1. The quadratic equation $kx^2 + (3k - 1)x - 4 = 0$ has no real roots. Find the set of possible values of k.

[7]

2. i. Express $3x^2 + 9x + 10$ in the form $3(x + p)^2 + q$.

[3]

ii. State the coordinates of the minimum point of the curve $y = 3x^2 + 9x + 10$.

[2]

iii. Calculate the discriminant of $3x^2 + 9x + 10$.

[2]

3. Solve the equation $8x^6 + 7x^3 - 1 = 0$.

[5]

4. Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$.

[5]

5. Express $5x^2 + 10x + 2$ in the form $p(x + q)^2 + r$, where p, q and r are integers.

[4]

6. Solve the equation $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$.

[5]

7. Find the set of values of k for which the equation $x^2 + 2x + 11 = k(2x - 1)$ has two distinct real roots.

[7]

8. i. Express $4 + 12x - 2x^2$ in the form $a(x + b)^2 + c$.

[4]

ii. State the coordinates of the maximum point of the curve $y = 4 + 12x - 2x^2$.

[2]

9. Solve the equation $2y^{\frac{1}{2}} - 7y^{\frac{1}{4}} + 3 = 0$.

[5]

10. Show that, for all values of k, the equation $x^2 + (k - 5)x - 3k = 0$ has real roots. [6]

11. (a) Express $2x^2 + 4x + 5$ in the form $p(x + q)^2 + r$, where p, q and r are integers. [4]

(b) State the coordinates of the turning point on the curve $y = 2x^2 + 4x + 5$. [2]

(c) Given that the equation $2x^2 + 4x + 5 = k$ has no real roots, state the set of possible values of the constant k. [1]

12. Find the roots of the equation $4t^{\frac{2}{3}} = 15 - 17t^{\frac{1}{3}}$. [5]

13. (a) Express $4x^2 - 12x + 11$ in the form $a(x + b)^2 + c$. [3]

[1]

(b) State the number of real roots of the equation $4x^2 - 12x + 11 = 0$.

(c) Explain fully how the value of r is related to the number of real roots of the equation $p(x + q)^2 + r = 0$ where p, q and r are real constants and $p > 0$.

[2]

14. (a) Express $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$. [4]

(b) Use your result to show that the equation $2x^2 - 12x + 23 = 0$ has no real roots. [1]

(c) Given that the equation $2x^2 - 12x + k = 0$ has repeated roots, find the value of the constant k. [2]

15. (a) Show that $4x^2 - 12x + 3 = 4\left(x - \frac{3}{2}\right)^2 - 6$. [3]

(b) State the coordinates of the minimum point of the curve $y = 4x^2 - 12x + 3$. [2]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			$(3k-1)^2 - 4 \times k \times -4$	*M1	Attempts $b^2 - 4ac$ or an equation or inequality involving b^2 and $4ac$. Must involve k^2 in first term (but no x anywhere). If $b^2 - 4ac$ not stated, must be clear attempt.	Must be working with the discriminant explicitly and not only as part of the quadratic formula. Allow $\sqrt{b^2-4ac}$ for first **M1 A1**
			$= 9k^2 + 10k + 1$	A1	Correct discriminant, simplified to 3 terms	
			$9k^2 + 10k + 1 < 0$	M1	States discriminant < 0 or $b^2 < 4ac$.	Can be awarded at any stage. Doesn't need first M1. No square root here
			$(9k+1)(k+1) < 0$	DM1	Correct method to find roots of a three term quadratic	
			$-1, -\dfrac{1}{9}$	A1	Both values of k correct	
			$-1 < k < -\dfrac{1}{9}$	M1	Chooses "inside region" of inequality	Allow correct region for their inequality
				A1	Allow "$k < -\dfrac{1}{9}$ **and** $k > -1$" etc. must be strict inequalities for A mark <u>Examiner's Comments</u> This unstructured question proved to be very demanding. Most candidates recognised the need to find the discriminant and the	Do not allow "$k < -\dfrac{1}{9}$ **or** $k > -1$";

					majority realised that this needed to be less than zero. Given that both terms involved algebraic manipulation, determining the discriminant proved challenging to a large number of candidates. Similarly, the solution of the resulting quadratic inequality proved challenging, with added difficulty seeming to result from the fact that both roots were negative; a significant number thought that $-\frac{1}{9}$ was less than –1, showing these roots in the wrong positions on the x-axis and getting the inequality the wrong way round when their intention was to choose the inside region. The best candidates handled all these obstacles well and produced short fluent solutions gaining all seven marks (as achieved by around one-third of candidates); some candidates were unable to start the question at all, instead trying to solve the equation using the quadratic formula.	
			Total	7		
2		i	$3(x^2 + 3x) + 10$ $= 3\left(x + \frac{3}{2}\right)^2 - \frac{27}{4} + 10$ $= 3\left(x + \frac{3}{2}\right)^2 + \frac{13}{4}$	B1 M1 A1	$\left(x + \frac{3}{2}\right)^2$ $10 - 3p^2$ or $\frac{10}{3} - p^2$ Allow $p = \frac{3}{2}$, $q = \frac{13}{4}$ **A1 www**	If p, q found correctly, then ISW slips in format. $3(x + 1.5)^2 - 3.25$ B1 M0 A0 $3(x + 1.5) + 3.25$ B1 M1 A1 (BOD) $3(x + 1.5x)^2 + 3.25$ B0 M1 A0 $3(x^2 + 1.5)^2 + 3.25$ B0 M1 A0 $3(x - 1.5)^2 + 3.25$ B0 M1 A1 (BOD) $3x(x + 1.5)^2 + 3.25$ B0M1A0
		i			Examiner's Comments The fact that the first digit was given in this "completing the square" question appeared to ease the difficulty somewhat, but this is still an area which many candidates find difficult with less than two-	

					thirds achieving full marks. Identifying the value of *p* was usually very well done; the problems usually occurred in the calculation of *q*, with both arithmetic problems, particularly with the squaring, and structural misunderstanding when the candidates failed to multiply by 3.	
		ii	$\left(-\frac{3}{2}, \frac{13}{4}\right)$	B1	FT i.e. – their *p*	If restarted e.g. by differentiation:
		ii		B1	FT i.e. their *q* **Examiner's Comments** This question provided a follow through from the previous part which enabled many candidates with poor arithmetic to earn credit for their understanding of the relationship between the format and the graph. Many secured both marks as a result. Those who re-started by differentiation were usually less successful, again due to the difficulties with the fraction work.	Correct method to find *x* value of minimum point **M1** Fully correct answer **www A1**
		iii	$9^2 - (4 \times 3 \times 10)$	M1	Uses $b^2 - 4ac$ Ignore > 0, < 0 etc. **ISW** comments about number of roots	Use of $\sqrt{b^2 - 4ac}$s **M0** unless recovered
		iii	$= -39$	A1	**Examiner's Comments** Most candidates are familiar with the term discriminant and only a few erroneously used $\sqrt{b^2 - 4ac}$. Around one in ten candidates substituted correctly but then made arithmetical errors. Commonly seen was $9^2 = 49$ and the subtraction 81 – 120 often resulted in 39 or ±49 or ±41.	
			Total	7		

3			$k = x^3$ $8k^2 + 7k - 1 = 0$	M1*	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing x^3	**No marks** if whole equation cube rooted etc. **No marks** if straight to formula with no evidence of substitution at start and no cube rooting / cubing at end
			$(8k - 1)(k + 1) = 0$	DM1*	Correct method to solve a quadratic	
			$k = \frac{1}{8},\ k = -1$	A1	Both values of k correct	**Spotted solutions:**
				M1	Attempt to cube root at least one value to obtain x	If **M0 DMO** or **M1 DM0** **SC B1** $x = -1$ **www**
			$x = \frac{1}{2},\ x = -1$	A1	Both values of x correct and no other values **Examiner's Comments** This disguised quadratic was approached well by the vast majority of candidates, with about twothirds achieving all five marks. It was very rare to see candidates leap straight to the quadratic formula with no attempt to find the cube roots at the end, which has been a problem with similar questions in the past. Most candidates opted to perform a substitution and then to factorise, although those who opted for a two-bracket approach using x^3 were also often successful. Some candidates who used the quadratic formula failed to deal accurately with $a = 8$, but most earned the first three marks with apparent ease. Thereafter a small number of candidates opted to cube rather than find cube roots, but the main loss of credit was due to lack of accuracy at the end. Theassertion "you can't cube root a negative number" was seen regularly; $\sqrt[3]{\frac{1}{8}} = \pm\frac{1}{2}$ was less common but not rare. This confusion between cube and square roots clearly needs addressing as it let down an otherwise well-answered question where only the very lowest-scoring candidates made no progress at all.	**SC B1** $x = \frac{1}{2}$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)
			Total	5		

4			$k = x^2$ $4k^2 + 3k - 1 = 0$	M1*	Substitute for x^2	**No marks** if whole equation square rooted etc.
			$(4k - 1)(k + 1) = 0$	M1dep*	Attempt to solve resulting quadratic	**No marks** if straight to formula with no evidence of substitution at start and no square rooting/squaring at end.
			$k = \frac{1}{4}, k = -1$	A1	Correct values of k **soi**	If factorising into two brackets: $(4x^2 - 1)(x^2 + 1) = 0$ **M1 A1** $(2x + 1)(2x - 1)(x^2 + 1) = 0$ **M1 A1 A1 as before** Spotted solutions: If **M0 DM0** or **M1 DM0** **SR B1** $x = \frac{1}{2}$ **www** **SC B1** $x = -\frac{1}{2}$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)
			$x = \pm\sqrt{\frac{1}{4}}$	M1	Attempt to square root	
			$x = \pm\frac{1}{2}$	A1	Final answers correct, no extras <u>Examiner's Comments</u> This disguised quadratic was well approached by the vast majority of candidates, with just over half achieving all 5 marks. This continued an improving trend over the last few sessions, with fewer candidates going straight to the quadratic formula with no attempt to square root at the end, which has been a problem with similar questions in the past. The most common approach was to perform a substitution and then to factorise, although those who opted for a two-bracket approach using x^2	

					were also often successful. Fewer candidates than in previous sessions used the quadratic formula; those that did were usually successful. Completing the square was rarely seen. Only a small number of candidates opted to square rather than square root, but the main loss of credit was due to lack of accuracy at the end. Although most candidates correctly ismissed any roots from $x^2 = -1$, some did not. More common was the absence of the negative square root	Quadratic Functions
			Total	5		
5			$5x^2 + 10x + 2 = 5(x^2 + 2x) + 2$	B1	$p = 5$	If p, q and r found correctly, then **ISW** slips in format. $5(x + 1)^2 + 3$ **B1 B1 M0 A0** $5(x + 1) - 3$ **B1 B1 M1 A1 (BOD)** $5(x + 1\ x)^2 - 3$ **B1 B0 M1 A0** $5(x^2 + 1)^2 - 3$ **B1 B0 M1 A0** $5(x - 1)^2 - 3$ **B1 B0 M1 A0** $5\ x\,(x + 1)^2 - 3$ **B0 B1 M1 A0**
			$= 5[(x + 1)^2 - 1] + 2$	B1	$q = 1$ $2 - 5$ "their q"^{2} or $\frac{2}{5} -$ "their q"^{2} **Must be evidence of squaring** $r = -3$ <u>Examiner's Comments</u>	
			$= 5(x + 1)^2 - 3$	M1	This "completing the square" question was tackled well by the majority of candidates, many securing all four marks. In keeping with previous sessions, almost all earned the first two marks, seeing that p was 5 and q was 1, although $q = 5$ was seen relatively often among weaker candidates. Failure to multiply by five when working out the constant was the most common error amongst candidates who did not achieve full marks. Those	

				A1	who took out 5 as a factor of the full expression often made errors with the resulting fractions.	
			Total	4		
6			$k = x^{\frac{1}{3}}$	M1*	Use a substitution to obtain a quadratic, or factorise into 2 brackets each containing $x^{\frac{1}{3}}$	**No marks** if whole equation cubed / rooted etc. No marks if straight to quadratic formula with no evidence of substitution at start and no cube rooting / cubing at end.
			$k^2 - k - 6 = 0$ $(k - 3)(k + 2) = 0$	M1dep	Attempt to solve resulting three-term quadratic – see guidance in appendix 1	
			$k = 3$, $k = -2$	A1	Correct values of k	Spotted solutions: If M0 DMO or M1 DM0
			$x = 3^3$, $x = -2^3$	M1	Attempt to cube at least one value	SC B1 $x = 27$ www
			$x = 27$, $x = -8$	A1	Final answers correct ISW Examiner's Comments This disguised quadratic was well approached by the vast majority of candidates. The most common approach was to perform a substitution and then to factorise, although some candidates did make their choice of substitution clear, which made it difficult to award partial credit in the cases where errors then occurred. As the resulting quadratic was simple, very few candidates used the quadratic formula and factorisation was usually successful with only a few sign errors seen. Some candidates stopped after solving the quadratic and the number who tried to cube root, rather than to cube, their solutions was comparatively large.	SC B1 $x = -8$ www (Can then get 5/5 if both found www and exactly two solutions justified)
			Total	5		

7			$x^2 + (2 - 2k)x + 11 + k = 0$	M1*	Attempt to rearrange to a three-term quadratic	Each Ms depend on the previous M
			$(2 - 2k)^2 - 4(11 + k)$	M1dep*	Uses $b^2 - 4ac$, involving k and not involving x	
			$4k^2 - 12k - 40 > 0$	A1	Correct simplified inequality obtained **www**	
			$k^2 - 3k - 10 > 0$			
			$(k - 5)(k + 2)$	M1dep*	Correct method to find roots of 3-term quadratic	
				A1	5 and − 2 seen as roots	
			$k < -2$, $k > 5$	M1dep*	$b^2 - 4ac > 0$ and chooses "outside region"	$-2 > k > 5$ scores **M1A0** Allow "$k < -2$ or $k > 5$" for A1 **Do not allow** "$k < -2$ **and** $k > 5$" **Examiner's Comments** Although this question proved demanding to many candidates, there were a large number of neat solutions. Most candidates understood the nature of the question and many gained at least three of the four method marks available. Accuracy was the main barrier to complete success as a large number either failed to rearrange the given equation correctly or to substitute correctly into the discriminant. Repeated sign errors often resulted in apparently correct critical values for k that did not receive credit as they were from wrong working.
				A1	Fully correct, strict inequalities.	
			Total	7		
8		i	$-2(x^2 - 6x - 2)$	B1	or $a = -2$	$-2(x - 3)^2 - 22$ **B1 B1 M0 A0**
		i	$= -2[(x - 3)^2 - 2 - 9)]$	B1	$b = -3$	$-2(x - 3) + 22$ **4/4 (BOD)**
		i		M1	$4 + 2b^2$	$-2(x - 3x)^2 + 22$ **B1 B0 M1 A0**

						$-2(x^2-3)^2+22$ **B1 B0 M1 A0** $-2(x+3)^2+22$ **B1 B0 M1 A0** $-2x(x-3)^2+22$ **B0 B1 M1 A0** $-2(x^2-3)+22$ **B1 B0 M1 A0** **Examiner's Comments** The negative coefficient of x^2 generally did not daunt candidates and there were many clear and accurate solutions, aided by the integer arithmetic. There were the usual errors when trying to find the constant term and these were exacerbated by the need to multiply two negative numbers together. Some candidates however, chose to change all the signs to make the question easier; this approach earned a maximum of one mark in this part, with the possibility of follow through marks in part (i). Others treated the expression as an equation to achieve the same effect; at this level it is expected that candidates should know the difference.
		i	$= -2(x-3)^2+22$	A1	$c = 22$ If *a, b* and *c* found correctly, then **ISW** slips in format. **If signs of all terms changed at start, can only score SC B1** for fully correct working to obtain $2(x-3)^2-22$ If done correctly and then signs changed at end, do not **ISW**, award **B1B1M1A0**	
		ii	(3, 22)	B1ft	Allow follow through "– their *b*"	May restart. Follow through marks are for **their final answer** to (i) **Examiner's Comments**
		ii		B1ft	Allow follow through "their *c*"	This part was sometimes omitted with candidates apparently not seeing connection between the parts. Others found the coordinates by differentiation and substitution but most used (i) and so were allowed follow-through marks had they made errors in the previous part.

			Total	6		
9			Let $y^{\frac{1}{4}} = x$	M1*	Use a substitution to obtain a quadratic or	**No marks** if whole equation raised to fourth power etc.
			$2x^2 - 7x + 3 = 0$		factorise into two brackets each containing $y^{\frac{1}{4}}$	
			$(2x - 1)(x - 3) = 0$	M1dep*	Correct method to solve resulting quadratic	**No marks** if straight to formula with no evidence of substitution at start and no raising to fourth power / fourth rooting at end.
			$x = \frac{1}{2}, x = 3$	A1	Both values correct	
			$y = \left(\frac{1}{2}\right)^4, y = 3^4$	M1dep*	Attempt to raise to the fourth power	**No marks** if $y^{\frac{1}{4}} = x$ and then $2x - 7x^2 + 3 = 0$.
			$y = \frac{1}{16}, y = 81$	A1	Correct final answers	
			Alternative by rearrangement and squaring:			
			$2y^{\frac{1}{2}} - 7y^{\frac{1}{4}} + 3 = 0,\ 7y^{\frac{1}{4}} = 2y^{\frac{1}{2}} + 3$ $49y^{\frac{1}{2}} = 4y + 12y^{\frac{1}{2}} + 9,\ 37y^{\frac{1}{2}} = 4y + 9$	M2*	Rearrange and square both sides twice	If **M0 DM0** or **M1 DM0** **SC B1** $y = 81$ **www** **SC B1** $y = \frac{1}{16}$ **www**
			$16y^2 - 1297y + 81 = 0$	A1	Correct quadratic obtained	(Can then get 5/5 if both found **www** and exactly two solutions justified)
			$(16y - 1)(y - 81) = 0$	M1dep*	Correct method to solve resulting quadratic	
			$y = \frac{1}{16}, y = 81$	A1	Correct final answers	

			OR methods may be combined:			
			e.g. after $37y^{\frac{1}{2}} = 4y + 9$			
			$4y - 37y^{\frac{1}{2}} + 9 = 0$	M1*	Rearrange, square both sides and substitute	
			$4x^2 - 37x + 9 = 0$			
			$(4x - 1)(x - 9) = 0$	M1dep*	Correct method to solve resulting quadratic	
			$x = \frac{1}{4}, x = 9$	A1		
				M1dep*	Attempt to square	
			$y = \left(\frac{1}{4}\right)^2, y = 9^2$	A1	Correct final answers	**Examiner's Comments** This disguised quadratic was well approached by the vast majority of candidates, with around three-quarters of candidates achieving all 5 marks. A very small number of candidates factorised into two brackets, but the most common approach was as usual to perform a substitution and then to factorise. There was some confusion with choice of substitution with many incorrectly obtaining $2x - 7x^2 + 3$; this earned no credit. Again, very few candidates used the quadratic formula and factorisation was usually successful with only a few sign errors seen. Some candidates stopped after solving the quadratic and a small number tried to take the fourth root, rather than to raise to the power four. Some only squared, implying an incorrect

						substitution. A few gave answers like ±81, which lost the final accuracy mark, as did poor attempts at $\left(\frac{1}{2}\right)^4$ which was variously seen as $\frac{1}{32}$, $\frac{1}{64}$ or $\frac{1}{256}$.
			Total	5		
10			$\Delta = (k-5)^2 - 4(1)(-3k)$	M1 (AO3.1a)	Attempt at discriminant	
			$= k^2 + 2k + 25$	A1 (AO1.1)	Obtain correct 3-term quadratic	
				M1 (AO3.1a)	Complete the square on their 3-term quadratic	**OR**: differentiate and solve = 0
			$= (k+1)^2 + 24$	A1 (AO1.1)		Obtain $k = -1$
			Condition for real roots is $\Delta \geq 0$	M1 (AO2.1)	For '$b^2 - 4ac \geq 0$' condition **OR** for explanation that their $\Delta \geq 0$	Substitute $k = -1$ and explain that the result is the minimum value of their k-quadratic
			$(k+1)^2 \geq 0$ for all k so $(k+1)^2 + 24 > 0$ and hence the equation has real roots for all values of k	A1 (AO2.2a) [6]	$(k+1)^2 \geq 0$ with complete	Correct numerical values and complete

					argument and conclusion	argument using 24 > 0 plus conclusion	
			Total	6			
11		a	$2x^2 + 4x + 5 = 2(x^2 + 2x) + 5$ $= 2[(x + 1)^2 - 1] + 5$ $= 2(x + 1)^2 + 3$	B1(AO2.2a) B1(AO1.1) M1(AO1.1a) A1(AO1.1) [4]	$p = 2$ $q = 1$ Attempt r $r = 3$	The values of p, q and r could be stated explicitly or could be implied by an answer in completed square form	
		b	Vertex is at (–1, 3)	B1ft(AO1.1) B1ft(AO1.1) [2]	Correct x coordinate Correct y coordinate	FT their **(a)**	
		c	$k < 3$	B1ft(AO3.1a) [1]	State $k < 3$, ft their **(a)**	Must be strict inequality	
			Total	7			
12			$k = t^{\frac{1}{3}}$	M1*	Substitute for $t^{\frac{1}{3}}$ to obtain a	Alternative**: M2** Rearrange and factorise into two	

			$4k^2+17k-15=0$ $(4k-3)(k+5)=0$ $k=\frac{3}{4}, k=-5$ $t=\frac{27}{64}, t=-125$	M1 *dep A1 M1 A1 [5]	quadratic expression Rearrange and attempt to solve resulting quadratic equation. **See appendix 1.** Correct values of *k* Attempt to cube at least one value Final answers correct **Examiner's Comments** Although this disguised quadratic needed rearrangement as well as substitution, this question was well approached by the vast majority	brackets containing $t^{\frac{1}{3}}$. **See appendix 1.** **SC** If straight to formula with no evidence of substitution at start and no cubing / cube rooting at end, then **B1** for $\frac{-17\pm\sqrt{(17^2-4\times4\times-15)}}{2\times4}$ or better **No marks** if whole equation cubed etc. **Spotted solutions:** If **M0 DM0 or M1 DM0** **SC B1** $t=\frac{27}{64}$ **www** **SC B1** $t=-125$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)

					of candidates, with around 70% achieving all 5 marks. As in previous sessions, some candidates did not make their choice of substitution clear which made it difficult to award marks. The question was best approached by factorisation, and those who opted to use the quadratic formula were often unable to deal with the required arithmetic. Most remembered to cube their solutions to the quadratic, although some did so inaccurately, particularly the fractional solution.	
			Total	5		
13		a	$4\left[x^2-3x\right]+11$ $4\left[\left(x-\frac{3}{2}\right)^2-\frac{9}{4}\right]+11$ — $a=4$ $(x-3/2)^2$ $4\left(x-\frac{3}{2}\right)^2+2$ — $c=2$	B1 (AO 1.1) B1 (AO 1.1) B1 (AO 1.1) [3]	No marks until attempt to complete the square Must be of the form $4(x \pm a)^2 \pm \ldots$ **Examiner's Comments** This was done very well. Candidates seemed to be very familiar with completing the square. The most common simple numerical error was to have $c = 8.75$. $(2x-3)^2 + 2$ was seen occasionally.	
		b	No real roots	B1 (AO 2.2a)	Zero, none, 0, … if not 'no real	

				[1]	roots' must be consistent with their **(a)** Examiner's Comments Many candidates did this by evaluating the discriminant rather than using the result they had just obtained. 'State' indicates neither working nor justification is required (cf Specification Document).	
		c	$r = 0 \Rightarrow$ 1 real root or 1 repeated root $r < 0 \Rightarrow$ 2 real roots $r > 0 \Rightarrow$ no real roots	M1 (AO 2.4) A1 (AO 2.4) [2]	Attempt to relate the value of r to the number of real roots (this can be implied with at least one correct statement) All three statements correct Examiner's Comments This part proved less successful. Many candidates were not able to start an argument. Some attempted to evaluate $b^2 - 4ac$ but this was rarely done accurately. Those who recognised how to use the given form of the equation made the most progress, occasionally confusing the $r > 0$ and $r < 0$ cases.	
			Total	6		

14		a	$2(x^2 - 6x + 11.5)$ $2((x - 3)^2 + 11.5 - 9)$ $2(x - 3)^2 + 5$	B1 (AO 1.1a) B1 (AO 1.1) M1 (AO 1.1) A1 (AO 1.1) [4]	or $a = 2$ or $b = -3$ $23 - 2(\text{their } b)^2$ or $c = 5$ Examiner's Comments Most candidates answered this correctly. A few found a and b correctly but made an error in finding c. This most frequently came from an incorrect first step such as $2(x - 3)^2 + 11.5 - 9$ or $2(x - 3)^2 + 23 - 9$.	
		b	$2(x + 3)^2 + 5$ is always +ve or $2(x + 3)^2 + 5 > 0$ or $2(x + 3)^2 + 5 \geq 5$ Hence no real roots	B1f (AO 1.1) [1]	or $2(x + 3)^2 = -5$, which is impossible or "+ve quadratic" and min on $y = 5$ or "+ve" quadratic; TP at (3, 5). Both Hence no real roots Must use (a), not use D $2(x + 3)^2 + 5 = 0$ $\Rightarrow x = \sqrt{\text{neg}}$ or $x + 3 = \sqrt{\text{neg}}$ ft their (a) (a & $c > 0$) Examiner's Comments Most candidates answered this correctly. Some of those who	

					discussed the turning point lost a mark because they merely stated that it is a minimum, rather than showing that this is so.	
		c	$2(x-3)^2 = 2(x^2-6x+9)$ $k = 18$	M1 (AO 1.1a) A1 (AO 2.2a) [2]	or $12^2 - 8k = 0$ Examiner's Comments This question was very well answered.	
			Total	7		
15		a	$4x^2 - 12x + 3 = 4(x^2 - 3x) + 3$ $= 4\left[\left(x-\frac{3}{2}\right)^2 - \frac{9}{4}\right] + 3$ $4\left(x-\frac{3}{2}\right)^2 - 4\times\frac{9}{4} + 3 = 4\left(x-\frac{3}{2}\right)^2 - 6$ AG Alternative method $4\left(x-\frac{3}{2}\right)^2 - 6 = 4\left[x^2 - 3x + \frac{9}{4}\right] - 6$ $= 4x^2 - 4\times 3x + 4\times\frac{9}{4} - 6$ $4x^2 - 12x + 3 = 4(x^2 - 3$ AG	M1 (AO1.1) A1 (AO1.1) A1 (AO2.1) M1 (AO1.1) A1 (AO1.1) A1 (AO2.1) [3]	Take out a factor of 4: $4x^2 - 12x = 4(x^2 - 3x)$ $x^2 - 3x = \left(x-\frac{3}{2}\right)^2 - \frac{9}{4}$ multiply out square bracket: $x^2 - 3x = \left(x-\frac{3}{2}\right)^2 - \frac{9}{4}$ intermediate step: $4x^2 - 12x = 4(x^2 - 3x)$	
		b	Minimum point is $\left(\frac{3}{2}, -6\right)$	B1		

				(AO1.1) B1 (AO1.1) [2]		Quadratic Functions
			Total	5		