1. Solve the inequalities

i. $3 - 8x > 4$,

[2]

ii. $(2x - 4)(x - 3) \leqslant 12$.

[5]

2. Solve the following inequalities.

(i) $5 < 6x + 3 < 14$

[3]

(ii) $x(3x - 13) \geq 10$

[5]

3. i. Sketch the curve $y = 2x^2 - x - 3$, giving the coordinates of all points of intersection with the axes.

[4]

ii. Hence, or otherwise, solve the inequality $2x^2 - x - 3 > 0$.

[2]

iii. Given that the equation $2x^2 - x - 3 = k$ has no real roots, find the set of possible values of the constant k.

[3]

4. Find the set of values of x for which

$$x^2 < x + 6 \quad \text{or} \quad 3x + 2 \geqslant 20 - x.$$

Give your answer in set notation. [6]

5. **In this question you must show detailed reasoning.**

A gardener is planning the design for a rectangular flower bed. The requirements are:

- the length of the flower bed is to be 3 m longer than the width,
- the length of the flower bed must be at least 14.5 m,
- the area of the flower bed must be less than 180 m^2.

The width of the flower bed is x m.

By writing down and solving appropriate inequalities in x, determine the set of possible values for the width of the flower bed. [6]

6. (a) The equation $x^2 + 3x + k = 0$ has repeated roots. Find the value of the constant k. [2]

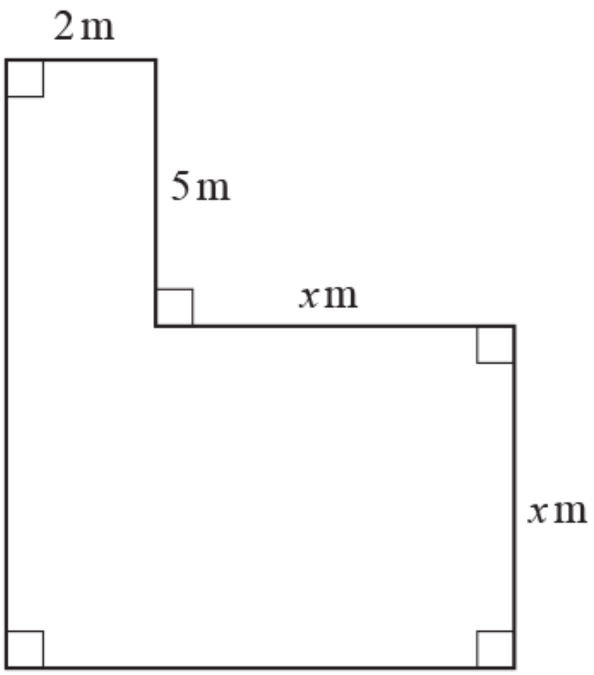
(b) Solve the inequality $6 + x - x^2 > 0$. [2]

7. Solve the following inequalities.

(a) $-5 < 3x + 1 < 14$ [2]

(b) $4x^2 + 3 > 28$ [3]

8.



The diagram shows a patio.

The perimeter of the patio has to be less than 44 m.

The area of the patio has to be at least 45 m^2.

(a) Write down, in terms of x, an inequality satisfied by

(i) the perimeter of the patio, [1]

(ii) the area of the patio. [1]

[4]

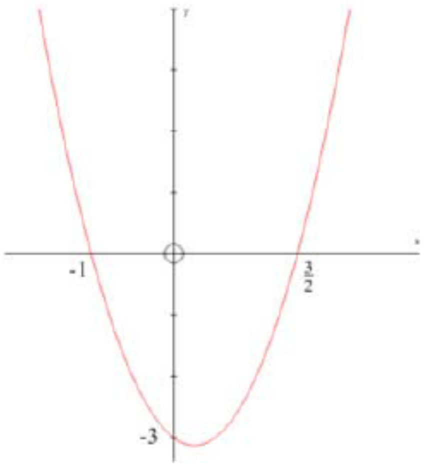
(b) Hence determine the set of possible values of x.

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$8x < -1$	B1	soi, allow $-8x > 1$ but not just $8x + 1 < 0$	Allow ≤ or ≥ for first mark
		i	$x < -\frac{1}{8}$	B1	Correct working only, allow $-\frac{1}{8} > x$ Do not allow $\frac{1}{-8}$	Do not **ISW** if contradictory
		i			**Examiner's Comments** The negative x coefficient increased the difficulty of this linear inequality so that only two-thirds of candidates secured both marks.	Do not allow ≤ or ≥
		ii	$2x^2 - 10x \le 0$	M1*	Expand brackets and rearrange to collect all terms on one side	No more than one incorrect term
		ii	$2x(x - 5) \le 0$	DM1*	Correct method to find roots of resulting quadratic	Allow $(2x + 0)(x - 5)$ **Do not allow $(2x - 4)(x - 3)$,** this is the original expression.
		ii		A1	0, 5 seen as roots – could be on sketch graph	
		ii		DM1*	Chooses “inside region” for their roots of their **resulting** quadratic (not the original) Do **not** accept strict inequalities for final mark	Dependent on first M1 only
		ii	$0 \le x \le 5$	A1	**Examiner's Comments** Less than half of candidates provided fully correct solutions to this quadratic inequality. Some failed to expand and rearrange initially and thus earned no credit. Most were able to complete both first stages accurately, but on	Allow “$x \ge 0$, $x \le 5$”, “$x \ge 0$ **and** $x \le 5$” but do not allow “$x \ge 0$ **or** $x \le 5$”

					reaching $2x^2 - 10x \le 0$ many "cancelled" x and thus could get no further. Where both roots were found, choosing the correct region still proved difficult, with some choosing the "outside" and other candidates writing $x \le 0$, $x \le 5$.	Inequalities
			Total	7		
2		i	$5 - 3 < 6x < 14-3$	M1	Attempt to solve two equations/inequalities **each involving all 3 terms**	
		i	$2 < 6x < 11$	A1	2, 11 seen from correct inequalities	Allow "$\frac{1}{3} < x$ **and** $x < \frac{11}{6}$"
		i	$\frac{1}{3} < x < \frac{11}{6}$	A1	**www** Award full marks if initially working with equations but final answer correct. **Examiner's Comments** This simple "double inequality" was well tackled by almost all candidates. Only the very weakest either tried to combine it into a single inequality and/or made arithmetical errors.	"$\frac{1}{3} < x$, $x < \frac{11}{6}$" but do not allow "$\frac{1}{3} < x$ **or** $x < \frac{11}{6}$"
		ii	$3x^2 - 13x - 10 \ge 0$	M1*	Expands and rearranges to collect all terms on one side	
		ii	$(3x + 2)(x - 5) \ge 0$	M1dep*	Correct method to find roots	
		ii		A1	$-\frac{2}{3}$, 5 seen as roots	
		ii	$x \le -\frac{2}{3}$, $x \ge 5$	M1	Chooses "outside region" for their roots of their quadratic	e.g. $-\frac{2}{3} \ge x \ge 5$ scores **M1A0**
		ii		A1	Do not allow strict inequalities for final mark **Examiner's Comments** Just under half of candidates provided fully correct solutions to this quadratic inequality. Some failed to expand and rearrange at the start and thus earned	Allow "$x \le -\frac{2}{3}$, $x \ge 5$", "$x \le -\frac{2}{3}$ or

					no credit. Most were able to complete the first stage accurately, but the resulting quadratic proved more difficult to handle. Those who factorised were usually successful, whilst those who attempted to use the quadratic formula were often correct in performing the substitution but unable to find the square root of 289. Most of those who found the correct roots also chose the correct region, but there were a significant number who expressed this incorrectly, often as $-\frac{2}{3} \geq x \geq 5$.	$x \geq 5$” but do not allow “$x \leq -\frac{2}{3}$ and $x \geq 5$” **SC** If question “misread” as $x(3x - 13) \geq 0$ Roots found as 0, $\frac{13}{3}$ B1 $x \leq 0$, $x \geq \frac{13}{3}$ etc. as above **B1, max 2/5**
			Total	8		
3		i	$(2x - 3)(x + 1) = 0$	M1	Correct method to find roots – **see appendix 1**	
		i	$x = \frac{3}{2}, x = -1$	A1	Correct roots	
		i		A1ft	Good curve: • Correct shape, symmetrical positive quadratic • Minimum point in the correct quadrant for their roots **(ft)** • their x intercepts correctly labelled **(ft)**	

		i		B1	y intercept at (0, −3). Must have a graph. **Examiner's Comments** Most candidates recognised this as a quadratic and provided an appropriate sketch, although there was a tendency for some to become steep/vertical extremely quickly rather indicate increasing gradient. The points of intersection on the x-axis were usually accurate with the occasional sign swaps. Although the y-intercept was usually correctly identified as –3, it was very common to see this as vertex of the graph which lost an accuracy mark; candidates were expected to indicate the vertex would be in the correct quadrant for their roots	
		ii	$x < -1, x > \frac{3}{2}$	M1	Choises the "outside region"	If restarted, fully correct method for solving a quadratic inequality including choosing "outside region" needed for **M1**
		ii		A1ft	Follow through x-values in (i). Allow "$x < -1, x > \frac{3}{2}$", "$x < -1$ **or** $x > \frac{3}{2}$" but do not allow "$x < -1$ **and** $x > \frac{3}{2}$" **Examiner's Comments** Most candidates used their answer to part (i) and chose the correct outside region, although choosing the inside region was a frequently seen error. The notation used to describe the region was usually correct; incorrect language such as joining the two sections with the word 'and' lost the accuracy mark.	**NB** e.g. $-1 > x > \frac{3}{2}$ scores **M1A0** Must be strict inequalities for **A** mark
		iii	$b^2 - 4ac = 1^2 - 4 \times 2 \times -(3 + k)$	M1	Rearrangement and use of $b^2 - 4ac < 0$, must involve 3 and k in constant term (not $3k$)	Alt for first two marks: **M1** Attempt to find turning point and form inequality $k < y_{min}$
		iii	$25 + 8k < 0$	A1	$p + 8k < 0$ oe found, any constant p. p need not be simplified	**A1** turning point correct $(\frac{1}{4}, -\frac{25}{8})$

		iii	$k < -\frac{25}{8}$	A1	Correct final answer **Examiner's Comments** This proved demanding for many candidates. Although some secured all three marks, many earned no credit as they either put the discriminant equal to zero or, as was frequently seen, to *k*, making no attempt to rearrange the given equation. Accuracy marks were often lost as candidates failed to deal with the minus signs both in the discriminant and in the expression for *c*. A few candidates found the turning point of their graph either by differentiation or by completing the square but these approaches were far less common.	If M0 (either scheme) SC B1 $k = -\frac{25}{8}$ or $k > -\frac{25}{8}$ seen
			Total	9		
4			$3x + 2 \geq 20 - x \Rightarrow 4x \geq 18$ $x \geq \frac{9}{2}$ $x^2 < x + 6 \Rightarrow x^2 - x - 6 < 0$ Critical values 3, −2 $-2 < x < 3$ $\{x : -2 < x < 3\} \cup \left\{x : x \geq \frac{9}{2}\right\}$	M1 (AO1.1a) A1 (AO1.1) M1 (AO1.1a) A1 (AO1.1) A1FT (AO1.1) A1 (AO2.5) [6]	Rearranging to the form $ax \geq b$ — Allow one error Rearrange and attempt to solve resulting 3-term quadratic BC Correct region for their critical values Dependent on both M marks	

			Total	6	Inequalities
5			DR $x + 3 \geq 14.5$ $x \geq 11.5$ $x(x + 3) < 180$ $x^2 + 3x - 180 (< 0) \Rightarrow (x - 12)(x + 15) (< 0)$ $-15 < x < 12$ $11.5 \leq x < 12$	M1(AO 3.1b)E A1(AO 1.1)E M1(AO 3.1b)E M1(AO 1.1)E A1(AO 1.1)E B1(AO 1.1)E [6]	Accept any inequality or equals and any letter for the width Correct inequality (seen or implied) Accept any inequality or equals Correct expansion and attempt to solve three term quadratic Correct inequalities (seen or implied) M1A1 correct answer with no working SC B1: $x < \sqrt{60}$ B1: $x \geq 29/6$ Examiner's Comments As this was a detailed reasoning question it was expected that candidates would do just that and show sufficient reasoning so that examiners could see that a complete analytical method had been employed. So it was therefore not possible to award full marks to those candidates who wrote statements such as $x(x + 3) < 180 \Rightarrow -15 < x < 12$. While many candidates correctly found that $11.5 \leq x < 12$ a small proportion of candidates stated that $\frac{29}{6} \leq x < \sqrt{60}$. This incorrect answer came from misreading the question and considering the length of the flower bed to be three times longer than its width (and not just 3m longer than the width).

			Total	6		
6		a	$3^2 - 4k = 0$ $k = \frac{9}{4}$ or 2.25	M1 (AO1.2) A1 (AO1.1) [2]	$x^2 + 3x + k = (x + a)^2 = x^2 + 2ax + a^2$ $\Rightarrow a = 1.5$ $\Rightarrow k = 1.5^2$ or $(x + 1.5)^2 - 2.25 + k = 0$ **Examiner's Comments** This question was answered well. Use of the discriminant was the more popular approach, but some candidates used the "completing the square" method. . A few candidates started with 9 – 4k > 0. Others used $b^2 + 4ac$. In general the "completing the square" method was less successfully applied, with mistakes in the algebraic manipulation more common.	
		b	$(3 - x)(2 + x) > 0$ or $(x - 3)(x + 2) < 0$ $-2 < x < 3$ or $3 > x > -2$ ISW or $x \in (-2, 3)$	M1 (AO1.1a) M1 (AO2.2a) [2]	oe Allow $(3 - x)(2 + x)$ or $(x - 3)(x + 2)$ or -2 and 3 seen Allow $x > -2$, $x < 3$ or $x > -2$ and $x < 3$ Correct ans: BOD M1A1 $x > -2$ **or** $x < 3$ M1A0 unless followed by ans **Examiner's Comments** Many candidates were unable to deal with the signs. Some wrote (x – 3)(x + 2) > 0 or (–x – 3)(x + 2) > 0. Many eventually obtained either {x < –2 and x > 3} or {–3 < x < 2}. A few candidates gave correct working, but gave their solution as two separate regions: x > –2, x < 3.	
			Total	4		

7		a	$-6 < 3x < 13$ $-2 < x < \frac{13}{3}$	M1 (AO 1.1) A1 (AO 1.1) [2]	Attempt to solve two equations / inequalities each involving all three terms Obtain correct inequality | Correct order of operations	
		b	$4x^2 > 25$ $-\frac{5}{2}, \frac{5}{2}$ $x < -\frac{5}{2}$ or $x > \frac{5}{2}$	M1 (AO 1.1) M1 (AO 1.1) A1FT (AO 2.2a) [3]	Rearrange to useable form | Or $4x^2 - 25 > 0$ Attempt to find critical values | or **BC** Choose 'outside' region for inequality FT their critical values | or **BC**	
			Total	5		
8		a	(i) $2 + 5 + x + x + (x + 2) + (x + 5) < 44$ oe (ii) $x(x + 2) + 10 \geq 45$ oe	B1(AO 1.1) [1] B1(AO 1.1) [1]	Correct inequality | Must be < only Correct inequality relating to area | Must be ≥ only	
		b	$x < 7.5$ $x^2 + 2x - 35 \geq 0$	B1FT(AO 1.1) M1(AO 1.1a)	Obtain $x < 7.5$ from linear inequality FT their linear inequality in **(a)** Attempt to solve three term quadratic	

			critical values are – 7 and – 5 $x \le -7$, $x \ge 5$ but x is a length so $x \ge 5$ $\{x: 5 \le x < 7.5\}$ or $[5, 7.5)$	A1FT(AO 2.4) B1(AO 2.2a) [4]	Choose 'outside' region for inequality FT their quadratic inequality in **(b)**, as long as one positive root and one negative root Single correct interval – any correct notation B1M1A0B1 possible if no reason for rejecting –7	BC Both values of x needed –7 must be seen and discarded with a reason Condone $5 \le x < 7.5$	Inequalities
			Total	6		