1.   i.   Express $125\sqrt{5}$ in the form 5^k.

[2]

      ii.   Simplify $10 + 7\sqrt{5} + \dfrac{38}{1 - 2\sqrt{5}}$, giving your answer in the form $a + b\sqrt{5}$.

[3]

2.   i.   Evaluate $(0.2)^{-2}$.

[2]

      ii.   Simplify $\left(16a^{12}\right)^{\frac{3}{4}}$.

[3]

3.   i.   Expand and simplify $\left(7 - 2\sqrt{3}\right)^2$.

[3]

      ii.   Express $\dfrac{20\sqrt{6}}{\sqrt{50}}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible.

[2]

4.   Find the value of each of the following.

      i. $\left(\dfrac{5}{3}\right)^{-2}$

[2]

      ii. $81^{\frac{3}{4}}$

[2]

5.   Simplify $\dfrac{\left(4x^5y\right)^3}{\left(2xy^2\right)\times\left(8x^{10}y^4\right)}$.

[3]

6.   i.   Express $\sqrt{48} + \sqrt{75}$ in the form $a\sqrt{b}$, where a and b are integers.

[2]

     ii.   Simplify $\dfrac{7 + 2\sqrt{5}}{7 + \sqrt{5}}$, expressing your answer in the form $\dfrac{a + b\sqrt{5}}{c}$, where a, b and c are integers.

[3]

7.   i.   Evaluate $\left(\dfrac{1}{27}\right)^{\frac{2}{3}}$.

[2]

     ii.   Simplify $\dfrac{(4a^2c)^3}{32a^4c^7}$.

[3]

8.   i.   Express $\sqrt{50} + 3\sqrt{8}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible.

[2]

     ii.   Express $\dfrac{5 + 2\sqrt{3}}{4 - \sqrt{3}}$ in the form $(5c^2d)^3 \times \dfrac{2c^4}{d^5}$, where c and d are integers.

[3]

9.   Find the value of each of the following.

     i. 3°

[1]

     ii. $9^{\frac{3}{2}}$

[2]

     iii.

         $\left(\dfrac{4}{5}\right)^{-2}$

[2]

10. Simplify $\dfrac{\left(2x^2y\right)^3 \times 4x^3y^5}{2xy^{10}}$. [2]

11. (See Insert for Specimen 64003.) Show that the two values of b given on line 34 are equivalent. [3]

12. (See Insert for Specimen 64003.) On a unit circle, the inscribed regular polygon with 12 edges gives a lower bound for π, and the escribed regular polygon with 12 edges gives an upper bound for π. Calculate the values of these bounds for π, giving your answers:

    (A) in surd form

    (B) correct to 2 decimal places. [3]

13. (i) Find the value of $\left(1\frac{7}{9}\right)^{-\frac{1}{2}}$. [3]

    (ii) Simplify $\dfrac{\left(6x^5y^2\right)^3}{18y^{10}}$. [2]

14. (i) Simplify $\dfrac{5-2\sqrt{7}}{3+\sqrt{7}}$, giving your answer in the form $\dfrac{a-b\sqrt{7}}{c}$, where a, b and c are integers. [3]

    (ii) Simplify $\dfrac{12}{\sqrt{2}} + \sqrt{98}$, giving your answer in the form $d\sqrt{2}$ where d is an integer. [2]

15. Show that $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ can be written in the form $a+\sqrt{b}$ where a and b are integers. [3]

16.   Show that $\sqrt{27} + \sqrt{192} = a\sqrt{b}$, where a and b are prime numbers to be determined.   [2]

17.   (See Insert for Practice2 64003.) Let a_1 and a_2 be the two values of y referred to in line 38 with $a_1^3 = \dfrac{5 + \sqrt{29}}{2}$ and $a_2^3 = \dfrac{5 - \sqrt{29}}{2}$.

(a)   Show that $a_2^3 = -\dfrac{1}{a_1^3}$.   [1]

(b)   Deduce that $a_1 - \dfrac{1}{a_1} = a_2 - \dfrac{1}{a_2}$ as stated in line 38.   [2]

18.   Write $\dfrac{8}{3 - \sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers to be found.   [2]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1	i		$5^{3.5}$ oe or $k = 7/2$ oe	2	M1 for $125 = 5^3$ or $\sqrt{5} = 5^{\frac{1}{2}}$ soi **Examiner's Comments** This question was found to be difficult by many candidates. In the first part, although the correct answer was seen fairly frequently, a significant number of candidates, having correctly shown 125 and $\sqrt{5}$ to be 5^3 and $5^{\frac{1}{2}}$ respectively, then multiplied the indices to give an answer of $5^{\frac{3}{2}}$. Others found one of the indices correctly, but not the other. Some candidates treated it as though the square root applied to 125 as well.	M0 for just answer of 5^3 with no reference to 125
	ii		attempting to multiply numerator and denominator of fraction by $1 + 2\sqrt{5}$	M1		some cands are incorporating the $10 + 7\sqrt{5}$ into the fraction. The M1s are available even if this is done wrongly or if $10 + 7\sqrt{5}$ is also multiplied by $1 + 2\sqrt{5}$
	ii		denominator = −19 soi	M1	must be obtained correctly, but independent of first M1 **Examiner's Comments** Few correct answers were seen in the second part. Being in a different format from usual, many candidates did not know how to cope with the initial $10 + 7\sqrt{5}$. Many multiplied the '$10 + 7\sqrt{5}$' term by $2 + \sqrt{5}$, sometimes losing the denominator altogether. Those who knew they should rationalise the denominator of the fraction often made	e.g. M1 for denominator of 19 with a minus sign in front of whole expression or with attempt to change signs in numerator

	i i				errors in multiplying the denominator, with 9, −9 or 19 often seen (19 often following the correct 1 − 20). Some who correctly reached this point then only divided the first term in the numerator by −19.
	i i	$8 + 3\sqrt{5}$	A1		

		Total	5		

2	i	25	2	M1 for $\left(\dfrac{10}{2}\right)^2$ or $\left(\dfrac{1}{0.2}\right)^2$ oe soi or for $\dfrac{1}{0.04}$ oe **Examiner's Comments** In evaluating $(0.2)^{-2}$, many stopped after evaluating $\dfrac{1}{0.2^2}$ as $\dfrac{1}{0.04}$ (or, sadly often, as $\dfrac{1}{0.4}$). Those who converted to fractions first were more successful in reaching 25.	ie M1 for one of the two powers used correctly M0 for just $\dfrac{1}{0.4}$ with no other working
	i i	$8a^9$	3	B2 for 8 or M1 for $16^{\frac{1}{4}} = 2$ soi and B1 for a^9 **Examiner's Comments** In the second part, the majority found the power of a	ignore ± eg M1 for 2^3; M0 for just 2

					correctly, but the $16^{\frac{3}{4}}$ proved more challenging. A surprising number did $\frac{3}{4} \times 16 = 12$ to obtain $12a^b$.	
		Total		5		
3	i	$61 - 28\sqrt{3}$		3	B2 for 61 or B1 for 49 + 12 found in expansion (may be in a grid) and B1 for $-28\sqrt{3}$ if B0, allow M1 for at least three terms correct in $49 - 14\sqrt{3} - 14\sqrt{3} + 12$ the correct answer obtained then spoilt earns SC2 only	
	ii	$4\sqrt{3}$		2	M1 for $\sqrt{50} = 5\sqrt{2}$ or $\sqrt{300} = 10\sqrt{3}$ or $20\sqrt{300} = 200\sqrt{3}$ or $\sqrt{48} = 2\sqrt{12}$ seen **Examiner's Comments** Most candidates gained at least one mark in the first part for $-28\sqrt{3}$. Those who failed to reach the correct final answer often incorrectly expanded the last terms of the brackets, obtaining $\pm 4\sqrt{3}$, 6 or 12 rather than +12. For most candidates the second part was more challenging than the first part. Errors tended to be introduced when rationalising the denominator, with many choosing to multiply by $\sqrt{50}$ or $-5\sqrt{2}$. Those that did rationalise were then unsure how to simplify the numerator, often obtaining large roots which they were unable to simplify accurately. Those that had the most success in this question expressed the $\sqrt{50}$ in the	

				denominator as $5\sqrt{2}$ and were then comfortable dividing surds and cancelling fractions.	
		Total	5		
4	i	$\dfrac{9}{25}$ or 0.36 isw	2	M1 for numerator or denominator correct or for squaring correctly or for inverting correctly **Examiner's Comments** The first part was very well answered on the whole, with the majority scoring full marks. Most inverted first and attempted to square second.	$\dfrac{1}{\left(\frac{25}{9}\right)}$ or $\left(\dfrac{25}{9}\right)^{-1}$ or $\dfrac{25}{9}$ M1 for eg $\left(\dfrac{25}{9}\right)$ or for $\left(\dfrac{3}{5}\right)^{2}$ or $\dfrac{3}{5}$ M0 for just $\dfrac{1}{\left(\frac{5}{3}\right)^{2}}$
	ii	27	2	M1 for $81^{\frac{1}{4}}=3$ soi **Examiner's Comments** Again a high proportion of correct answers was seen. Among the common errors were responses from candidates who either thought that $81^{\frac{1}{4}}=\sqrt{3}$ or that they needed to find $\left(\sqrt[3]{81}\right)^{4}$. Regrettably, the error $3^{3}=9$ was not rare.	eg M1 for 3^{3} M0 for $81^{3}=531441$ (true but not helpful)
		Total	4		
5		$4x^{4}y^{-3}$ or $\dfrac{4x^{4}}{y^{3}}$ as final answer	3	B1 each 'term'; or M1 for numerator $=64x^{15}y^{3}$ and M1 for denominator $=16x^{11}y^{6}$ **Examiner's Comments**	B0 if obtained fortuitously mark B scheme or M scheme to advantage of candidate, but not a mixture of both schemes

				Whereas the numerical work with indices is good, the algebraic work is definitely weaker – as was seen in this question. There were still a pleasing number of correct solutions, but quite a few dropped a mark or two here – often for not cubing the 4 in the numerator – and/or for having x^{10} in the denominator.		
		Total	**3**			
6	i	$9\sqrt{3}$ www oe as final answer	2	M1 for $\sqrt{48} = 4\sqrt{3}$ **or** $\sqrt{75} = 5\sqrt{3}$ soi		
	ii	$\dfrac{39 + 7\sqrt{5}}{44}$ www as final answer	3	M1 for attempt to multiply numerator and denominator by $7 - \sqrt{5}$ B1 for each of numerator and denominator correct (must be simplified) **Examiner's Comments** Simplifying and adding the surds was done correctly by a high proportion of candidates. Most candidates knew how to rationalise a denominator for the second part but mistakes in implementation were common, the denominator being more frequently correct than the numerator.	condone $\dfrac{39}{44} + \dfrac{7\sqrt{5}}{44}$ for 3 marks eg M0B1 if denominator correctly rationalised to 44 but numerator not multiplied	
		Total	**5**			
7	i	$\dfrac{1}{9}$	2	isw conversion to decimal		
	ii			M1 for 9 or for 3^{-2} or for $\dfrac{1}{3}$	ie M1 for evidence of $\left(\sqrt[3]{27}\right)^2$ or $1/\left(\sqrt[3]{27}\right)$ found correctly	

	i				Except M0 for 9 from 27/3 or $\sqrt[3]{27}$
	i i	$2a^2c^{-4}$ or $\dfrac{2a^2}{c^4}$ as final answer	3	B1 for each element; must be multiplied if B0, allow SC1 for $64a^6c^3$ obtained from numerator or for all elements correct but added **Examiner's Comments** Most candidates knew what to do and handled the indices well. Errors such as $\sqrt[3]{27} = 9$ were seen occasionally in the first part. In the second, the most frequent errors came from failing to cube the 4 or the a^2 correctly.	
		Total	**5**		
8	i	$11\sqrt{2}$	2	**M1** for $\left[\sqrt{50} = \right]5\sqrt{2}$ or $\left[3\sqrt{8} = \right]6\sqrt{2}$	
	i i	attempting to multiply numerator and denominator of fraction by $4+\sqrt{3}$	M1		
	i i	$2+\sqrt{3}$ or $2+1\sqrt{3}$ or $c=2$ and $d=1$	A2	or **B1** for denominator = 13 soi or numerator $26+13\sqrt{3}$	
	i i	**or**			
	i i	cross-multiplying by $4-\sqrt{3}$ and forming a pair of simultaneous equations in c and d, with at most one error	M1		
	i i	$c=2$ and $d=1$	A2	**A1** for one correct	**Examiner's Comments**

The first part was nearly always correct with the vast majority scoring at least one mark for correctly stating that $\sqrt{50} = 5\sqrt{2}$. Some candidates had difficulty with $3\sqrt{8}$ and a number incorrectly gave this as $5\sqrt{2}$ which typically came from the incorrect working of

$$3\sqrt{8} = 3(2\sqrt{2}) = (3 + 2)\sqrt{2}$$

In the second part, most candidates clearly knew how to rationalise the denominator with nearly all correctly indicating the need to multiply both numerator and denominator by $(4 + \sqrt{3})$; only a small minority incorrectly multiplied by either $(4 - \sqrt{3})$ or $\sqrt{3}$. Nearly all correctly achieved a value of 13 for the denominator but some had issues with either expanding or simplifying the numerator. A significant minority who achieved

$$\frac{26 + 13\sqrt{3}}{13}$$

did not simplify this correctly with $2 + 13\sqrt{3}$ being a common incorrect answer.

		Total		5	
9	i	1		1	
	ii	27		2	condone ±27;
	iii				**B1** for [±]3^3 or $\sqrt{729}$

	i i i			or for $\left[9^{\frac{1}{2}} = \right]3$ or ± 3 soi	Surds and Indices
	i i i	$\dfrac{25}{16}$ or $1\dfrac{9}{16}$ isw		2	**B1** for $\dfrac{5}{4}$ or $\dfrac{1}{\frac{16}{25}}$ or $\dfrac{16}{25}$ oe	B0 for 1.5625 without fractions seen; if this is found, check for possible use of calculator throughout the paper **Examiner's Comments** Nearly all candidates interpreted the zero power correctly in the first part. Most interpreted the fractional power correctly in the second part, although a number of candidates began by attempting to cube 9, which usually ran into difficulties as candidates did not have the assistance of their calculators; they had similar issues when attempting to find the square root 729. The most common error was candidates believing that 3 cubed was 9. Coping with the fraction and negative power in the last part was usually done correctly; notable errors were inverting the fraction whilst losing the power altogether or losing the power from either the numerator or denominator.
		Total		5		
1 0		$16x^8y^{-2}$ OR $\dfrac{16x^8}{y^2}$	B2(AO1.1) (AO1.1) [2]	**B1** for two elements correct out of coefficient, power of x, power of y as part of a product	
		Total		2		

1 1		$\left(\dfrac{\sqrt{6}-\sqrt{2}}{2}\right)^2 = \dfrac{8-2\sqrt{12}}{4}$ $= \dfrac{8-4\sqrt{3}}{4} = 2-\sqrt{3}$ $\dfrac{\sqrt{6}-\sqrt{2}}{2}$ s positive so it is equal to $\sqrt{2-\sqrt{3}}$	M1(AO3. 1a) A1(AO1.1) E1(AO2.1) [3]	Attempt to square Answer in exact form Completion of argument to show the two values are equal		
		Total	3			
1 2		*(A)* Lower bound: $3\left(\sqrt{6}-\sqrt{2}\right)$ Upper bound: $24-12\sqrt{3}$ *(B)* = 3.11 and 3.22	B1(AO1. 1) B1(AO1. 1) B1(AO1. 1) [3]	Half perimeter (from text) Both as decimals		
		Total	3			

13	i	$\dfrac{3}{4}$ oe	3	**B2** for $\dfrac{3}{a}$ or $\dfrac{c}{4}$ or $\pm\dfrac{3}{4}$ or **M2** for $\left(\dfrac{4}{3}\right)^{-1}$ or $\left(\dfrac{9}{16}\right)^{\frac{1}{2}}$ or $\sqrt{\dfrac{9}{16}}$ or **M1** for $\dfrac{1}{\left(1\frac{7}{9}\right)^{\frac{1}{2}}}$ or $\left(\dfrac{16}{9}\right)^{-\frac{1}{2}}$ or $\dfrac{4}{3}$	isw wrong conversion to decimals
			[3]	**Examiner's Comments** Not many candidates dropped marks in the first part. Those who did usually lost out due to their inability to convert a mixed number into an improper fraction, preventing them from scoring any of the marks. Candidates scoring 0 often seemed to have little idea with indices, but these were a minority. Some candidates reached $\dfrac{1}{\left(1\frac{7}{9}\right)}$ gaining a mark for this, but then did not know how to proceed with their triple-decker fraction.	
	ii	$12x^{15}y^{-4}$ or $\dfrac{12x^{15}}{y^4}$	2	**B1** for two elements correct if B0, allow **M1** for expanded numerator	

			[2]	$= 6^3 x^{15} y^6$ or $216 x^{15} y^6$	

Examiner's Comments

In the second part, the vast majority of candidates coped well, the main mistakes were usually due to the misapplication of the rules of indices, adding when the powers should be multiplied. What was concerning was the minority of candidates who could not multiply or divide the numerical values forming the coefficient.

	Total		5	

14	i	$\dfrac{29 - 11\sqrt{7}}{2}$ sw	3	**B1** for each element; condone written as two separate fractions if 0, allow **M1** for three terms correct in $15 - 5\sqrt{7} - 6\sqrt{7} + 14$ or for attempt to multiply both denominator and numerator by $3 - \sqrt{7}$	
			[3]	**Examiner's Comments**	
				In the first part, the vast majority of candidates understood the need to multiply	

the numerator and denominator by $\left(3-\sqrt{7}\right)$, however a few tried to multiply both parts of the fraction by $\sqrt{7}$, or by $\left(3+\sqrt{7}\right)$, or to 'cancel' the $\sqrt{7}$ in the numerator and denominator. The most common error was in determining $-2\sqrt{7} \times \sqrt{7}$ which commonly retained a multiple of $\sqrt{7}$.
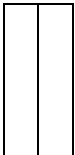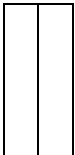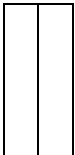
	i i	$13\sqrt{2}$	2 [2]	**M1** for $\dfrac{12}{\sqrt{2}} = 6\sqrt{2}$ soi or for $\sqrt{98} = 7\sqrt{2}$ soi or for $\dfrac{12+14}{\sqrt{2}}$ oe **Examiner's Comments** In the second part, most candidates could simplify $\sqrt{98}$ to $7\sqrt{2}$ (so scoring at least one mark) but many had difficulties with $\dfrac{12}{\sqrt{2}}$ with some multiplying the $\sqrt{2}$ by the $\sqrt{98}$ or leaving their answer as $\dfrac{26}{\sqrt{2}}$.	
		Total	5		
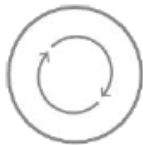
15		multiply by $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ $\dfrac{(\sqrt{5})^2 + 2\sqrt{3}\sqrt{5} + (\sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2}$ oe $4 + \sqrt{15}$	M1(AO1.1b) M1(AO1.1b) A1(AO1.1b) [3]	Attempt to expand brackets in numerator or denominator	Allow one slip, eg sign error	
		Total	3			
16		$3\sqrt{3}$ or $8\sqrt{3}$ seen $[3\sqrt{3} + 8\sqrt{3}] = 11\sqrt{3}$	M1 (AO 1.1) A1 (AO 2.1) [2]	**Examiner's Comments** This is a show that question; simply obtaining the values for a and b from the calculator, without any written justification, would not gain full credit.		
		Total	2			
17	a	$-\dfrac{1}{a_1^3} = -\dfrac{2}{5+\sqrt{29}} = -\dfrac{2(5-\sqrt{29})}{25-29} = \dfrac{5-\sqrt{29}}{2} = a_2^3$	B1(AO2.1) B1 [1]	AG Convincingly shown		

		Alternative solution $$a_1^3 a_2^3 = \frac{5+\sqrt{29}}{2} \times \frac{5-\sqrt{29}}{2} = \frac{25-29}{4} = -1 \Rightarrow a_2^3 = -\frac{1}{a_1^3}$$		AG Convincingly shown		
	b	$$a_2^3 = -\frac{1}{a_1^3} \Rightarrow a_2 = -\frac{1}{a_1}$$ $$a_2 - \frac{1}{a_2} = -\frac{1}{a_1} + a_1 = a_1 - \frac{1}{a_1}$$	B1(AO2. 2a) B1(AO2. 1) [2]	AG Convincing completion		
		Total	3			
18		$$\frac{8}{(3-\sqrt{5})} \times \frac{(3+\sqrt{5})}{(3+\sqrt{5})} = 6 + 2\sqrt{5}$$	M1 (AO1.1a) A1 (AO1.1b) [2]	Attempt to rationalize the denominator Must be in correct notation	Allow full credit for correct answer	

Examiner's Comments

This was well answered, although some arithmetic mistakes were seen in the simplifying.

Check numerical answers with a calculator.

		Total	2	