

d $= 4[x^2 + 6x] + 11$
$= 4[(x+3)^2 - 9] + 11$
$= 4(x+3)^2 - 25$

e $= -[x^2 + 2x] - 5$
$= -[(x+1)^2 - 1] - 5$
$= -(x+1)^2 - 4$

f $= -[x^2 - 10x] + 1$
$= -[(x-5)^2 - 25] + 1$
$= -(x-5)^2 + 26$

g $= 2[x^2 + x] - 1$
$= 2[(x+\frac{1}{2})^2 - \frac{1}{4}] - 1$
$= 2(x+\frac{1}{2})^2 - \frac{3}{2}$

h $= 3[x^2 - 3x] + 5$
$= 3[(x-\frac{3}{2})^2 - \frac{9}{4}] + 5$
$= 3(x-\frac{3}{2})^2 - \frac{7}{4}$

i $= 3[x^2 - 8x] + 48$
$= 3[(x-4)^2 - 16] + 48$
$= 3(x-4)^2$

j $= 3[x^2 - 5x]$
$= 3[(x-\frac{5}{2})^2 - \frac{25}{4}]$
$= 3(x-\frac{5}{2})^2 - \frac{75}{4}$

k $= 5[x^2 + 8x] + 70$
$= 5[(x+4)^2 - 16] + 70$
$= 5(x+4)^2 - 10$

l $= 2[x^2 + \frac{5}{2}x] + 2$
$= 2[(x+\frac{5}{4})^2 - \frac{25}{16}] + 2$
$= 2(x+\frac{5}{4})^2 - \frac{9}{8}$

m $= 4[x^2 + \frac{3}{2}x] - 7$
$= 4[(x+\frac{3}{4})^2 - \frac{9}{16}] - 7$
$= 4(x+\frac{3}{4})^2 - \frac{37}{4}$

n $= -2[x^2 - 2x] - 1$
$= -2[(x-1)^2 - 1] - 1$
$= -2(x-1)^2 + 1$

o $= -3[x^2 + \frac{2}{3}x] + 4$
$= -3[(x+\frac{1}{3})^2 - \frac{1}{9}] + 4$
$= -3(x+\frac{1}{3})^2 + \frac{13}{3}$

p $= \frac{1}{3}[x^2 + \frac{3}{2}x] - \frac{1}{4}$
$= \frac{1}{3}[(x+\frac{3}{4})^2 - \frac{9}{16}] - \frac{1}{4}$
$= \frac{1}{3}(x+\frac{3}{4})^2 - \frac{7}{16}$

3 **a** $(y-2)^2 - 4 + 2 = 0$
$(y-2)^2 = 2$
$y - 2 = \pm\sqrt{2}$
$y = 2 \pm \sqrt{2}$

b $(p+1)^2 - 1 - 2 = 0$
$(p+1)^2 = 3$
$p + 1 = \pm\sqrt{3}$
$p = -1 \pm \sqrt{3}$

c $(x-3)^2 - 9 + 4 = 0$
$(x-3)^2 = 5$
$x - 3 = \pm\sqrt{5}$
$x = 3 \pm \sqrt{5}$

d $(r+5)^2 - 25 + 7 = 0$
$(r+5)^2 = 18$
$r + 5 = \pm\sqrt{18} = \pm 3\sqrt{2}$
$r = -5 \pm 3\sqrt{2}$

e $(x-1)^2 - 1 = 11$
$(x-1)^2 = 12$
$x - 1 = \pm\sqrt{12} = \pm 2\sqrt{3}$
$x = 1 \pm 2\sqrt{3}$

f $(a-6)^2 - 36 - 18 = 0$
$(a-6)^2 = 54$
$a - 6 = \pm\sqrt{54} = \pm 3\sqrt{6}$
$a = 6 \pm 3\sqrt{6}$

g $(m-\frac{3}{2})^2 - \frac{9}{4} + 1 = 0$
$(m-\frac{3}{2})^2 = \frac{5}{4}$
$m - \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$
$m = \frac{1}{2}(3 \pm \sqrt{5})$

h $(t-\frac{7}{2})^2 - \frac{49}{4} + 9 = 0$
$(t-\frac{7}{2})^2 = \frac{13}{4}$
$t - \frac{7}{2} = \pm\frac{\sqrt{13}}{2}$
$t = \frac{1}{2}(7 \pm \sqrt{13})$

i $(u + \frac{7}{2})^2 - \frac{49}{4} = 44$
$(u + \frac{7}{2})^2 = \frac{225}{4}$
$u + \frac{7}{2} = \pm\frac{15}{2}$
$u = -\frac{7}{2} \pm \frac{15}{2}$
$u = -11$ or 4

j $y^2 - 2y + \frac{1}{2} = 0$
$(y - 1)^2 - 1 + \frac{1}{2} = 0$
$(y - 1)^2 = \frac{1}{2}$
$y - 1 = \pm\frac{1}{\sqrt{2}} = \pm\frac{1}{2}\sqrt{2}$
$y = 1 \pm \frac{1}{2}\sqrt{2}$

k $p^2 + 6p = -\frac{23}{3}$
$(p + 3)^2 - 9 = -\frac{23}{3}$
$(p + 3)^2 = \frac{4}{3}$
$p + 3 = \pm\frac{2}{\sqrt{3}} = \pm\frac{2}{3}\sqrt{3}$
$p = -3 \pm \frac{2}{3}\sqrt{3}$

l $x^2 + 6x = \frac{9}{2}$
$(x + 3)^2 - 9 = \frac{9}{2}$
$(x + 3)^2 = \frac{27}{2}$
$x + 3 = \pm\sqrt{\frac{27}{2}} = \pm\frac{3}{2}\sqrt{6}$
$x = -3 \pm \frac{3}{2}\sqrt{6}$

m $m^2 - m = 1$
$(m - \frac{1}{2})^2 - \frac{1}{4} = 1$
$(m - \frac{1}{2})^2 = \frac{5}{4}$
$m - \frac{1}{2} = \pm\frac{\sqrt{5}}{2}$
$m = \frac{1}{2}(1 \pm \sqrt{5})$

n $4x^2 - 28x + 49 = 0$
$x^2 - 7x + \frac{49}{4} = 0$
$(x - \frac{7}{2})^2 - \frac{49}{4} + \frac{49}{4} = 0$
$(x - \frac{7}{2})^2 = 0$
$x = \frac{7}{2}$

o $t^2 + \frac{1}{3}t = \frac{1}{3}$
$(t + \frac{1}{6})^2 - \frac{1}{36} = \frac{1}{3}$
$(t + \frac{1}{6})^2 = \frac{13}{36}$
$t + \frac{1}{6} = \pm\frac{\sqrt{13}}{6}$
$t = \frac{1}{6}(-1 \pm \sqrt{13})$

p $a^2 - \frac{7}{2}a + 2 = 0$
$(a - \frac{7}{4})^2 - \frac{49}{16} + 2 = 0$
$(a - \frac{7}{4})^2 = \frac{17}{16}$
$a - \frac{7}{4} = \frac{\sqrt{17}}{4}$
$a = \frac{1}{4}(7 \pm \sqrt{17})$

4 **a** $y = (x - 1)^2 - 1 + 7$
$y = (x - 1)^2 + 6$
$y = 6$ at $x = 1$, minimum

b $y = (x + 1)^2 - 1 - 3$
$y = (x + 1)^2 - 4$
$y = -4$ at $x = -1$, minimum

c $y = (x - 3)^2 - 9 + 1$
$y = (x - 3)^2 - 8$
$y = -8$ at $x = 3$, minimum

d $y = (x + 5)^2 - 25 + 35$
$y = (x + 5)^2 + 10$
$y = 10$ at $x = -5$, minimum

e $y = -[x^2 - 4x] + 4$
$y = -[(x - 2)^2 - 4] + 4$
$y = -(x - 2)^2 + 8$
$y = 8$ at $x = 2$, maximum

f $y = (x + \frac{3}{2})^2 - \frac{9}{4} - 2$
$y = (x + \frac{3}{2})^2 - \frac{17}{4}$
$y = -\frac{17}{4}$ at $x = -\frac{3}{2}$, minimum

g $y = 2[x^2 + 4x] + 5$
$y = 2[(x + 2)^2 - 4] + 5$
$y = 2(x + 2)^2 - 3$
$y = -3$ at $x = -2$, minimum

h $y = -3[x^2 - 2x]$
$y = -3[(x - 1)^2 - 1]$
$y = -3(x - 1)^2 + 3$
$y = 3$ at $x = 1$, maximum

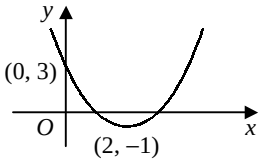
i $y = -[x^2 + 5x] + 7$
$y = -[(x + \frac{5}{2})^2 - \frac{25}{4}] + 7$
$y = -(x + \frac{5}{2})^2 + \frac{53}{4}$
$y = \frac{53}{4}$ at $x = -\frac{5}{2}$, maximum

j $y = 4[x^2 - 3x] + 9$
$y = 4[(x - \frac{3}{2})^2 - \frac{9}{4}] + 9$
$y = 4(x - \frac{3}{2})^2$
$y = 0$ at $x = \frac{3}{2}$, minimum

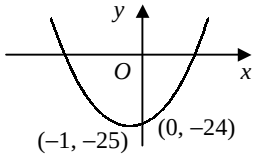
k $y = 4[x^2 + 5x] - 8$
$y = 4[(x + \frac{5}{2})^2 - \frac{25}{4}] - 8$
$y = 4(x + \frac{5}{2})^2 - 33$
$y = -33$ at $x = -\frac{5}{2}$, minimum

l $y = -2[x^2 + x] + 17$
$y = -2[(x + \frac{1}{2})^2 - \frac{1}{4}] + 17$
$y = -2(x + \frac{1}{2})^2 + \frac{35}{2}$
$y = \frac{35}{2}$ at $x = -\frac{1}{2}$, maximum

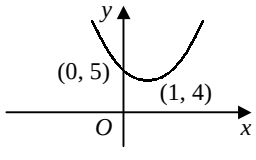
5 **a** $y = (x - 2)^2 - 4 + 3$
$y = (x - 2)^2 - 1$
minimum $(2, -1)$



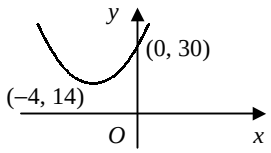
b $y = (x + 1)^2 - 1 - 24$
$y = (x + 1)^2 - 25$
minimum $(-1, -25)$



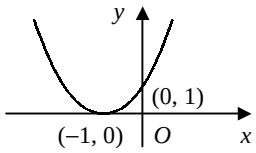
c $y = (x - 1)^2 - 1 + 5$
$y = (x - 1)^2 + 4$
minimum $(1, 4)$



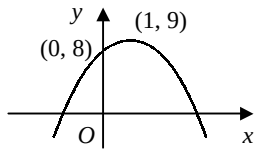
d $y = (x + 4)^2 - 16 + 30$
$y = (x + 4)^2 + 14$
minimum $(-4, 14)$



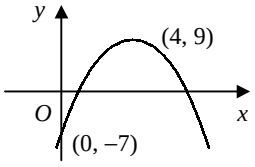
e $y = (x + 1)^2 - 1 + 1$
$y = (x + 1)^2$
minimum $(-1, 0)$



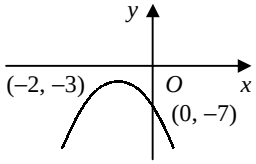
f $y = -[x^2 - 2x] + 8$
$y = -[(x - 1)^2 - 1] + 8$
$y = -(x - 1)^2 + 9$
maximum $(1, 9)$



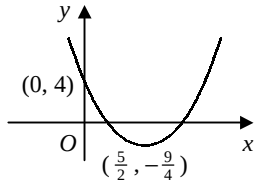
g $y = -[x^2 - 8x] - 7$
$y = -[(x - 4)^2 - 16] - 7$
$y = -(x - 4)^2 + 9$
maximum $(4, 9)$



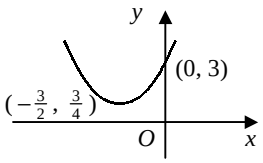
h $y = -[x^2 + 4x] - 7$
$y = -[(x + 2)^2 - 4] - 7$
$y = -(x + 2)^2 - 3$
maximum $(-2, -3)$



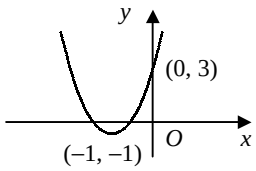
i $y = (x - \frac{5}{2})^2 - \frac{25}{4} + 4$
$y = (x - \frac{5}{2})^2 - \frac{9}{4}$
minimum $(\frac{5}{2}, -\frac{9}{4})$



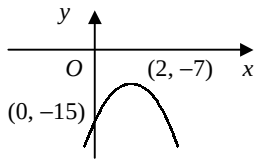
j $y = (x + \frac{3}{2})^2 - \frac{9}{4} + 3$
$y = (x + \frac{3}{2})^2 + \frac{3}{4}$
minimum $(-\frac{3}{2}, \frac{3}{4})$



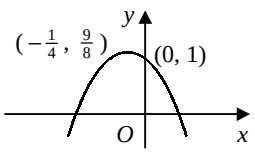
k $y = 4[x^2 + 2x] + 3$
$y = 4[(x + 1)^2 - 1] + 3$
$y = 4(x + 1)^2 - 1$
minimum $(-1, -1)$



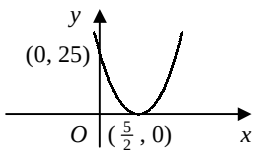
l $y = -2[x^2 - 4x] - 15$
$y = -2[(x - 2)^2 - 4] - 15$
$y = -2(x - 2)^2 - 7$
maximum $(2, -7)$



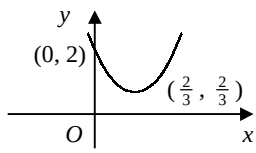
m $y = -2[x^2 + \frac{1}{2}x] + 1$
$y = -2[(x + \frac{1}{4})^2 - \frac{1}{16}] + 1$
$y = -2(x + \frac{1}{4})^2 + \frac{9}{8}$
maximum $(-\frac{1}{4}, \frac{9}{8})$



n $y = 4[x^2 - 5x] + 25$
$y = 4[(x - \frac{5}{2})^2 - \frac{25}{4}] + 25$
$y = 4(x - \frac{5}{2})^2$
minimum $(\frac{5}{2}, 0)$



o $y = 3[x^2 - \frac{4}{3}x] + 2$
$y = 3[(x - \frac{2}{3})^2 - \frac{4}{9}] + 2$
$y = 3(x - \frac{2}{3})^2 + \frac{2}{3}$
minimum $(\frac{2}{3}, \frac{2}{3})$



6 **a** $= (x - 2\sqrt{2})^2 - 8 + 5$
$= (x - 2\sqrt{2})^2 - 3$
b $x = 2\sqrt{2}$

7 $x^2 + 2kx - 3 = 0$
$(x + k)^2 - k^2 - 3 = 0$
$(x + k)^2 = k^2 + 3$
$x + k = \pm\sqrt{k^2 + 3}$
$x = -k \pm \sqrt{k^2 + 3}$