

1. The quadratic equation $kx^2 + (3k - 1)x - 4 = 0$ has no real roots. Find the set of possible values of k . [7]
2. i. Express $3x^2 + 9x + 10$ in the form $3(x + p)^2 + q$. [3]
- ii. State the coordinates of the minimum point of the curve $y = 3x^2 + 9x + 10$. [2]
- iii. Calculate the discriminant of $3x^2 + 9x + 10$. [2]
3. Solve the equation $8x^6 + 7x^3 - 1 = 0$. [5]
4. Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$. [5]
5. Express $5x^2 + 10x + 2$ in the form $p(x + q)^2 + r$, where p , q and r are integers. [4]
6. Solve the equation $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$. [5]
7. Find the set of values of k for which the equation $x^2 + 2x + 11 = k(2x - 1)$ has two distinct real roots. [7]
8. i. Express $4 + 12x - 2x^2$ in the form $a(x + b)^2 + c$. [4]

ii. State the coordinates of the maximum point of the curve $y = 4 + 12x - 2x^2$.

[2]

9. Solve the equation $2y^{\frac{1}{2}} - 7y^{\frac{1}{4}} + 3 = 0$.

[5]

10. Show that, for all values of k , the equation $x^2 + (k - 5)x - 3k = 0$ has real roots.

[6]

11. (a) Express $2x^2 + 4x + 5$ in the form $p(x + q)^2 + r$, where p , q and r are integers.

[4]

(b) State the coordinates of the turning point on the curve $y = 2x^2 + 4x + 5$.

[2]

(c) Given that the equation $2x^2 + 4x + 5 = k$ has no real roots, state the set of possible values of the constant k .

[1]

12. Find the roots of the equation $4t^{\frac{2}{3}} = 15 - 17t^{\frac{1}{3}}$.

[5]

13. (a) Express $4x^2 - 12x + 11$ in the form $a(x + b)^2 + c$.

[3]

(b) State the number of real roots of the equation $4x^2 - 12x + 11 = 0$.

[1]

(c) Explain fully how the value of r is related to the number of real roots of the equation $p(x + q)^2 + r = 0$ where p , q and r are real constants and $p > 0$.

[2]

14. (a) Express $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$. [4]
- (b) Use your result to show that the equation $2x^2 - 12x + 23 = 0$ has no real roots. [1]
- (c) Given that the equation $2x^2 - 12x + k = 0$ has repeated roots, find the value of the constant k . [2]
15. (a) Show that $4x^2 - 12x + 3 = 4\left(x - \frac{3}{2}\right)^2 - 6$. [3]
- (b) State the coordinates of the minimum point of the curve $y = 4x^2 - 12x + 3$. [2]

END OF QUESTION paper

Mark scheme

Question	Answer/Indicative content	Marks	Part marks and guidance
1	$(3k - 1)^2 - 4 \times k \times -4$ $= 9k^2 + 10k + 1$ $9k^2 + 10k + 1 < 0$ $(9k + 1)(k + 1) < 0$ $-1, -\frac{1}{9}$ $-1 < k < -\frac{1}{9}$	<p>*M1</p> <p>A1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Attempts $b^2 - 4ac$ or an equation or inequality involving b^2 and $4ac$. Must involve k^2 in first term (but no x anywhere). If $b^2 - 4ac$ not stated, must be clear attempt.</p> <p>Correct discriminant, simplified to 3 terms</p> <p>States discriminant < 0 or $b^2 < 4ac$.</p> <p>Correct method to find roots of a three term quadratic</p> <p>Both values of k correct</p> <p>Chooses "inside region" of inequality</p> <p>Allow</p> <p>"$k < -\frac{1}{9}$ and $k > -1$"</p> <p>etc. must be strict inequalities for A mark</p> <p><u>Examiner's Comments</u></p> <p>This unstructured question proved to be very demanding. Most candidates recognised the need to find the discriminant and the</p> <p>Must be working with the discriminant explicitly and not only as part of the quadratic formula.</p> <p>Allow</p> <p>$\sqrt{b^2 - 4ac}$ or first M1 A1</p> <p>Can be awarded at any stage. Doesn't need first M1. No square root here</p> <p>Allow correct region for their inequality</p> <p>Do not allow</p> <p>"$k < -\frac{1}{9}$ or $k > -1$";</p>

			majority realised that this needed to be less than zero. Given that both terms involved algebraic manipulation, determining the discriminant proved challenging to a large number of candidates. Similarly, the solution of the resulting quadratic inequality proved challenging, with added difficulty seeming to result from the fact that both roots were negative; a significant number thought that $-\frac{1}{9}$ was less than -1 , showing these roots in the wrong positions on the x-axis and getting the inequality the wrong way round when their intention was to choose the inside region. The best candidates handled all these obstacles well and produced short fluent solutions gaining all seven marks (as achieved by around one-third of candidates); some candidates were unable to start the question at all, instead trying to solve the equation using the quadratic formula.	
		Total	7	
2	i	$3(x^2 + 3x) + 10$ $= 3\left(x + \frac{3}{2}\right)^2 - \frac{27}{4} + 10$ i $= 3\left(x + \frac{3}{2}\right)^2 + \frac{13}{4}$	B1 $\left(x + \frac{3}{2}\right)^2$ M1 $10 - 3p^2 \text{ or } \frac{10}{3} - p^2$ A1 $\text{Allow } p = \frac{3}{2}, q = \frac{13}{4} \text{ A1 www}$	If p, q found correctly, then ISW slips in format. $3(x + 1.5)^2 - 3.25$ B1 M0 A0 $3(x + 1.5) + 3.25$ B1 M1 A1 (BOD) $3(x + 1.5x)^2 + 3.25$ B0 M1 A0 $3(x^2 + 1.5)^2 + 3.25$ B0 M1 A0 $3(x - 1.5)^2 + 3.25$ B0 M1 A1 (BOD) $3 \times (x + 1.5)^2 + 3.25$ B0M1A0
	i		Examiner's Comments The fact that the first digit was given in this "completing the square" question appeared to ease the difficulty somewhat, but this is still an area which many candidates find difficult with less than two-	

				thirds achieving full marks. Identifying the value of p was usually very well done; the problems usually occurred in the calculation of q , with both arithmetic problems, particularly with the squaring, and structural misunderstanding when the candidates failed to multiply by 3.	
	ii	$\left(-\frac{3}{2}, \frac{13}{4}\right)$	B1	FT i.e. – their p FT i.e. their q Examiner's Comments	If restarted e.g. by differentiation:
	ii		B1	This question provided a follow through from the previous part which enabled many candidates with poor arithmetic to earn credit for their understanding of the relationship between the format and the graph. Many secured both marks as a result. Those who re-started by differentiation were usually less successful, again due to the difficulties with the fraction work.	Correct method to find x value of minimum point M1 Fully correct answer www A1
	iii	$9^2 - (4 \times 3 \times 10)$	M1	Uses $b^2 - 4ac$ Ignore $> 0, < 0$ etc. ISW comments about number of roots Examiner's Comments	Use of $\sqrt{b^2 - 4ac}$ s M0 unless recovered
	iii	$= -39$	A1	Most candidates are familiar with the term discriminant and only a few erroneously used $\sqrt{b^2 - 4ac}$. Around one in ten candidates substituted correctly but then made arithmetical errors. Commonly seen was $9^2 = 49$ and the subtraction $81 - 120$ often resulted in 39 or ± 49 or ± 41 .	
		Total	7		

3		$k = x^3$ $8k^2 + 7k - 1 = 0$ $(8k - 1)(k + 1) = 0$ $k = \frac{1}{8}, k = -1$ $x = \frac{1}{2}, x = -1$	M1* DM1* A1 M1 A1	<p>Use a substitution to obtain a quadratic or factorise into 2 brackets each containing x^3</p> <p>Correct method to solve a quadratic</p> <p>Both values of k correct</p> <p>Attempt to cube root at least one value to obtain x</p> <p>Both values of x correct and no other values</p> <p>Examiner's Comments</p> <p>This disguised quadratic was approached well by the vast majority of candidates, with about two-thirds achieving all five marks. It was very rare to see candidates leap straight to the quadratic formula with no attempt to find the cube roots at the end, which has been a problem with similar questions in the past. Most candidates opted to perform a substitution and then to factorise, although those who opted for a two-bracket approach using x^3 were also often successful. Some candidates who used the quadratic formula failed to deal accurately with $a = 8$, but most earned the first three marks with apparent ease. Thereafter a small number of candidates opted to cube rather than find cube roots, but the main loss of credit was due to lack of accuracy at the end. The assertion "you can't cube root a negative number" was seen regularly;</p> $\sqrt[3]{\frac{1}{8}} = \pm \frac{1}{2}$ <p>was less common but not rare. This confusion between cube and square roots clearly needs addressing as it let down an otherwise well-answered question where only the very lowest-scoring candidates made no progress at all.</p>	<p>No marks if whole equation cube rooted etc.</p> <p>No marks if straight to formula with no evidence of substitution at start and no cube rooting / cubing at end</p> <p>Spotted solutions:</p> <p>If M0 DMO or M1 DMO SC B1 $x = -1$ www</p> <p>SC B1 $x = \frac{1}{2}$ www</p> <p>(Can then get 5/5 if both found www and exactly two solutions justified)</p>
		Total	5		

4		$k = x^2$ $4k^2 + 3k - 1 = 0$ $(4k - 1)(k + 1) = 0$ $k = \frac{1}{4}, k = -1$ $x = \pm \sqrt{\frac{1}{4}}$ $x = \pm \frac{1}{2}$	<p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Substitute for x^2</p> <p>Attempt to solve resulting quadratic</p> <p>Correct values of k soi</p> <p>Attempt to square root</p> <p>Final answers correct, no extras</p> <p><u>Examiner's Comments</u></p> <p>This disguised quadratic was well approached by the vast majority of candidates, with just over half achieving all 5 marks. This continued an improving trend over the last few sessions, with fewer candidates going straight to the quadratic formula with no attempt to square root at the end, which has been a problem with similar questions in the past. The most common approach was to perform a substitution and then to factorise, although those who opted for a two-bracket approach using x^2</p>	<p>No marks if whole equation square rooted etc.</p> <p>No marks if straight to formula with no evidence of substitution at start and no square rooting/squaring at end.</p> <p>If factorising into two brackets: $(4x^2 - 1)(x^2 + 1) = 0$ M1 A1 $(2x + 1)(2x - 1)(x^2 + 1) = 0$ M1 A1 A1 as before</p> <p>Spotted solutions: If M0 DMO or M1 DMO</p> $\frac{1}{2}$ <p>SR B1 $x = \frac{1}{2}$ www</p> $-\frac{1}{2}$ <p>SC B1 $x = -\frac{1}{2}$ www</p> <p>(Can then get 5/5 if both found www and exactly two solutions justified)</p>
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				<p>were also often successful. Fewer candidates than in previous sessions used the quadratic formula; those that did were usually successful. Completing the square was rarely seen. Only a small number of candidates opted to square rather than square root, but the main loss of credit was due to lack of accuracy at the end. Although most candidates correctly dismissed any roots from $x^2 = -1$, some did not. More common was the absence of the negative square root</p>	
		Total	5		
5		$5x^2 + 10x + 2 = 5(x^2 + 2x) + 2$ $= 5[(x + 1)^2 - 1] + 2$ $= 5(x + 1)^2 - 3$	<p>B1</p> <p>$p = 5$</p> <p>B1</p> <p>$q = 1$</p> $2 - 5 \frac{2}{5}$ <p>“their q^2 or $\frac{2}{5}$ “their q^2”</p> <p>Must be evidence of squaring $r = -3$</p> <p><u>Examiner's Comments</u></p> <p>This “completing the square” question was tackled well by the majority of candidates, many securing all four marks. In keeping with previous sessions, almost all earned the first two marks, seeing that p was 5 and q was 1, although $q = 5$ was seen relatively often among weaker candidates. Failure to multiply by five when working out the constant was the most common error amongst candidates who did not achieve full marks. Those</p>	<p>If p, q and r found correctly, then ISW slips in format.</p> <p>$5(x + 1)^2 + 3$ B1 B1 M0 A0</p> <p>$5(x + 1) - 3$ B1 B1 M1 A1 (BOD)</p> <p>$5(x + 1)x^2 - 3$ B1 B0 M1 A0</p> <p>$5(x^2 + 1)^2 - 3$ B1 B0 M1 A0</p> <p>$5(x - 1)^2 - 3$ B1 B0 M1 A0</p> <p>$5x(x + 1)^2 - 3$ B0 B1 M1 A0</p>	

				who took out 5 as a factor of the full expression often made errors with the resulting fractions.	
			A1		
		Total	4		
6		$k = x^{\frac{1}{3}}$ $k^2 - k - 6 = 0$ $(k - 3)(k + 2) = 0$ $k = 3, k = -2$ $x = 3^3, x = -2^3$ $x = 27, x = -8$	M1* M1dep A1 M1 A1	Use a substitution to obtain a quadratic, or factorise into 2 brackets each containing $x^{\frac{1}{3}}$ Attempt to solve resulting three-term quadratic – see guidance in appendix 1 Correct values of k Attempt to cube at least one value Final answers correct ISW Examiner's Comments This disguised quadratic was well approached by the vast majority of candidates. The most common approach was to perform a substitution and then to factorise, although some candidates did make their choice of substitution clear, which made it difficult to award partial credit in the cases where errors then occurred. As the resulting quadratic was simple, very few candidates used the quadratic formula and factorisation was usually successful with only a few sign errors seen. Some candidates stopped after solving the quadratic and the number who tried to cube root, rather than to cube, their solutions was comparatively large.	No marks if whole equation cubed / rooted etc. No marks if straight to quadratic formula with no evidence of substitution at start and no cube rooting / cubing at end. Spotted solutions: If M0 DMO or M1 DMO SC B1 $x = 27$ www SC B1 $x = -8$ www (Can then get 5/5 if both found www and exactly two solutions justified)
		Total	5		

7			$x^2 + (2 - 2k)x + 11 + k = 0$ $(2 - 2k)^2 - 4(11 + k)$ $4k^2 - 12k - 40 > 0$ $k^2 - 3k - 10 > 0$ $(k - 5)(k + 2)$ $k < -2, k > 5$	M1* M1dep* A1 M1dep* A1 M1dep* A1	Attempt to rearrange to a three-term quadratic Uses $b^2 - 4ac$, involving k and not involving x Correct simplified inequality obtained www Correct method to find roots of 3-term quadratic 5 and -2 seen as roots $b^2 - 4ac > 0$ and chooses "outside region" Fully correct, strict inequalities.	Each Ms depend on the previous M -2 > k > 5 scores M1A0 Allow " $k < -2$ or $k > 5$ " for A1 Do not allow " $k < -2$ and $k > 5$ " Examiner's Comments Although this question proved demanding to many candidates, there were a large number of neat solutions. Most candidates understood the nature of the question and many gained at least three of the four method marks available. Accuracy was the main barrier to complete success as a large number either failed to rearrange the given equation correctly or to substitute correctly into the discriminant. Repeated sign errors often resulted in apparently correct critical values for k that did not receive credit as they were from wrong working.
			Total	7		
8	i	$-2(x^2 - 6x - 2)$	B1	or $a = -2$	$-2(x - 3)^2 - 22$ B1 B1 M0 A0	
	i	$= -2[(x - 3)^2 - 2 - 9]$	B1	$b = -3$	$-2(x - 3) + 22$ 4/4 (BOD)	
	i		M1	$4 + 2b^2$	$-2(x - 3)^2 + 22$ B1 B0 M1 A0	

	i	$= -2(x - 3)^2 + 22$	A1	<p>$c = 22$</p> <p>If a, b and c found correctly, then ISW slips in format.</p> <p>If signs of all terms changed at start, can only score SC B1 for fully correct working to obtain $2(x - 3)^2 - 22$</p> <p>If done correctly and then signs changed at end, do not ISW, award B1B1M1A0</p>	<p>$-2(x^2 - 3)^2 + 22$ B1 B0 M1 A0</p> <p>$-2(x + 3)^2 + 22$ B1 B0 M1 A0</p> <p>$-2x(x - 3)^2 + 22$ B0 B1 M1 A0</p> <p>$-2(x^2 - 3) + 22$ B1 B0 M1 A0</p> <p>Examiner's Comments</p> <p>The negative coefficient of x^2 generally did not daunt candidates and there were many clear and accurate solutions, aided by the integer arithmetic. There were the usual errors when trying to find the constant term and these were exacerbated by the need to multiply two negative numbers together. Some candidates however, chose to change all the signs to make the question easier; this approach earned a maximum of one mark in this part, with the possibility of follow through marks in part (i). Others treated the expression as an equation to achieve the same effect; at this level it is expected that candidates should know the difference.</p>
	ii	(3, 22)	B1ft	Allow follow through “– their b ”	May restart.
	ii		B1ft	Allow follow through “their c ”	<p>Follow through marks are for their final answer to (i)</p> <p>Examiner's Comments</p> <p>This part was sometimes omitted with candidates apparently not seeing connection between the parts. Others found the coordinates by differentiation and substitution but most used (i) and so were allowed follow-through marks had they made errors in the previous part.</p>

		Total	6		
9		<p>Let $y^{\frac{1}{4}} = x$</p> <p>$2x^2 - 7x + 3 = 0$</p> <p>$(2x - 1)(x - 3) = 0$</p> <p>$x = \frac{1}{2}, x = 3$</p> <p>$y = \left(\frac{1}{2}\right)^4, y = 3^4$</p> <p>$y = \frac{1}{16}, y = 81$</p> <p><u>Alternative by rearrangement and squaring:</u></p> <p>$2y^{\frac{1}{2}} - 7y^{\frac{1}{4}} + 3 = 0, 7y^{\frac{1}{4}} = 2y^{\frac{1}{2}} + 3$</p> <p>$49y^{\frac{1}{2}} = 4y + 12y^{\frac{1}{2}} + 9, 37y^{\frac{1}{2}} = 4y + 9$</p> <p>$16y^2 - 129y + 81 = 0$</p> <p>$(16y - 1)(y - 81) = 0$</p> <p>$y = \frac{1}{16}, y = 81$</p>	<p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>M2*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p>	<p>Use a substitution to obtain a quadratic or factorise into two brackets each containing $y^{\frac{1}{4}}$</p> <p>Correct method to solve resulting quadratic</p> <p>Both values correct</p> <p>Attempt to raise to the fourth power</p> <p>Correct final answers</p> <p>Rearrange and square both sides twice</p> <p>Correct quadratic obtained</p> <p>Correct method to solve resulting quadratic</p> <p>Correct final answers</p>	<p>No marks if whole equation raised to fourth power etc.</p> <p>No marks if straight to formula with no evidence of substitution at start and no raising to fourth power / fourth rooting at end.</p> <p>No marks if $y^{\frac{1}{4}} = x$ and then $2x - 7x^2 + 3 = 0$.</p> <p>If M0 DM0 or M1 DM0 SC B1 $y = 81$ www</p> <p>SC B1 $y = \frac{1}{16}$ www</p> <p>(Can then get 5/5 if both found www and exactly two solutions justified)</p>

OR methods may be combined:

e.g. after $37y^{\frac{1}{2}} = 4y + 9$

$$4y - 37y^{\frac{1}{2}} + 9 = 0$$

$$4x^2 - 37x + 9 = 0$$

$$(4x - 1)(x - 9) = 0$$

$$x = \frac{1}{4}, x = 9$$

$$y = \left(\frac{1}{4}\right)^2, y = 9^2$$

M1*

Rearrange, square both sides and substitute

M1dep*

Correct method to solve resulting quadratic

A1

M1dep*

Attempt to square

A1

Correct final answers

Examiner's Comments

This disguised quadratic was well approached by the vast majority of candidates, with around three-quarters of candidates achieving all 5 marks. A very small number of candidates factorised into two brackets, but the most common approach was as usual to perform a substitution and then to factorise. There was some confusion with choice of substitution with many incorrectly obtaining $2x - 7x^2 + 3$; this earned no credit. Again, very few candidates used the quadratic formula and factorisation was usually successful with only a few sign errors seen. Some candidates stopped after solving the quadratic and a small number tried to take the fourth root, rather than to raise to the power four. Some only squared, implying an incorrect

					substitution. A few gave answers like ± 81 , which lost the final accuracy mark, as did poor attempts at $\left(\frac{1}{2}\right)^4$	
					which was variously seen as $\frac{1}{32}$, $\frac{1}{64}$ or $\frac{1}{256}$.	
		Total	5			
10		$\Delta = (k-5)^2 - 4(1)(-3k)$ $= k^2 + 2k + 25$ $= (k+1)^2 + 24$ Condition for real roots is $\Delta \geq 0$ $(k+1)^2 \geq 0$ for all k so $(k+1)^2 + 24 > 0$ and hence the equation has real roots for all values of k	M1 (AO3.1a) A1 (AO1.1) M1 (AO3.1a) A1 (AO1.1) M1 (AO2.1) A1 (AO2.2a)	Attempt at discriminant Obtain correct 3-term quadratic Complete the square on their 3-term quadratic For ' $b^2 - 4ac \geq 0$ ' condition OR for explanation that their $\Delta \geq 0$ $(k+1)^2 \geq 0$ with complete	 OR: differentiate and solve = 0 Obtain $k = -1$ Substitute $k = -1$ and explain that the result is the minimum value of their k -quadratic Correct numerical values and complete	

					argument and conclusion	argument using $24 > 0$ plus conclusion						
			Total	6								
11	a	<table border="1"> <tr> <td>$2x^2 + 4x + 5$</td> <td>$= 2(x^2 + 2x) + 5$</td> </tr> <tr> <td></td> <td>$= 2[(x + 1)^2 - 1] + 5$</td> </tr> <tr> <td></td> <td>$= 2(x + 1)^2 + 3$</td> </tr> </table>	$2x^2 + 4x + 5$	$= 2(x^2 + 2x) + 5$		$= 2[(x + 1)^2 - 1] + 5$		$= 2(x + 1)^2 + 3$	B1(AO2.2a) B1(AO1.1) M1(AO1.1a) A1(AO1.1) [4]	$p = 2$ $q = 1$ Attempt r $r = 3$	The values of p , q and r could be stated explicitly or could be implied by an answer in completed square form	
$2x^2 + 4x + 5$	$= 2(x^2 + 2x) + 5$											
	$= 2[(x + 1)^2 - 1] + 5$											
	$= 2(x + 1)^2 + 3$											
	b	Vertex is at $(-1, 3)$	B1ft(AO1.1) B1ft(AO1.1) [2]	Correct x coordinate Correct y coordinate	FT their (a)							
	c	$k < 3$	B1ft(AO3.1a) [1]	State $k < 3$, ft their (a)	Must be strict inequality							
			Total	7								
12		$k = t^{\frac{1}{3}}$	M1*	Substitute for $t^{\frac{1}{3}}$ to obtain a	Alternative: M2 Rearrange and factorise into two							

$$4k^2 + 17k - 15 = 0$$

$$(4k - 3)(k + 5) = 0$$

$$k = \frac{3}{4}, k = -5$$

$$t = \frac{27}{64}, t = -125$$

M1 *dep

quadratic expression

brackets containing $t^{\frac{1}{3}}$. See appendix 1.

Rearrange and attempt to solve resulting quadratic equation. See appendix 1.

SC If straight to formula with no evidence of substitution at start and no cubing / cube rooting at end, then

B1 for
$$\frac{-17 \pm \sqrt{(17^2 - 4 \times 4 \times -15)}}{2 \times 4}$$

or better

A1

Correct values of k

No marks if whole equation cubed etc.

M1

Spotted solutions:

A1

Attempt to cube at least one value

If M0 DM0 or M1 DM0

SC B1
$$t = \frac{27}{64} \text{www}$$

SC B1 $t = -125 \text{www}$

(Can then get 5/5 if both found www and exactly two solutions justified)

Final answers correct

Examiner's Comments

[5]

Although this disguised quadratic needed rearrangement as well as substitution, this question was well approached by the vast majority

of candidates, with around 70% achieving all 5 marks. As in previous sessions, some candidates did not make their choice of substitution clear which made it difficult to award marks. The question was best approached by factorisation, and those who opted to use the quadratic formula were often unable to deal with the required arithmetic. Most remembered to cube their solutions to the quadratic, although some did so inaccurately, particularly the fractional solution.

Total

5

13

a

$$4[x^2 - 3x] + 11$$

$$4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 11$$

$$a = 4$$

$$\left(x - \frac{3}{2}\right)^2$$

$$4\left(x - \frac{3}{2}\right)^2 + 2$$

$$c = 2$$

B1
(AO 1.1)

B1
(AO 1.1)

B1
(AO 1.1)

[3]

No marks until attempt to complete the square

Must be of the form $4(x \pm a)^2 \pm \dots$

Examiner's Comments

This was done very well. Candidates seemed to be very familiar with completing the square. The most common simple numerical error was to have $c = 8.75$. $(2x - 3)^2 + 2$ was seen occasionally.

b

No real roots

B1
(AO 2.2a)

Zero, none, 0, ...
if not 'no real

			[1]	<div style="border: 1px solid black; padding: 5px;"> roots' must be consistent with their (a) </div> <p>Examiner's Comments</p> <p>Many candidates did this by evaluating the discriminant rather than using the result they had just obtained. 'State' indicates neither working nor justification is required (cf Specification Document).</p>
	c	$r = 0 \Rightarrow 1$ real root or 1 repeated root $r < 0 \Rightarrow 2$ real roots $r > 0 \Rightarrow$ no real roots	M1 (AO 2.4) A1 (AO 2.4) [2]	<div style="border: 1px solid black; padding: 5px;"> Attempt to relate the value of r to the number of real roots (this can be implied with at least one correct statement) All three statements correct </div> <p>Examiner's Comments</p> <p>This part proved less successful. Many candidates were not able to start an argument. Some attempted to evaluate $b^2 - 4ac$ but this was rarely done accurately. Those who recognised how to use the given form of the equation made the most progress, occasionally confusing the $r > 0$ and $r < 0$ cases.</p>
		Total	6	

14	a	$2(x^2 - 6x + 11.5)$ $2((x - 3)^2 + 11.5 - 9)$ $2(x - 3)^2 + 5$	<p>B1 (AO 1.1a) B1 (AO 1.1) M1 (AO 1.1) A1 (AO 1.1)</p> <p>[4]</p>	<table border="1" data-bbox="1057 73 1525 416"> <tr> <td data-bbox="1057 73 1292 172">or $a = 2$</td> <td data-bbox="1292 73 1525 172"></td> </tr> <tr> <td data-bbox="1057 172 1292 271">or $b = -3$</td> <td data-bbox="1292 172 1525 271"></td> </tr> <tr> <td data-bbox="1057 271 1292 370">$23 - 2(\text{their } b)^2$</td> <td data-bbox="1292 271 1525 370"></td> </tr> <tr> <td data-bbox="1057 370 1292 416">or $c = 5$</td> <td data-bbox="1292 370 1525 416"></td> </tr> </table> <p><u>Examiner's Comments</u></p> <p>Most candidates answered this correctly. A few found a and b correctly but made an error in finding c. This most frequently came from an incorrect first step such as $2(x - 3)^2 + 11.5 - 9$ or $2(x - 3)^2 + 23 - 9$.</p>	or $a = 2$		or $b = -3$		$23 - 2(\text{their } b)^2$		or $c = 5$		
or $a = 2$													
or $b = -3$													
$23 - 2(\text{their } b)^2$													
or $c = 5$													
	b	$2(x + 3)^2 + 5$ is always +ve or $2(x + 3)^2 + 5 > 0$ or $2(x + 3)^2 + 5 \geq 5$ Hence no real roots	<p>B1f (AO 1.1)</p> <p>[1]</p>	<table border="1" data-bbox="1057 746 1637 1257"> <tr> <td data-bbox="1057 746 1348 1257"> or $2(x + 3)^2 = -5$, which is impossible or "+ve quadratic" and min on $y = 5$ or "+ve" quadratic; TP at (3, 5). Both Hence no real roots Must use (a), not use D </td> <td data-bbox="1348 746 1637 1257"> $2(x + 3)^2 + 5 = 0$ $\Rightarrow x = \sqrt{\text{neg}}$ or $x + 3 = \sqrt{\text{neg}}$ ft their (a) (a & $c > 0$) </td> </tr> </table> <p><u>Examiner's Comments</u></p> <p>Most candidates answered this correctly. Some of those who</p>	or $2(x + 3)^2 = -5$, which is impossible or "+ve quadratic" and min on $y = 5$ or "+ve" quadratic; TP at (3, 5). Both Hence no real roots Must use (a), not use D	$2(x + 3)^2 + 5 = 0$ $\Rightarrow x = \sqrt{\text{neg}}$ or $x + 3 = \sqrt{\text{neg}}$ ft their (a) (a & $c > 0$)							
or $2(x + 3)^2 = -5$, which is impossible or "+ve quadratic" and min on $y = 5$ or "+ve" quadratic; TP at (3, 5). Both Hence no real roots Must use (a), not use D	$2(x + 3)^2 + 5 = 0$ $\Rightarrow x = \sqrt{\text{neg}}$ or $x + 3 = \sqrt{\text{neg}}$ ft their (a) (a & $c > 0$)												

				discussed the turning point lost a mark because they merely stated that it is a minimum, rather than showing that this is so.		
		c	$2(x-3)^2 = 2(x^2 - 6x + 9)$ $k = 18$	M1 (AO 1.1a) A1 (AO 2.2a) [2]	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> or $12^2 - 8k = 0$ </div> <u>Examiner's Comments</u> This question was very well answered.	
		Total		7		
15		a	$4x^2 - 12x + 3 = 4(x^2 - 3x) + 3$ $= 4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 3$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $4\left(x - \frac{3}{2}\right)^2 - 4 \times \frac{9}{4} + 3 = 4\left(x - \frac{3}{2}\right)^2 - 6$ AG </div> Alternative method $4\left(x - \frac{3}{2}\right)^2 - 6 = 4\left[x^2 - 3x + \frac{9}{4}\right] - 6$ $= 4x^2 - 4 \times 3x + 4 \times \frac{9}{4} - 6$ $4x^2 - 12x + 3 = 4(x^2 - 3x) + 3$ AG	M1 (AO1.1) A1 (AO1.1) A1 (AO2.1) M1 (AO1.1) A1 (AO1.1) A1 (AO2.1) [3]	<div style="border: 1px solid black; padding: 5px;"> Take out a factor of 4 $4x^2 - 12x = 4(x^2 - 3x)$ $x^2 - 3x = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$ multiply out square bracket $x^2 - 3x = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$ intermediate step $4x^2 - 12x = 4(x^2 - 3x)$ </div>	
		b	Minimum point is $\left(\frac{3}{2}, -6\right)$	B1	<div style="border: 1px solid black; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center;"> <div style="border-right: 1px solid black; width: 20px; height: 20px;"></div> <div style="width: 20px; height: 20px;"></div> </div>	

				(AO1.1) B1 (AO1.1) [2]		
			Total	5		