

1. Solve the equations

i. $3^n = 1,$

[1]

ii. $t^3 = 64,$

[2]

iii. $(8p^6)^{\frac{1}{3}} = 8.$

[3]

2. Express each of the following in the form $a\sqrt{5}$, where a is an integer.

i. $4\sqrt{15} \times \sqrt{3}$

[2]

ii. $\frac{20}{\sqrt{5}}$

[1]

iii. $5^{\frac{3}{2}}$

[1]

3. Express each of the following in the form 5^k .

i. 25^4

[1]

ii. $\frac{1}{\sqrt[4]{5}}$

[2]

iii. $(5\sqrt{5})^3$

[2]

4. Express $\frac{8}{\sqrt{3}-1}$ in the form $a\sqrt{3}+b$, where a and b are integers. [3]

5. Express the following in the form 2^p .

i. $(2^5 \div 2^7)^3$ [2]

ii. $5 \times 4^{\frac{2}{3}} + 3 \times 16^{\frac{1}{3}}$ [3]

6. Express $\frac{3 + \sqrt{20}}{3 + \sqrt{5}}$ in the form $a + b\sqrt{5}$. [4]

7. Simplify fully.

(a) $\sqrt{a^3} \times \sqrt{16a}$. [2]

(b) $(4b^6)^{\frac{5}{2}}$ [2]

8. Simplify

(a) $\frac{(3x)^3 \times 2x^{-1}}{9x^2}$, [2]

(b) $(49x^{-4})^{-\frac{1}{2}}$.

[2]

9. In this question you must show detailed reasoning.

Express each of the following in the form $a + b\sqrt{2}$, where a and b are integers.

(a) $\sqrt{3}(\sqrt{12} + \sqrt{54})$

[3]

(b) $\frac{6}{2 + \sqrt{2}}$

[3]

10. Express $\frac{2 + \sqrt{7}}{\sqrt{7} - 2}$ in the form $a + b\sqrt{7}$, where a and b are rational numbers. [3]

11. In this question you must show detailed reasoning. [2]

- (a) Express $3^{\frac{7}{2}}$ in the form $a\sqrt{b}$, where a is an integer and b is a prime number.

- (b) Express $\frac{\sqrt{2}}{1 - \sqrt{2}}$ in the form $c + d\sqrt{e}$, where c and d are integers and e is a prime number. [3]

END OF QUESTION paper

Mark scheme

Question		Answer/Indicative content	Marks	Part marks and guidance	
1	i	$n = 0$	B1	<p>Allow 3^0</p> <p><u>Examiner's Comments</u></p> <p>Only a tiny number of candidates failed to secure the mark for this simple recall of index notation; $\frac{1}{3}$ and 1 were occasionally seen.</p>	
	ii	$\frac{1}{t^3} = 64$ (or 4^3)	M1	<p>or $t^3 = \frac{1}{64}$ or $64t^3 = 1$ or $\left(\frac{1}{t}\right)^3 = 64$</p> <p>$4^{-1}$ is A0 $t = \pm \frac{1}{4}$ is A0</p> <p><u>Examiner's Comments</u></p>	Allow embedded
	ii	$t = \frac{1}{4}$	A1	<p>Most candidates knew how to deal with the negative index and rewrote the equation as $\frac{1}{t^3} = 64$ or</p> <p>equivalent. Thereafter, however, a significant number could not proceed further, with -4 being a common wrong answer.</p>	4^{-1} www alone implies M1 A0
	iii	$2p^2 = 8$	M1	<p>or $8p^6 = 8^3$. Allow $2p^{\frac{6}{3}} = 8$ for M1</p>	If not 512, evidence of $8 \times 8 \times 8$ needed.
	iii	$p = 2$	A1	www	SC Spotted B1 for 2, B1 for -2 , B1 for justifying exactly 2 solutions
	iii	or $p = -2$	A1	www	SC $8p^2 = 8$, $p = \pm 1$ B1

				<u>Examiner's Comments</u>	
				Although a large majority of candidates realised the need to find a cube root, many applied this only to the ρ^6 term and not to the 8. Those that were successful often omitted the negative solution thus surrendering the final mark.	
		Total	6		
2	i	$4\sqrt{45}$	M1	or $4\sqrt{5}\sqrt{3} \times \sqrt{3}$ (not just $4\sqrt{5 \times 3} \times \sqrt{3}$) or $\sqrt{720}$ or $\sqrt{240} \times \sqrt{3}$ or better	For method mark, makes a correct start to manipulate the expression i.e. at least combines two parts correctly or splits one part correctly
	i	$=12\sqrt{5}$	A1	Correctly simplified answer <u>Examiner's Comments</u> Most candidates were successful with this easy starter, but a significant minority found it quite challenging. Most earned at least a method mark for correct surd manipulation of some kind, but the accuracy was more of a problem, with some arithmetic errors and also conceptual ones such as $4 \times 3\sqrt{5} = 7\sqrt{5}$.	
	ii	$\frac{20\sqrt{5}}{5} = 4\sqrt{5}$	B1	cao , do not allow unsimplified, do not allow if clearly from wrong working <u>Examiner's Comments</u> Around 85% of candidates were successful in rationalising the denominator. Where no credit was earned, this was usually due to a lack of understanding rather than arithmetical error with a significant minority appearing not to know how to rationalise the given expression. Simply rewriting it as $20(\sqrt{5})^{-1}$ was quite common.	
	iii	$5\sqrt{5}$	B1	cao www , do not allow unsimplified, do not allow if clearly from wrong working	

				<p>Examiner's Comments</p> <p>This was generally less successful than parts (i) and (ii), with just under three-quarters of candidates earning the mark. Many of those who did not give the answer in the required form did at least understand the notation as $(\sqrt{5})^3$ was often seen, but then simplified to $3\sqrt{5}$.</p>	
		Total	4		
3	i	5^8	B1	<p>cao</p> <p>Examiner's Comments</p> <p>Almost all candidates secured this easy mark, but the error of $(5^2)^4 = 5^6$ was quite common.</p>	
	ii	$5^{-\frac{1}{4}}$	M1	<p>Fourth root $\equiv \frac{1}{4}$ soi</p> <p>cao www</p>	
	ii		A1	<p>Examiner's Comments</p> <p>Again, most candidates were able to gain both marks dealing with both the fractional and negative elements of the index.</p>	
	iii	$5^{\frac{9}{2}}$	M1	<p>$(5^{\frac{3}{2}})^3$ or $5^3 \times 5^{\frac{3}{2}}$ or other correct product of two simplified powers of 5</p> <p>oe cao www</p>	
	iii		A1	<p>Examiner's Comments</p> <p>This part of the question proved rather more demanding with a minority of</p>	

				<p>candidates securing both marks. Those who recognised that $\sqrt{5} = 5^{\frac{1}{2}}$ were usually able to go on and complete the question successfully; those who tried to multiply out were less successful.</p>	
		Total	5		
4		$\frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $\frac{8\sqrt{3}+8}{3-1}$ $4\sqrt{3}+4$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>Multiply top and bottom by $\sqrt{3}+1$ or $-\sqrt{3}-1$ evidence of multiplying out needed</p> <p>Either numerator or denominator correct</p> <p>Final answer cao</p> <p>Examiner's Comments</p> <p>Most candidates recognised the need to rationalise the denominator and did so efficiently and accurately, with many candidates securing all three marks. Errors were sometimes seen both in evaluating the numerator and the denominator, and occasionally in performing the final division</p>	<p>Alternative:</p> <p>M1 Correct method to solve</p> <p>simultaneous equations formed from equating expression to $a\sqrt{3}+b$</p> <p>A1 Either a or b correct</p> <p>A1 Both correct</p>
		Total	3		
5	i	$(2^{-2})^3$ or $2^{15} \div 2^{21}$	B1	Valid attempt to simplify	Correct use of either index law
	i	2^{-6}	B1	Correct answer. Accept $p = -6$.	$\left(\frac{1}{2}\right)^6$ oe is B1

					Examiner's Comments
					This simple index question was very well done, with around 90% securing both marks.
ii	$5 \times (2^2)^{\frac{2}{3}} + 3 \times (2^4)^{\frac{1}{3}}$	M1	Attempts to express both terms or a combined term as a power of 2	e.g. Both $4 = 2^2$ and $16 = 2^4$ soi	
ii	$= 5 \times 2^{\frac{4}{3}} + 3 \times 2^{\frac{4}{3}} \text{ or } 10 \times 2^{\frac{1}{3}} + 6 \times 2^{\frac{1}{3}}$	B1	Correctly obtains $2^{\frac{4}{3}}$ or $2^{\frac{1}{3}}$ for either term	If MO	
ii	$= 8 \times 2^{\frac{4}{3}}$			Examiner's Comments	
ii	$= 2^{\frac{13}{3}}$	A1	Correct final answer	Although there were a significant number of excellent solutions, this question proved much more demanding than expected with less than a third of candidates securing all three marks. Many reached $5 \times 2^{\frac{4}{3}} + 3 \times 2^{\frac{4}{3}}$ but then went no further, or even "simplified" this to $10^{\frac{4}{3}} + 6^{\frac{4}{3}}$. Many of those who did obtain $8 \times 2^{\frac{4}{3}}$ appeared not to realise 8 was a power of 2. Some of those who did then made errors adding the powers, either through incorrect addition or multiplying so that $5 \times 2^3 \times 2^{\frac{4}{3}} = 2^4$	
Total			5		

6		$\frac{3 + \sqrt{20}}{3 + \sqrt{5}} \times \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$ $\frac{-1 + 3\sqrt{5}}{9 - 5}$ $-\frac{1}{4} + \frac{3}{4}\sqrt{5}$	<p>M1</p> <p>B1</p> <p>A1</p> <p>A1</p>	<p>Attempt to rationalise the denominator – must attempt to multiply</p> $\sqrt{20} = 2\sqrt{5} \text{ soi}$ <p>Either numerator or denominator correct and simplified to no more than two terms</p> <p>Fully correct and fully simplified. Allow $\frac{-1 + 3\sqrt{5}}{4}$ order reversed etc.</p> <p>Do not ISW if then multiplied by 4 etc.</p>	<p>Alternative:</p> <p>M1 Correct method to solve simultaneous equations formed from equating expression to $a\sqrt{5} + b$</p> <p>B1 $\sqrt{20} = 2\sqrt{5}$ soi</p> <p>A1 Either a or b correct</p> <p>A1 Both correct</p> <p>Examiner's Comments</p> <p>Most candidates recognised the need to rationalise the denominator and did so efficiently and accurately, with many candidates securing all four marks. The conversion from $\sqrt{20}$ to $2\sqrt{5}$ was usually well done; most errors that occurred were seen when expanding and simplifying the numerator. Some candidates obtained the correct answer but then, seemingly unsatisfied with the fractional values of a and b found, multiplied by 4; this lost the final mark.</p>		
Total		4					
7	a	$\sqrt{16a^4} \text{ or } 4\sqrt{a^4} \text{ or } a\sqrt{a} \times 4\sqrt{a}$ $= 4a^2$	<p>M1(AO1.1)</p> <p>A1(AO1.1)</p> <p>[2]</p>	<table border="1" style="width: 100%; height: 100%;"> <tr> <td style="width: 80%; padding: 5px;">Any correct first step</td> <td style="width: 20%;"></td> </tr> </table>	Any correct first step		
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	b	$32b^{15}$	B2(AO1.1) (AO1.1) [2]	B1 for 32 and B1 for b^{15}	
		Total	4		
8	a	$\frac{(3x)^3 \times 2x^{-1}}{9x^2} = \frac{27x^3 \times 2x^{-1}}{9x^2}$ $= \frac{54x^2}{9x^2} = 6$	B1 (AO1.1) B1 (AO1.1) [2]	Correctly expands $(3x)^3$ as $27x^3$	
	b	$(49x^{-4})^{-\frac{1}{2}} = \frac{1}{7}x^2$	B1 (AO1.1) B1 (AO1.1) [2]	For $\frac{1}{7}$, independent of power of x For x^2	
		Total	4		
9	a	DR $\sqrt{36} + \sqrt{162}$ oe	M1(AO1.1a)	Attempt to expand	

		$\sqrt{6^2} + \sqrt{9^2 \times 2} \text{ oe}$ $= 6 + 9\sqrt{2}$	<p>A1(AO1.1)</p> <p>A1(AO1.1)</p> <p>[3]</p>	<p>bracket</p> <p>Obtain 6</p> <p>Obtain $9\sqrt{2}$</p>	<p>Must show sufficient method</p>	
	b	<p>DR</p> $\frac{6(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})} = \frac{12 - 6\sqrt{2}}{2} = 6 - 3\sqrt{2}$	<p>M1(AO1.1a)</p> <p>A1(AO1.1)</p> <p>A1(AO1.1)</p> <p>[3]</p>	<p>Multiply numerator and denominator by $2 - \sqrt{2}$</p> <p>Either numerator or denominator correct</p> <p>Fully correct expression</p>	<p>Must be simplified</p> <p>Must show sufficient method</p>	
		Total	6			
10		$\frac{2 + \sqrt{7}}{\sqrt{7} - 2} \times \frac{\sqrt{7} + 2}{\sqrt{7} + 2}$ $\frac{11 + 4\sqrt{7}}{7 - 4}$	<p>M1</p> <p>A1</p>	<p>Attempt to rationalise the denominator – must attempt to multiply. (May use $-\sqrt{7} - 2$)</p> <p>Either numerator or denominator correct and simplified to no more than two terms</p>	<p>Alternative: M1 Correct method to solve simultaneous equations formed from</p> <p>equating expression to $a + b\sqrt{7}$</p>	

		$\frac{11}{3} + \frac{4\sqrt{7}}{3}$	A1 [3]	<p>Fully correct and simplified.</p> $\frac{11 + 4\sqrt{7}}{3},$ <p>Allow</p> <p>terms in any order</p> <p>Do not ISW if then incorrect</p>	<p>A1 Either a or b correct</p> <p>A1 Both correct</p> <p>Do not allow</p> $\frac{-11 - 4\sqrt{7}}{-3}$ <p>for last A1.</p>								
		Total	3										
11	a	<p>DR</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td>$(\sqrt{3})^7$</td> <td>or</td> <td>$\sqrt{3^7}$</td> <td>or</td> <td>$3^3 \times \sqrt{3}$</td> <td>or</td> <td>$3\sqrt{243}$</td> </tr> </table>	$(\sqrt{3})^7$	or	$\sqrt{3^7}$	or	$3^3 \times \sqrt{3}$	or	$3\sqrt{243}$	M1 (AO1.1a)	<p>or any correct intermediate step</p>	<p>If this step is not seen, MOAO</p>	
$(\sqrt{3})^7$	or	$\sqrt{3^7}$	or	$3^3 \times \sqrt{3}$	or	$3\sqrt{243}$							

	$27\sqrt{3}$	<p>A1 (AO1.1)</p> <p>[2]</p>	<table border="1" style="width: 100%;"> <tr> <td style="width: 50%; padding: 5px;"> using $\sqrt{\quad}$ or $3^3 \times 3^{\frac{1}{2}}$ or $a = 27, b = 3$ </td> <td style="width: 50%;"></td> </tr> </table> <p><u>Examiner's Comments</u></p> <p>Many candidates answered this question correctly. A few made a correct first step, for example $\sqrt{3^7}$, but could not continue correctly. Some candidates gave the correct answer with no working or with incorrect working. These scored no marks.</p>	using $\sqrt{\quad}$ or $3^3 \times 3^{\frac{1}{2}}$ or $a = 27, b = 3$							
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	<p>DR</p> $\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$ <p>b</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td style="padding: 5px;">$= \frac{\sqrt{2}+2}{1-2}$</td> <td style="padding: 5px;">or</td> <td style="padding: 5px;">$\frac{\sqrt{2}+2}{-1}$</td> <td style="padding: 5px;">or</td> <td style="padding: 5px;">$\frac{\sqrt{2}+2}{1+\sqrt{2}-\sqrt{2}-2}$</td> </tr> </table> $= -2 - \sqrt{2} \quad \text{ISW}$	$= \frac{\sqrt{2}+2}{1-2}$	or	$\frac{\sqrt{2}+2}{-1}$	or	$\frac{\sqrt{2}+2}{1+\sqrt{2}-\sqrt{2}-2}$	<p>M1 (AO1.1a)</p> <p>A1 (AO1.1)</p> <p>A1 (AO1.1)</p> <p>[3]</p>	<table border="1" style="width: 100%;"> <tr> <td style="width: 50%; padding: 5px;"> A1 for correct num OR denom or $-2 + (-1\sqrt{2})$ or $c = -2, d = -1$ and $e = 2$ </td> <td style="width: 50%; padding: 5px;"> If this step is not seen, MOA0 Allow $-(2 + \sqrt{2})$ </td> </tr> </table> <p><u>Examiner's Comments</u></p> <p>Many candidates answered this question correctly. A few made a correct</p>	A1 for correct num OR denom or $-2 + (-1\sqrt{2})$ or $c = -2, d = -1$ and $e = 2$	If this step is not seen, MOA0 Allow $-(2 + \sqrt{2})$	
$= \frac{\sqrt{2}+2}{1-2}$	or	$\frac{\sqrt{2}+2}{-1}$	or	$\frac{\sqrt{2}+2}{1+\sqrt{2}-\sqrt{2}-2}$							
A1 for correct num OR denom or $-2 + (-1\sqrt{2})$ or $c = -2, d = -1$ and $e = 2$	If this step is not seen, MOA0 Allow $-(2 + \sqrt{2})$										

				<p>first step, multiplying numerator and denominator by $1 + \sqrt{2}$, but made a subsequent error. Some candidates gave the correct answer with no working or with incorrect working. These scored no marks.</p>	
			Total	5	