i. Sketch the curve y = (1 + x)(2 - x)(3 + x), giving the coordinates of all points of intersection with the axes.

ii. Describe the transformation that transforms the curve y = (1 + x)(2 - x)(3 + x) to the curve y = (1 - x)(2 + x)(3 - x).

[2]

- 2. i. Sketch the curve $y = 2x^2 x 6$, giving the coordinates of all points of intersection with the axes.
 - [5]

ii. Find the set of values of x for which $2x^2 - x - 6$ is a decreasing function.

- [3]
- iii. The line y = 4 meets the curve $y = 2x^2 x 6$ at the points *P* and *Q*. Calculate the distance *PQ*.

[4	4]
	_

З.

1.

i. Sketch the curve $y = \frac{2}{x^2}$.

[2]

ii. The curve $y = \frac{2}{x^2}$ is translated by 5 units in the negative *x*-direction. Find the equation of the curve after

it has been translated.

[2]

iii. Describe a transformation that transforms the curve $y = \frac{2}{x^2}$ to the curve $y = \frac{1}{x^2}$.

[2]

Graph Sketching and Transformations

i. Sketch the curve $y = 2x^2 - x - 3$, giving the coordinates of all points of intersection with the axes.

[4]

[2]

ii. Hence, or otherwise, solve the inequality $2x^2 - x - 3 > 0$.

Sketch the curve $y = -\frac{1}{x}$.

iii. Given that the equation $2x^2 - x - 3 = k$ has no real roots, find the set of possible values of the constant *k*.

[2]

[2]

- ii. The curve $y = -\frac{1}{x}$ is translated by 2 units parallel to the *x*-axis in the positive direction. State the equation of the transformed curve.
- iii. Describe a transformation that transforms the curve $y = -\frac{1}{x}$ to the curve $y = -\frac{1}{3x}$.
- 6. The curve y = f(x) passes through the point *P* with coordinates (2, 5).
 - (i) State the coordinates of the point corresponding to *P* on the curve y = f(x) + 2.
 - (ii) State the coordinates of the point corresponding to *P* on the curve y = f(2x).
- [1]

[1]

(iii) Describe the transformation that transforms the curve y = f(x) to the curve y = f(x + 4). [2]

4.

5.

i.

7. i. Sketch the curve $y = x^2(3 - x)$ stating the coordinates of points of intersection with the axes.

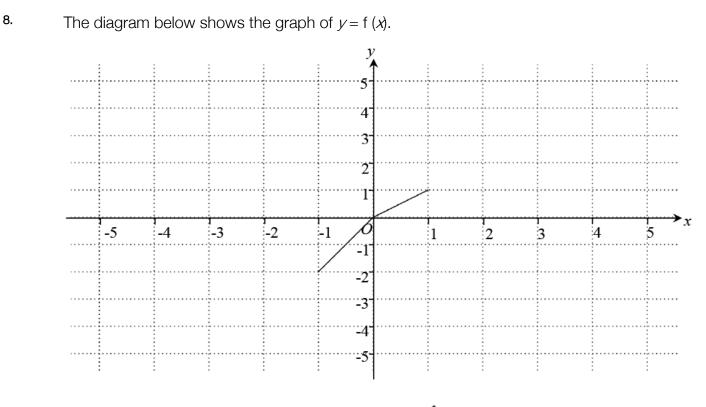
[3]

ii. The curve $y = x^2(3 - x)$ is translated by 2 units in the positive direction parallel to the x-axis. State the equation of the curve after it has been translated.

[2]

iii. Describe fully a transformation that transforms the curve $y = x^2(3 - x)$ to $y = \frac{1}{2}x^2(3 - x)$.

[2]



(a) On the diagram above, draw the graph of $y = f(\frac{1}{2}x)$ [1]

(b) On the diagram above, in a different colour, draw the graph of y = f(x - 2) + 1. [2]

х

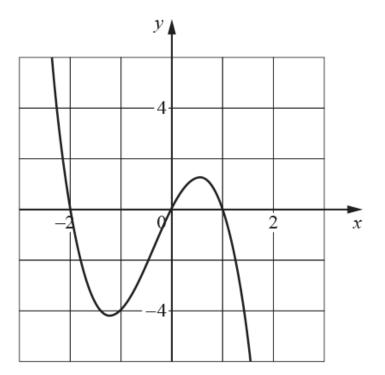
[6]

(a) Sketch the curves
$$y = \frac{3}{x^2}$$
 and $y = x^2 - 2$ on the axes provided below. [3]

(b) In this question you must show detailed reasoning. Find the exact coordinates of the points of intersection of the curves $y = \frac{3}{x^2}$ and $y = x^2 - 2$.

0

10. Part of the graph of y = f(x) is shown below, where f(x) is a cubic polynomial.



9.

	Graph Sketching and Transformations
(a) Find f(-1).	[1]
(b) Write down three linear factors of $f(x)$.	[1]
It is given that $f(x) \equiv ax^3 + bx^2 + cx + d$.	
(c) Show that $a = -2$.	[3]
(d) Find <i>b</i> , <i>c</i> and <i>d</i> .	[1]

^{11.} Show in a sketch the region of the x-y plane within which all three of the following inequalities are satisfied.

$$3y \ge 4x$$
 $y - x \le 1$ $y \ge (x - 1)^2$

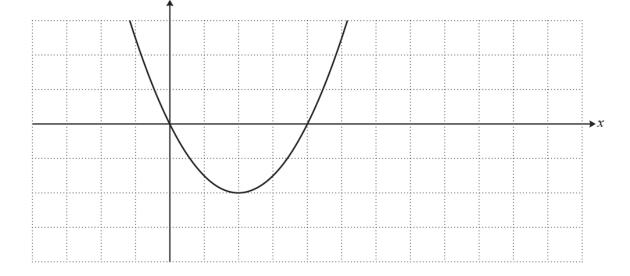
You should indicate the region for which the inequalities hold by labelling the region R. [4]

- 12. The cubic polynomial f(x) is defined by $f(x) = 2x^3 7x^2 + 2x + 3$.
 - (a) Given that (x 3) is a factor of f(x), express f(x) in a fully factorised form.
 - (b) Sketch the graph of y = f(x), indicating the coordinates of any points of intersection [2] with the axes.
 - (c) Solve the inequality f(x) < 0, giving your answer in set notation.
 - (d)

The graph of y = f(x) is transformed by a stretch parallel to the *x*-axis, scale factor $\overline{2}$. [2] Find the equation of the transformed graph.

[3]

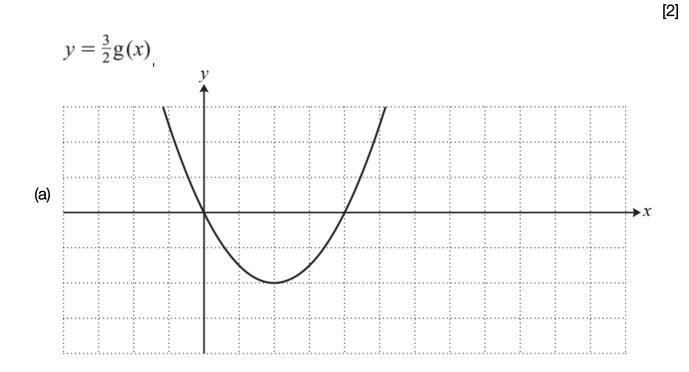
[2]



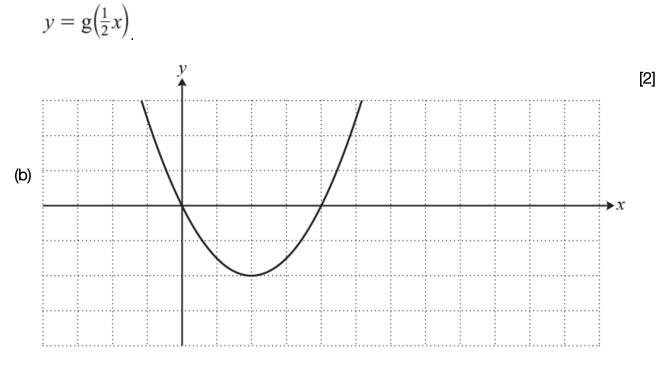
The diagram shows the graph of y = g(x).

y

Using the same scale as in this diagram, sketch, on the copy of the diagram below, the curves:



 $\rightarrow x$



- 14. f(x) is a cubic polynomial in which the coefficient of x^3 is 1. The equation f(x) = 0 has exactly two roots.
 - (a) Sketch a possible graph of y = f(x).

It is now given that the two roots are x = 2 and x = 3.

(b) Find, in expanded form, the two possible polynomials f(x).

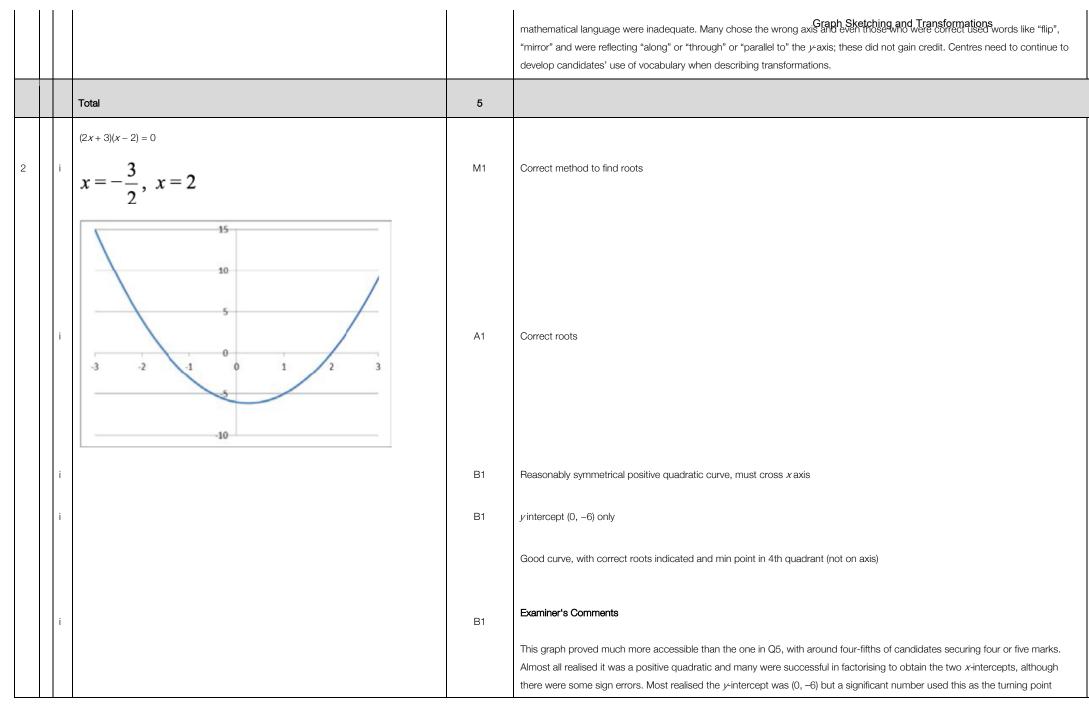
END OF QUESTION paper

[2]

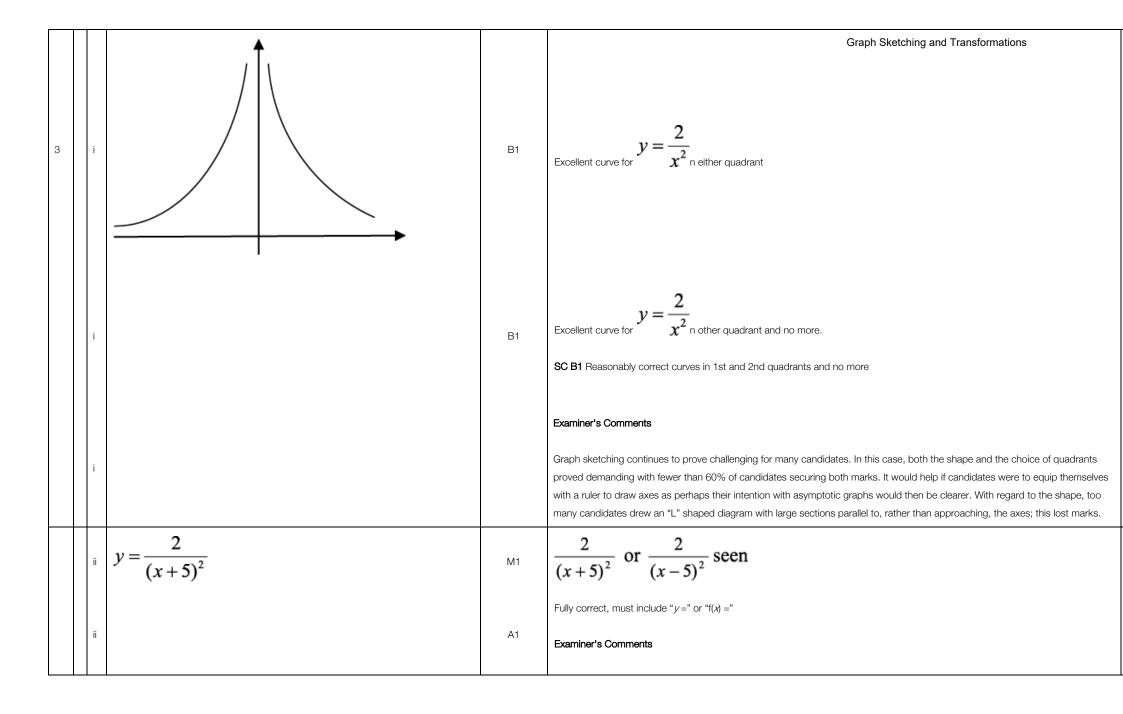
[3]

Mark scheme

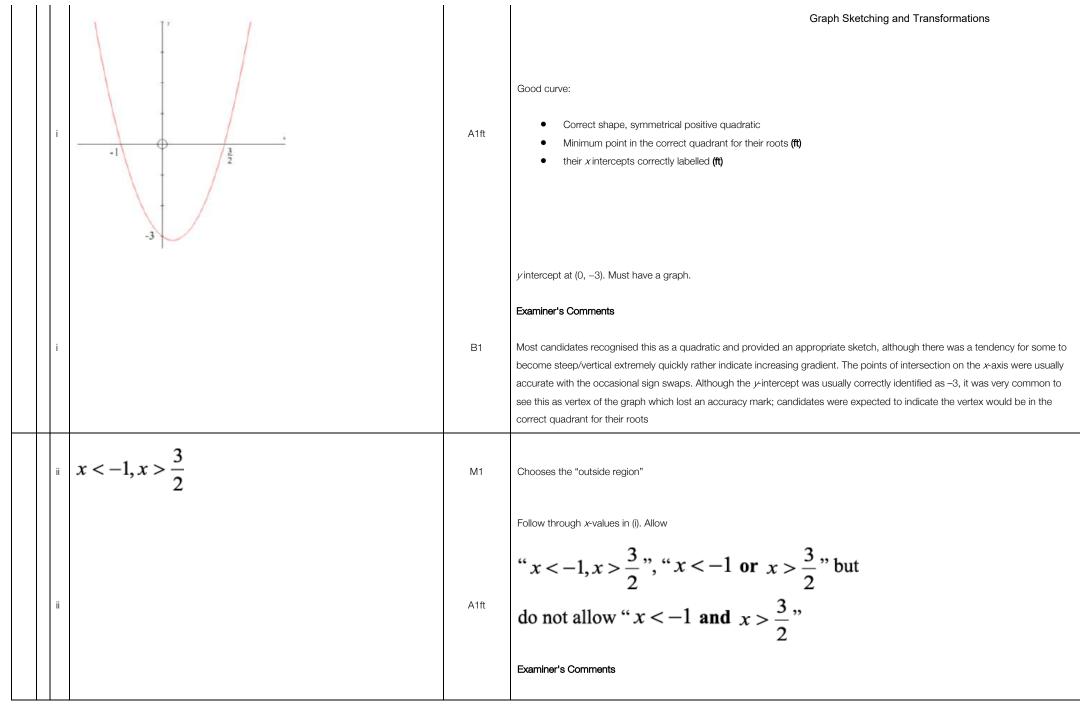
Ques	stion	Answer/Indicative content	Marks	Part marks and guidance
1	i		B1	-ve cubic with 3 distinct roots
	i		B1	(0, 6) labelled or indicated on y-axis – seen elsewhere not enough
				(-3, 0), (-1, 0) and (2, 0) labelled or indicated on x-axis and no other x-intercepts.
				Examiner's Comments
	i		B1	Most candidates recognised that the cubic was negative and sketched an appropriate curve with correct y- and x- intercepts clearly labelled. Some candidates who multiplied out the full expression made an error in finding the y-intercept; others with an otherwise correct solution omitted this. Those who drew a positive cubic did not normally see that this contradicted the required value on one of their axes.
	ii	Reflection	B1	Not mirrored / flipped etc.
	ï	in the <i>y</i> axis	B1	or $x = 0$. No / through / along etc. Must be "in". Cannot get 2 nd B1 without some indication of a reflection e.g. flip etc. Do not ISW if contradictory statement seen <u>Examiner's Comments</u>
				Although the majority of candidates recognised the transformation as being a reflection, their skills in describing this using correct



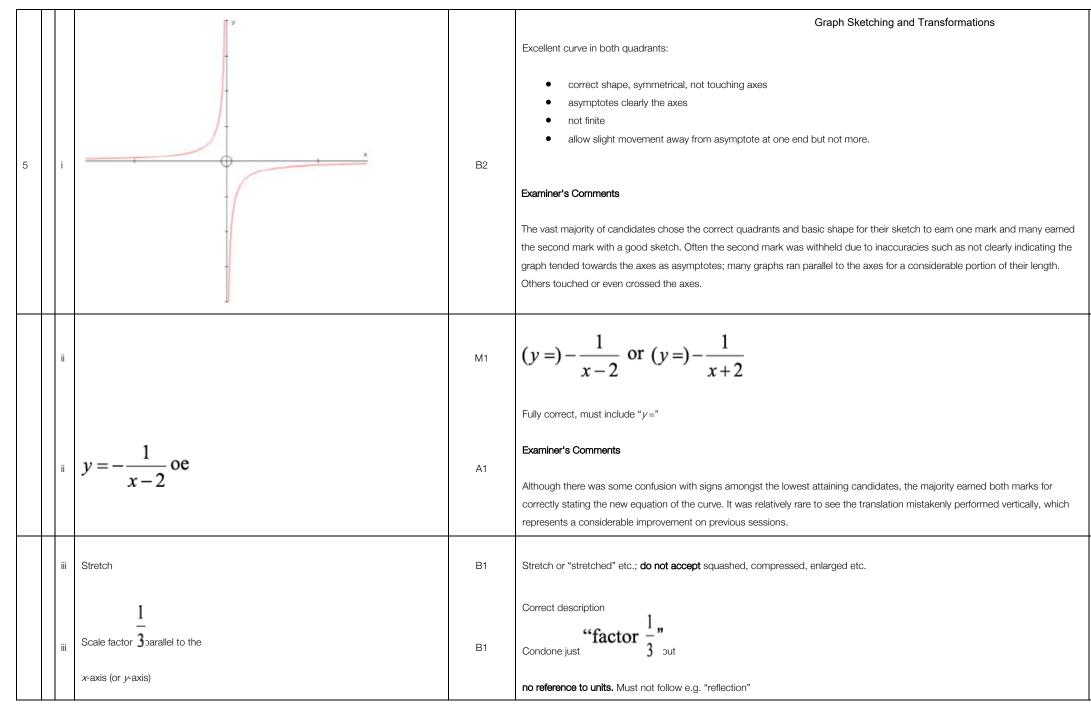
			Graph Sketching and Transformations rather than using the <i>x</i> -intercepts to locate the turning point. Some candidates who were successful in part (ii) had the wisdom to return and correct an error in this part.
ii	$\frac{dy}{dx} = 4x - 1 = 0$ Vertex when $x = \frac{1}{4}$	M1	Attempt to find <i>x</i> coordinate of vertex by differentiating and equating / comparing to zero, completing the square, finding the midpoint of their roots oe
ii	Vertex when $x = \frac{1}{4}$	A1	сао
			$x <$ their vertex, allow \leq
	$x < \frac{1}{2}$	A1FT	Examiner's Comments
II	4	AIFI	Many candidates did not understand the term "decreasing function" and instead gave the region between their roots found in part (i); indeed nearly half of candidates failed to score at all in this part. Those that approached by differentiation were usually more successful in finding the minimum point than those who completed the square.
iii	$2x^2 - x - 6 = 4$	M1	Set quadratic expression equal to 4
iii	$2x^2 - x - 10 = 0$		
iii	(2x-5)(x+2) = 0	M1	Correct method to solve resulting three term quadratic
iii	$x = \frac{5}{2}, x = -2$	A1	Must have both solutions – no mark for one spotted root
			FT from their x values found from their resulting quadratic, provided $y = 4$
			Examiner's Comments
iii	Distance $PQ = 4\frac{1}{2}$	B1FT	Around three quarters of candidates secured the first three marks of this question, equating to 4, simplifying and solving to find the <i>x</i> values of <i>P</i> and <i>Q</i> . Again, factorisation was both the most appropriate and most frequent approach. Rather than then subtracting these values, many saw the word "distance", used the "distance formula" and re-substituted to find y (not always getting 4) and then used Pythagoras' theorem. This led to a lot of unnecessary arithmetical difficulty.
	Total	12	



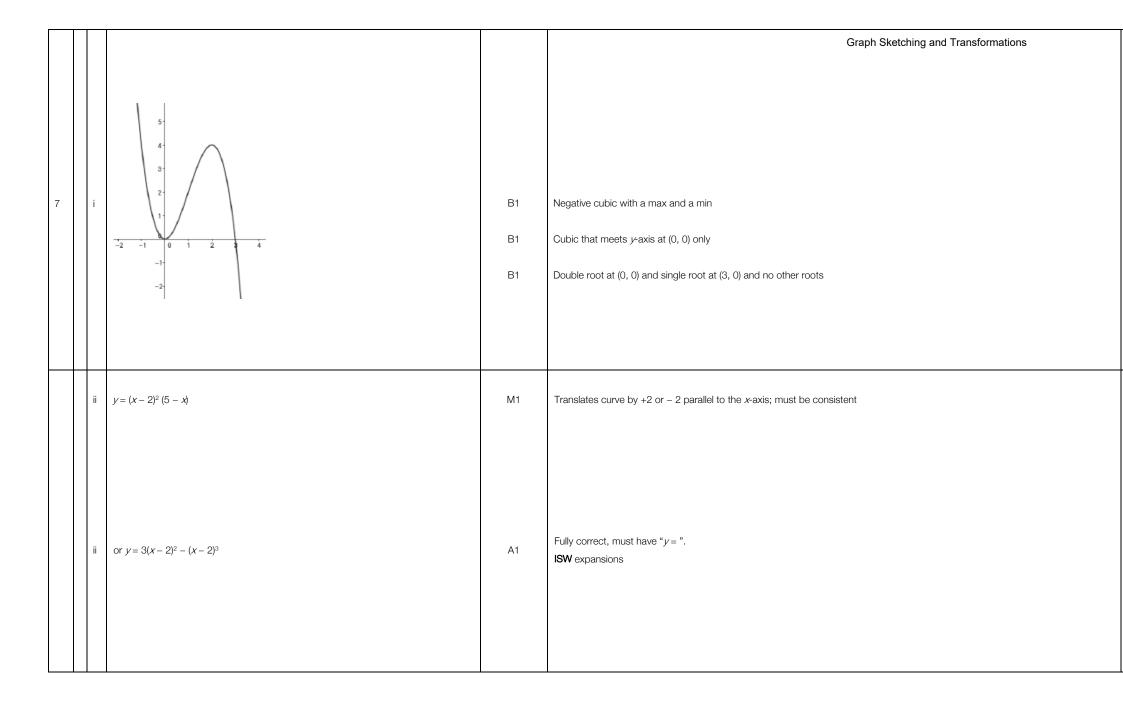
				Graph Sk2tching and Transformations Transformations of this type still prove difficult for candidates, with $y = \frac{x^2 + 5}{x^2 + 5}$ the most common incorrect answer. Again, fewer than 60% provided a fully correct equation.
		Stretch	B1	Or "stretched" etc; do not accept squashed, compressed etc.
	iii	scale facto $\frac{1}{2}$	B1	be e.g. scale factor 2/2 parallel to <i>x</i> -axis Examiner's Comments Mathematical description of transformations also continues to prove a challenge. Although around four in five candidates were successful in choosing the word stretch (many of those who did not used "squash" or similar colloquialisms), only about half of these could correctly describe its scale factor and direction. Many used incorrect language with references to "in/along/by/on/across the <i>y</i> axis", rather than "parallel to the <i>y</i> axis".
		Total	6	
4	i	(2x-3)(x+1) = 0	M1	Correct method to find roots - see appendix 1
	i	$x = \frac{3}{2}, x = -1$	A1	Correct roots



		Graph Sketching and Transformations. Most candidates used their answer to part (i) and chose the correct outside region, athough choosing the inside region was a frequently seen error. The notation used to describe the region was usually correct; incorrect language such as joining the two sections with the word 'and' lost the accuracy mark.
iii $b^2 - 4ac = 1^2 - 4 \times 2 \times -(3 + k)$	M1	Rearrangement and use of $b^2 - 4ac < 0$, must involve 3 and k in constant term (not 3k)
iii 25 + 8 <i>k</i> < 0	A1	p + 8k < 0 oe found, any constant p . p need not be simplified
$k < -\frac{25}{8}$	A1	Correct final answer Examiner's Comments This proved demanding for many candidates. Although some secured all three marks, many earned no credit as they either put the discriminant equal to zero or, as was frequently seen, to <i>k</i> , making no attempt to rearrange the given equation. Accuracy marks were often lost as candidates failed to deal with the minus signs both in the discriminant and in the expression for <i>c</i> . A few candidates found the turning point of their graph either by differentiation or by completing the square but these approaches were far less common.
Total	9	



				Graph Sketching and Transformations
				Examiner's Comments Most candidates knew that this was a stretch and used the correct word, although incorrect descriptions such as 'squash', 'squish' and 'enlargement' were seen again. The scale factor proved more difficult with many erroneously giving 3. There has been an improvement in describing the direction, with 'parallel to the <i>x</i> -axis'(or <i>y</i> -axis in this case as either is appropriate for this graph) often seen, although some candidates still use incorrect language such as 'along the axis.'
		Total	6	
6	i	(2, 7)	B1	Examiner's Comments This simple translation was recognised and the vast majority of candidates secured the mark.
	ii	(1, 5)	B1	Examiner's Comments This stretch proved much more demanding with only just over half of candidates obtaining the correct coordinate. Although $(4, 10)$ was the modal incorrect answer, there were many other common errors involving halving/doubling either of, or both, the x or y values
	iii	Translation	B1	Translation
	iii	– 4 units parallel to the <i>x</i> axis	B1	Correct description e.g. correct vector (not as a coordinate), "4 units to the left" Do not allow second B1 after incorrect type of transformation e.g. stretch/rotation etc. but allow after shift/move etc. Examiner's Comments Compared to previous sessions, candidates' attempts to describe a translation were better. The use of incorrect athematical language, e.g. using "shift" or "move" and/or expressions like "in/along/on the <i>x</i> axis" were common. Several candidates erroneously stated " – 4 units in the negative <i>x</i> direction"; other errors included "translated by scale factor 4" and translation parallel to the <i>y</i> axis.
		Total	4	



				Graph Sketching and Transformations
	iii	Stretch	B1	Must use the word "stretch"
	Ш	Scale factor one-half parallel to the <i>y</i> -axis	B1	Must have "factor" or "scale factor". For "parallel to the y axis" allow "vertically", "in the y direction".
		Total	7	
8	а	Coordinates of vertices seen at (0, 0), (-2, -2) and (2, 1)	B1(AO1.1)	Vertices must be clearly shown
			[1]	
			M1(AO1.1)	
	b	Coordinates of vertices seen at (2, 1), (3, 2) and $(1, -1)$		Clear attempt to translate graph to the right and to translate it vertically upwards
			A1(AO1.1)	All vertices correct
			[2]	
		Total	3	
		2	B1(AO1.1)	
9	а	Sketch of $y = \frac{3}{x^2}$		Must be in both quadrants, with axes intended as asymptotes

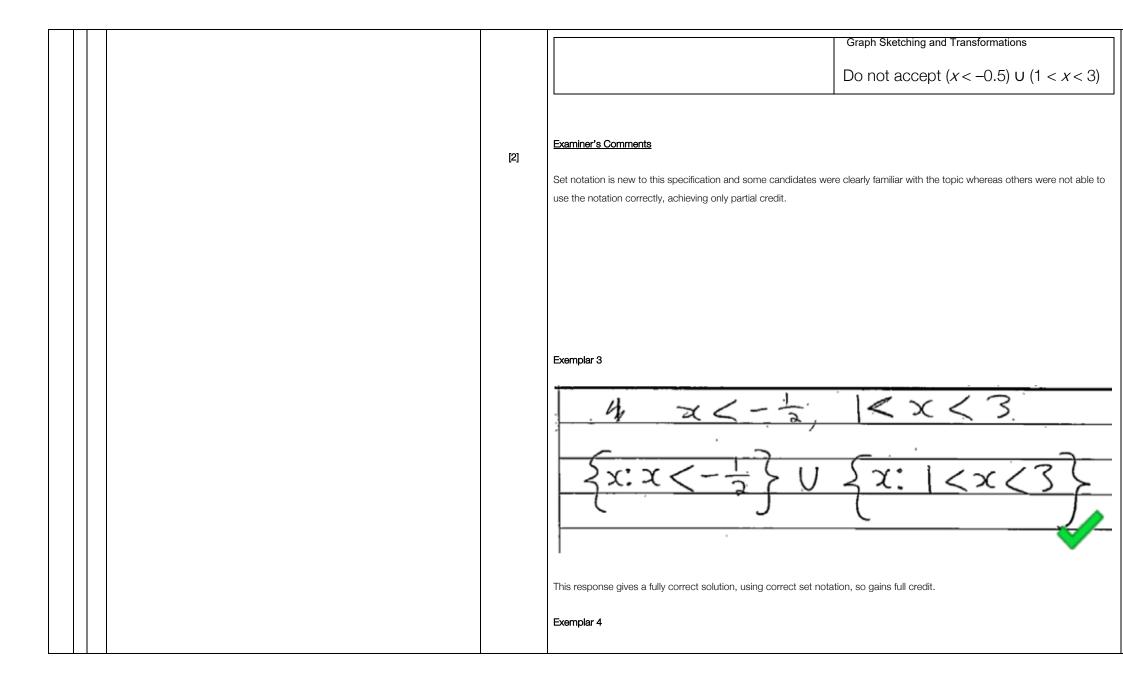
	Sketch of $y = x^2 - 2$ Intercepts of (0, -2), ($\sqrt{2}$, 0), (- $\sqrt{2}$, 0)	M1(AO1.1) A1(AO1.1) [3]	Positive quadratic, symmetrical in <i>y</i> -axis All 3 intercepts correct	Graph Sketching and Transformations Allow just eg $\sqrt{2}$ if marked on relevant axis
	DR $3 = x^4 - 2x^2$ $(x^2 - 3)(x^2 + 1) = 0$	M1(AO3.1a) M1(AO1.1a) M1(AO1.1a)	Equate and remove fractions Attempt to solve disguised quadratic Attempt to find <i>x</i> from their roots	Could use substitution
b	$x^2 = 3$ $x = \pm \sqrt{3}$ points are ($\sqrt{3}$, 1) and ($-\sqrt{3}$, 1) $x^2 = -1$ has no solutions as $x^2 > 0$	A1(AO1.1) A1(AO2.2a) B1(AO2.3) [6]	Correct <i>x</i> values Both correct coordinates Justify no roots for $x^2 = -1$	At least one value of <i>x</i> Allow A1 for one correct (<i>x</i> , <i>y</i>) coordinate A0 if extra roots, or if decimals
	Total	9		

10	а	-4	B1(AO 1.1) [1]	Graph Sketching and Transformations
	b	x, (x + 2), (x - 1)	B1(AO 1.1) [1]	
		y = ax(x-1)(x+2)	M1(AO 3.1a)	
	с	Subst (-1, -4) or from (a) $-4 = a(-1)(-2)(+1) \Rightarrow a = -2$	M1(AO 1.1) A1f(AO 2.2a) [3]	ft their (a) and (b)
	d	y = -2x(x-1)(x+2) $y = -2x^2 - 2x^2 + 4x$ or $b = -2$, $c = 4$, $d = 0$	B1ft (AO 1.1) [1]	ft their (b)
		Total	6	

		<i>y</i>	B1(AO1.1)		Graph Sketching and Transformations
			B1(AO1.1)	$y = (x - 1)^2$ drawn correctly	x-axis must be a tangent to the curve
			B1(AO1.1)	3y = 4x drawn correctly	Line must pass through the origin
11			B1(AO1.1)	y - x = 1 drawn correctly	Positive gradient and y-intercept
		-1 1 x		Correct identification of region (dependent on previous B marks); condone identification via shading so long as there is no ambiguity about the intended region	Note that both lines and curve must meet at the same point for this final mark to be awarded (ignore labelling on axes)
			[4]		·
		Total	4		
12	а	$f(x) = (x-3)(2x^2 - x - 1)$	M1 (AO 2.2a)	Attempt complete division by $(x - 3)$	Must be complete attempt Division – must be subtracting on each line (allow one error) Coefficient matching – valid attempt at all 3 coefficients Inspection – must give three correct terms on expansion Synthetic division – allow one error Could be seen in division Cannot be implied by $A = 2$ etc Must be as product of all 3 linear factors Correct answer gets full marks, but an incorrect factorisation such as (<i>x</i> –

		(AO 1.1)	Obtain correct quotient	Graph Sketching and Transformations 3) $(x + \frac{1}{2})(x - 1)$ is M0 unless method is seen
	f(x) = (x - 3)(2x + 1)(x - 1)	A1 (AO 1.1)	Obtain fully factorised f(x)	
		[3]		
			Examiner's Comments	
			method, which tended to be equally successful. They could then $f(x)$ to be given in fully factorised form, so candidates were expect	ted to give their final answer as the product of three factors. A empted to work backwards but no correct solutions were seen as
		B1		Ignore any intercepts for this mark
		(AO 1.2)	Three roots and two stationary points	ft their three factors Could be given as coordinates, or just
	Sketch of positive cubic	B1 (AO 2.2a)	All intercepts correctly indicated	values marked on relevant axes BOD if coordinates transposed as long as marked on correct axis
b				
	(-0.5, 0), (1, 0), (3, 0), (0, 3)		Examiner's Comments	
		[2]	Most candidates were able to provide a sketch of a positive cubic candidates marked the points on the axes first resulting in a disto Candidates were asked for the coordinates of any points of inters on the <i>y</i> -axis.	orted graph as they then tried to fit the cubic through the points.

			Whilst this question asked for the coordinates, it if not explicitly requested.	Graph Sketching and Transformations t is good practice to always provide them on a sketch graph even
			Guidance to offer for future teaching and learning	g practice
	M1 (AO 2.2a)	Identify one set of values	ft their cubic roots in (b) , even if not 3 real, distinct, roots Allow notation using just inequalities Allow interval notation eg $(-\infty, -0.5)$ and/or (1, 3) If both sets of values given then ignore linking sign for this mark ft their cubic roots in (b) , as long as 3	
c	{ <i>x</i> : <i>x</i> < -0.5} U { <i>x</i> :1 < <i>x</i> < 3}	A1ft (AO 2.5)		real, distinct, roots Each set should have the correct structure ie $\{x: \}$ with the sets linked by U Allow equivs
			Fully correct solution in set notation	eg { $x: x < -0.5$ } U { $x: x > 1$ } \cap { $x: x$ < 3} eg (- ∞ , -0.5) U (1, 3)



			$\frac{(2 \times +1)(2 \times -1)(2 \times -3)}{f(x) \in (-200, -0.5)}$ This response demonstrates an alternative, but equally, valid use of Exemplar 5 $\frac{f(x)(0)}{2x^3 - 7x^3 + 2x}$ $\frac{f(x)(0)}{x(-1)(x \times 3)}$ This response gains one mark for identifying at least one correct so familiarity with set notation, but the structure is not fully correct so familiarity with set notation, but the structure is not fully correct so	to f set notation and also gains full credit.
d	$y = 2(2x)^3 - 7(2x)^2 + 2(2x) + 3$ = 16x ³ - 28x ² + 4x + 3 OR	M1 (AO 1.2)	Attempt f(2 <i>x</i>) or f(0.5 <i>x</i>)	Condone lack of brackets as long as implied by later work M0 if each term just multiplied or divided by 2
		A1	Obtain correct equation	Must have $y = \dots$ Condone f(2x) =, or f(x) =

		y = (2x - 3)(4x + 1)(2x - 1)	(AO 1.1) [2]	Graph Sketching and Transformations Accept unsimplified equiv ISW an incorrect attempt to expand Examiner's Comments Most candidates were able to replace x with $2x$ to obtain a correct equation although a few gave it as an expression, omitting the 'y =' at the start. The best solutions made effective use of brackets to show x being replaced with $2x$, and then expanded them. Other candidates simply wrote down an equation; if this was wrong then there was no evidence to justify that they had been attempting the correct method, but with an error on substitution, as opposed to attempting to double each term.
		Total	9	
13	a	Tails up parabolaVertex (2, -3) $(\pm 2mm)$ cutting x-axis at (0, 0) and (4, 0) $(\pm 2mm)$	B1 (AO 1.1) B1 (AO 1.1) [2]	(scale: 1 cm = 1 unit)
	b	Tails up parabolavertex $(4, -2)$ $(\pm 2mm)$ cutting x-axis at $(0, 0)$ and $(8, 0)$ $(\pm 2mm)$	B1 (AO 1.1) B1 (AO 1.1) [2]	(scale: 1 cm = 1 unit)
		Total	4	

14	а	Curve touching <i>x</i> -axis once and cutting it once Roughly correct "positive" cubic shape	B1 (AO 1.2) B1 (AO 1.2) [2]	Graph Sketching and Transformations with no other implied meeting points with <i>x</i> -axis dep on 1st B1
	b	$(x - 2)^{2}(x - 3)$ $(x - 2)(x - 3)^{2}$ $x^{3} - 7x^{2} + 16x - 12 & x^{3} - 8x^{2} + 21x - 18$	M1 (AO 3.1a) M1 (AO 1.1) A1 (AO 1.1) [3]	Both
		Total	5	