Solution of Equations

- 1. Rearrange the equation 5c + 9t = a(2c + t) to make c the subject.
- 2. A circle has diameter *d*, circumference *C*, and area *A*. Starting with the standard formulae for a circle, show that Cd = kA, finding the numerical value of *k*.

[3]

[4]

3. Rearrange the following formula to make r the subject, where r > 0.

$$V = \frac{1}{3}\pi r^{2}(a+b)$$
[3]

4. Make *a* the subject of 3(a + 4) = ac + 5f.

5.
You are given that
$$a = \frac{3c+2a}{2c-5}$$
. Express *a* in terms of *c*.

6. Find the coordinates of the point of intersection of the lines 2x + 3y = 12 and y = 7 - 3x.

[4]

[4]

[4]

- 7. Find the coordinates of the point of intersection of the lines 2x + 5y = 5 and x 2y = 4. [4]
- 8. $r = \sqrt{\frac{V}{a+b}}$ [4] Rearrange the formula
- 9. Find the set of values of k for which the equation $2x^2 + kx + 8 = 0$ has distinct real roots. [3]
- ^{10.} In this question you must show detailed reasoning.

Find the set of values of x for which the line y = 5x - 6 lies below the curve $y = x^2$.

[4]

END OF QUESTION paper

Mark scheme

Qu	Question		Answer/Indicative content	Marks	Part marks and guidance		
1	1		5c + 9t = 2ac + at	M1	for correct expansion of brackets		
			5 <i>c</i> – 2 <i>ac</i> = 9 <i>t</i> oe	M1	for correct collection of terms, ft eg after M0 for $5c + 9t = 2ac + t$ allow this M1 for $5c - 2ac = -8t$ oe	for each M, ft previous errors if their eqn is of similar difficulty;	
			c(5-2a) = at - 9t oe	M1	for correctly factorising, ft; must be $c \times a$ two-term factor	may be earned before <i>t</i> terms collected	
					for correct division, ft their two-term factor		
			$\begin{bmatrix} c = \end{bmatrix} \frac{at - 9t}{5 - 2a} \text{ or } \frac{t(a - 9)}{5 - 2a}$ as final answer	M1	Examiner's Comments A good number were successful in the rearrangement, but some very poor work was also seen, revealing fundamental misconceptions about algebraic manipulation. Common errors included dividing some terms by <i>a</i> but not others, and confusion of division and subtraction.	treat as MR if <i>t</i> is the subject, with a penalty of 1 mark from those gained, marking similarly	
			Total	4			
2			obtaining a correct relationship in any 3 of <i>C</i> , <i>d</i> , <i>r</i> and <i>A</i>	M2	may substitute into given relationship;	eg M2 for $Cd = 4 \pi r^2$ or $\pi d^2 = k \pi r^2$ seen/obtained	
			or obtaining a correct relationship in <i>k</i> and no more than 2 other variables		or M1 for at least two of $A = \pi r^2$, $C = \pi d$, $C = 2\pi r$, $d = 2r$ or $r = \frac{d}{2}$ seen or used	condone eg Area = πl^2 ; $A = \pi \left(\frac{d}{2}\right)^2$ to imply $r = \frac{d}{2}$ and so earn	
			convincing argument leading to $k = 4$	A1	 must be from general argument, not just substituting values for <i>r</i> or <i>d</i>; may start from given relationship and derive <i>k</i> = 4 Examiner's Comments Many candidates did not know where to start. Having picked up on the keyword 'circle' many just wrote down the general equation of a circle and nothing else, or offered no response at all. For some 	M1, if M2 not earned eg M1only for eg $A = \pi/2$ and $C = \pi d$ and so $k = 4$ with no further evidence	

			l		Solution of Equations
				candidates, lack of real understanding of	
				algebra meant that when confronted with a	
				different style of question they were unable to	
				find an appropriate strategy. Some students	
				did not remember the required circle	
				formulae, eg $A = 2 \pi$ was not uncommon.	
				Those starting with the given form $Cd = kA$	
				and putting in the correct formulae were often	
				most successful. The squaring of	
				d	
				$\overline{2}$ formulae were often most successful. d^2	
				The squaring of 2 leading to $k = 2$.	
				Many had several attempts at this question	
				and solutions were often scrappily presented	
				and difficult to follow.	
		Total	3		
				M1 for dealing correctly with 3	
		$r = \sqrt{\frac{3V}{\pi(a+b)}}_{\text{oe www as}}$ final answer	3	and M1 for dealing correctly with $\pi(a + b)$, ft and M1 for correctly finding square root, ft <i>their</i> ' $i^2 = i$ '; square root symbol must extend below the fraction line Examiner's Comments There were many good answers in rearranging the formula. Most candidates managed at least one mark; some triple- decker fractions or the use of \div signs were seen. The π and the $(a + b)$ sometimes became separated. The radius was sometimes considered to be \pm , and the > sign was used on more than one occasion. It was encouraging to see very few penalties incurred due to a poor square root symbol.	M0 if triple-decker fraction, at the stage where it happens, then ft; condone missing bracket at rh end M0 if \pm or $r >$ for M3, final answer must be correct
		Total	3		
		3 <i>a</i> + 12 [= <i>ac</i> + 5 <i>t</i>]	M1	for expanding brackets correctly	annotate this question if partially correct
		3a - ac = 5f - 12 or ft	M1	for collecting <i>a</i> terms on one side, remaining terms on other	ft only if two <i>a</i> terms
		a(3 - c) = 5f - 12 or ft M1		for factorising <i>a</i> terms; may be implied by final answer	ft only if two <i>a</i> terms, needing factorising
			$r = \sqrt{\frac{3V}{\pi(a+b)}}_{\text{oe www as}}$ final answer inal answer $3a + 12 [= ac + 5I]$ $3a - ac = 5f - 12 or ft$	$r = \sqrt{\frac{3V}{\pi(a+b)}}_{\text{oe www as}}$ final answer $r = \sqrt{\frac{3V}{\pi(a+b)}}_{\text{oe www as}}$ 3 final answer 3 3 $3a + 12 [= ac + 5]$ $3a - ac = 5f - 12 \text{ or ft}$ M1	algebra meant that when contronted with a different style of queston they were unable to find an appropriate strategy. Some students did not remamber the required circle formulae, eg $A \ge 1$ twis not uncommon. Those starting with the given form $Cd = AA$ and putting in the correct formulae were often most successful. The squaring of $\frac{d^2}{2}$ formulae were often most successful. The squaring of $\frac{d^2}{2}$ leading to $k = 2$. Mary had soveral attempts at this question and solutions were often most successful. The squaring of $\frac{d^2}{2}$ leading to $k = 2$. Mary had soveral attempts at this question and solutions were often scappily presented and difficult to follow.Image: the squaring of $\frac{d^2}{2}$ leading to $k = 2$. Mary had soveral attempts at this question and solutions were often scappily presented and difficult to follow.Image: the squaring of $\frac{d^2}{2}$ leading to $k = 2$. Mary had soveral attempts at this question and solutions were often scappily presented and difficult to follow.Image: the squaring of $\frac{d^2}{2}$ leading to $k = 2$. Mary had soveral attempts at this question and solutions were often scappily presented and difficult to follow.Image: the squaring of $\frac{d^2}{2}$ leading to $k = 2$. Mary had soveral attempts of the square root, ft $\frac{d^2 + 1}{d^2 + \pi + (\pi + k), ft}$ and M1 for dealing correctly with $\pi(a + k), ft$ and M1 for dealing correctly with $\pi(a + k), ft$ and M1 for dealing correctly with $\pi(a + k), ft$ and M1 for dealing correctly with $\pi(a + k), ft$ and M1 for dealing correctly with $\pi(a + k), ft$ and M1 for dealing correctly with $\pi(a + k), ft$ and M1 for dealing correctly with $\pi(a + k), ft$ and M1 for dealing correctly with $\pi(a + k), ft$ and M1 for constanting the formula. Most candidates managed at kast ore mark; some triple- dealer fract

1 1		1				Solution of Equations
			$[a =] \frac{5f - 12}{3 - c}$ oe or ft as final answer	М1	for division by their two-term factor; for all 4 marks to be earned, work must be fully correct Examiner's Comments Rearranging the formula was usually done well. Those who found this difficult generally attempted to isolate just one a term and hence scored only the first mark. Other errors seen occasionally included sign errors and a final spoiling of the answer by invalidly 'cancelling' 3 into 12.	
			Total	4		
5			<i>a</i> (2 <i>c</i> – 5) = 3 <i>c</i> + 2 <i>a</i> or 2 <i>ac</i> – 5 <i>a</i> = 3 <i>c</i> + 2 <i>a</i>	M1	for multiplying up correctly (may also expand brackets)	annotate this question if partially correct
			<i>a</i> (2 <i>c</i> – 5) – 2 <i>a</i> = 3 <i>c</i> or 2 <i>ac</i> – 7 <i>a</i> = 3 <i>c</i> or ft	M1	for collecting <i>a</i> terms on one side, remaining term[s] on other [need not be simplified]	ft only if two or more <i>a</i> terms,
			<i>a</i> (2 <i>c</i> – 7) = 3 <i>c</i> or ft	M1	for factorising <i>a</i> terms, need not be simplified; may be implied by final answer	ft only if two or more a terms, needing factorising may be earned before 2 nd M1
			$[a=]\frac{3c}{2c-7}$	М1	for division by their two-term factor (accept a 3 term factor that would simplify to 2 terms); for all 4 marks to be earned, work must be	candidates whose final answer expresses <i>c</i> in terms of <i>a</i> : treat as MR after the first common M and mark equivalently, applying MR-1 if they gain further Ms. So that a final answer, correctly obtained, of $[c =] \frac{7a}{2a-3}$ or simplified equivalent earns 3 marks in total Examiner's Comments The majority of the candidates
			or simplified equivalent or ft as final answer		fully correct and simplified and not have a triple-or quadruple-decker answer	were very familiar with the topic of rearranging to make a different variable the subject of a formula, and coped well with this example. Nearly all candidates correctly multiplied by $(2c - 5)$ to give $a(2c - 5) = 3c + 2a$. However it was surprising that a large number of candidates went on to make <i>c</i> rather than <i>a</i> the subject of the formula (albeit the majority did this correctly and scored 3 of the 4 marks available). Where errors occurred it was usually sign errors
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Solution of Equations

I.	1	1			1	Solution of Equations
						from moving terms from one side
						to the other and a small minority
						did not simplify their answers fully,
						giving say an answer of $a = 3c/$
						(2c - 5 - 2). It was pleasing to see
						that the majority of candidates
						correctly factorised their a (or c)
						terms as this has in the past
						caused issues.
			Total	4		
					or multiplication or division to make one pair	
6			substitution to eliminate one variable	M1	of coefficients the same; condone one error	
					in either method	
			simplification to $ax = b$ or $ax - b = 0$ form, or		or appropriate subtraction (addition;	
					or appropriate subtraction / addition;	independent of first M1
			equivalent for y		condone one further error in either method	
						A0 for just rounded decimals or
						for $-9 / -7$ oe
						Examiner's Comments
						This was a good source of marks
						for the majority of candidates, who
						found the demand of solving a
						pair of simultaneous equations
						relatively straightforward, although
						errors in coping with the fractional
						answer to <i>x</i> to find the <i>y</i> -value
						were quite common, as was
						occasionally forgetting to find the
						y-value. Very few candidates
						found the <i>y</i> -value first. Those who
						used substitution and wrote down
			(9/7, 22/7) oe or $x = 9/7$ $y = 22/7$ oe isw	A2	A1 each	2x + 3(7 - 3x) = 12 nearly always
						went on to get the correct answer
						for x - although it was particularly
						disheartening the number of times
						that $7x = 9$ became $x = 7/9$.
						Those who substituted for y and
						had $y = 7 - 3((12 - 3y)/2)$ were
						usually less successful, due to the
						fraction and the number of
						negative terms in the equation.
						Elimination methods were less
						frequently seen and not as
						successful - candidates often did
						not multiply all values by the
						required constant or they added
						or subtracted their pair of
						equations incorrectly.
			Total	4		
				-		

			Solution of Equations
	2(4+2y) + 5y = 5 oe in x or $2x - 4y = 8$ oe	M1	for subst to eliminate one variable; condone one error; or for multn or divn of one or both eqns to get a pair of coeffts the same, condoning one error
7	9y = -3 or 9x = 30 oe	M1	for collecting terms and simplifying; condoning one error ft or for appropriate addn or subtn to eliminate a variable, condoning
	$\left(\frac{30}{9}, -\frac{3}{9}\right)$ oe isw	A2 [4]	an error in one term; if subtracting, condone eg x instead of 0 if no other errors or $x = 30/9$, y = -3/9 oe isw eg $x =$ 10/3, y = -1/3 allow A1 for each coordinate

						Solution of Equations
				Examiner's Comments		
				Candidates coped very well with equation and		
				fraction manipulation. Bo		
				were nearly always earne		
				A variety of methods were		
				substitution of $x = 2y + 4$		
				equation being the most		
				multiplied both equations		
				to use elimination. Where		
				equation was multiplied b		
				errors in subtracting the e	-	
				Rearranging both equation		
				= and then equating th		
				fairly common and, even		
				in fractions, was usually s	-	
				handling the signs when		
				second equation was a s	ource of error. A	
				minority of candidates sto		
				one of the values (usually		
				the coordinates, as reque		
				It should be noted that ve	ery few candidates	
				checked their answers ar	nd it is advisable to	
				do so in questions of this	nature.	
		Total	4			
8		$r^{2} = \frac{V}{a+b}$ $r^{2}(a+b) = V \text{ or } r^{2}a + r^{2}b = V$ $r^{2}b = V - r^{2}a \text{ or } a+b = \frac{V}{r^{2}}$	M1 M1	each M1 is for a correct, constructive step following through correctly from previous step for squaring both sides for multiplying both sides by denominator for this and all	allow candidates to combine two or three stages in one working statement eg award first two	
			M1	subsequent Ms, ft for equiv	Ms for $r^2 (a + b) =$ V seen as	

·		-	1		Solution of Equations
	$b = \frac{V - r^2 a}{r^2}$ or $b = \frac{V}{r^2} - a$		difficulty	first step	
	as final answer	[4]	for getting <i>b</i> term on one side, other terms on other side	3 rd and 4 th M1s may be earned in opposite order, as in second answer for	
			for dividing by coefficient of <i>b</i>	where rhs has two terms in the	
			award 4 marks only if working is fully correct, with at least one interim	numerator, the division line must clearly extend	
			step. Allow SC2 if there is no working, just the correct answer	under both terms	
			Examiner's Comments		
			There were many candid new subject both efficier was rare to find a candid to square both sides stra were a very small minori rails at that point, not co	ntly and accurately. It date who didn't know aight away but there ty who went off the	
			a denominator. A small r solved for <i>a</i> instead of <i>b</i> handful of candidates w a diagonal fraction line ir a horizontal one and this	ninority of candidates . There were a ho insisted on using nstead of	
			missteps when manipula A common error was to $b = \frac{V - r}{r^2}$ answer of $b = V - a$.	take the correct $r^2 a$ and cancel this	
			D = v - a.		

		Total	4	Solution of Equations
9		Discriminant $k^2 - 4 \times 2 \times 8 > 0$ k > 8 or $k < -8$	M1 (AO 1.1a) A1 (AO 1.1b) A1 (AO 1.1b) [3]	May be implied by $k^2 > 64$ oe withoutAllow as part of the quadratic used oeOe
		Total	3	
10		DR $x^2 > 5x - 6$ $x^2 - 5x + 6 > 0$ (x - 2)(x - 3) > 0 x < 2 or x > 3	M1(AO 3.1a) M1(AO 1.1) A1(AO 1.1) A1(AO 2.2a) [4]	Correct factors or values 2, 3 oe, e.g. $\{x : x \\ < 2 \cup x > 3\}$
		Total	4	