1. You are given that $f(x) = x^4 - x^3 + x^2 + 9x - 10$.

   i. Show that $x = 1$ is a root of $f(x) = 0$ and hence express $f(x)$ as a product of a linear factor and a cubic factor.

   [3]

   ii. Hence or otherwise find another root of $f(x) = 0$.

   [2]

   iii. Factorise $f(x)$, showing that it has only two linear factors. Show also that $f(x) = 0$ has only two real roots.

   [5]

2. You are given that $f(x) = x^2 + kx + c$.

   Given also that $f(2) = 0$ and $f(-3) = 35$, find the values of the constants k and c.

   [4]

3. You are given that $f(x) = 6x^3 - 25x^2 + 2x + 8$.

   (a) Evaluate $f(4)$. [1]

   (b) **In this question you must show detailed reasoning.**
   Express $f(x)$ as the product of three linear factors. [4]

4. (a) Show that $(x - 3)$ is a factor of $f(x) = 6x^3 - 17x^2 - 5x + 6$. [1]

   (b) Show that $f(x)$ can be written as the product of 3 linear factors. [4]

5. Show that $(x - 2)$ is a factor $3x^3 - 8x^2 + 3x + 2$. [3]

6. **In this question you must show detailed reasoning.**

   You are given that $f(x) = 4x^3 - 3x + 1$.

   (a) Use the factor theorem to show that $(x + 1)$ is a factor of $f(x)$. [2]

   (b) Solve the equation $f(x) = 0$. [3]

7. Find the quotient and the remainder when $x^3 - 2x + 3$ is divided by $x + 2$. [3]

8. Write down the roots of the equation $(x + 2)(x^2 - 25) = 0$. [2]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$f(1) = 1 - 1 + 1 + 9 - 10$ [= 0]	B1	allow for correct division of f(x) by (x – 1) showing there is no remainder, or for (x – 1) ($x^3 + x + 10$) found, showing it 'works' by multiplying it out	condone $1^4 - 1^3 + 1^2 + 9 - 10$
		i	attempt at division by (x – 1) as far as $x^4 - x^3$ in working	M1	allow equiv for (x + 2) as far as $x^4 + 2x^3$ in working or for inspection with at least two terms of cubic factor correct or $x^3 - 3x^2 + 7x - 5$	eg for inspection, M1 for two terms right and two wrong if M0 and this division / factorising is done in part (ii) or (iii), allow SC1 if correct cubic obtained there; attach the relevant part to (i) with a formal chain link if not already seen in the image zone for (i)
		i	correctly obtaining $x^3 + x + 10$	A1	**Examiner's Comments** A large number of candidates successfully used the factor theorem to score the first mark and many went on to find the correct cubic factor - the majority of these choosing to do long division rather than use the inspection method. Some did not use the factor theorem but still showed that $x = 1$ was a root by successful division with no remainder. Those who used inspection without first applying the factor theorem did not in general show enough working for a convincing argument that there was no remainder and therefore that $x = 1$ was a root. A small number did not appear to understand what was meant by 'express f(x) as the product of a linear factor and a quadratic factor' – some of these gained partial credit for the correct division seen in parts (ii) or (iii).	

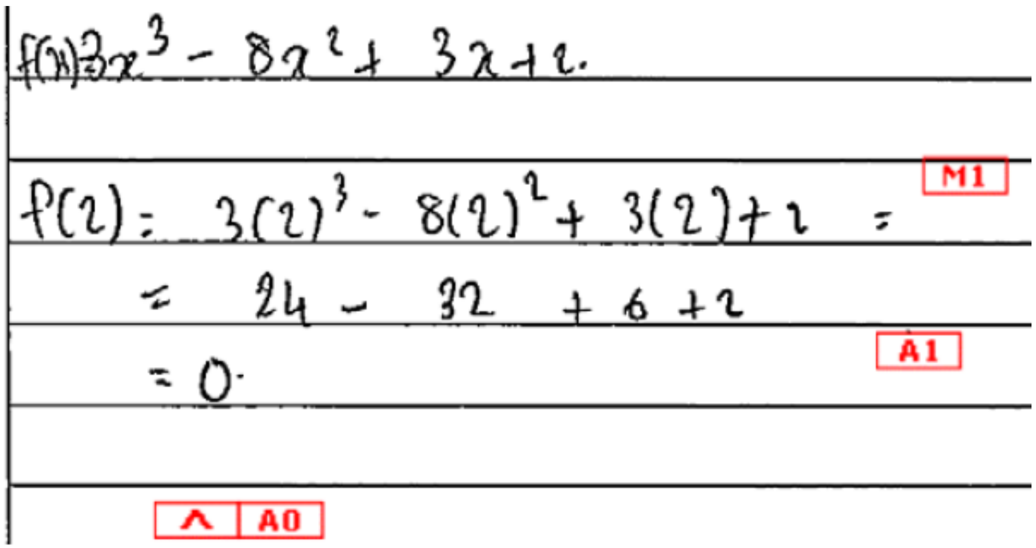
		ii	[g(− 2) =] − 8 − 2 + 10 or f(− 2) = 16 + 8 + 4 − 18 − 10	M1	[in this scheme g(x) = $x^3 + x + 10$] allow M1 for correct trials with at least two values of x (other than 1) using g(x) or f(x) or $x^3 - 3x^2 + 7x - 5$ (may allow similar correct trials using division or inspection)	eg f(2) = 16 − 8 + 4 + 18 − 10 or 20 f(3) = 81 − 27 + 9 + 27 − 10 or 80 f(0) = − 10 f(− 1) = 1 + 1 + 1 − 9 − 10 or − 16 No ft from wrong cubic 'factors' from (i)
		ii	$x = -2$ isw	A1	allow these marks if already earned in (i) **Examiner's Comments** Many used the correct method but made careless errors in calculations especially when trying negative values of x. Very few realised that they could use the factor theorem on the cubic they had found to obtain another root. Many confused 'root' with 'factor' and lost a mark.	NB factorising of $x^3 + x + 10$ or $x^3 - 3x^2 + 7x - 5$ in (ii) earns credit for (iii) [annotate with a yellow line in both parts to alert you – the image zone for (iii) includes part (ii)]
		iii	attempted division of $x^3 + x + 10$ by $(x + 2)$ as	M1	or $x^3 - 3x^2 + 7x - 5$ by $(x - 1)$ as far as $x^3 - x^2$ in working or inspection with at least two terms of quadratic factor correct	alt method: allow M1 for attempted

			far as $x^3 + 2x^2$ in working			division of quartic by $x^2 + x - 2$ as far as $x^4 + x^3 - 2x^2$ in working, or inspection etc
		iii	correctly obtaining $x^2 - 2x + 5$	A1	allow these first 2 marks if this has been done in (ii), even if not used here	or completing square form attempted
		iii	use of $b^2 - 4ac$ with $x^2 - 2x + 5$	M1	may be in attempt at formula (ignore rest of formula)	or attempt at calculus or symmetry to find min pt NB M0 for use of $b^2 - 4ac$ with cubic factor etc
		iii	$b^2 - 4ac = 4 - 20$ [= − 16]	A1	may be in formula;	or $(x - 1)^2 + 4$ or min = (1, 4)
		iii	so only two real roots[of f(x)] [and	A1	or no real roots of $x^2 - 2x + 5 = 0$; allow this last mark if clear use of $x^2 - 2x + 5 = 0$, even if error in $b^2 - 4ac$, provided result negative, but no ft from wrong factor	or $(x - 1)^2 + 4$ is always positive so

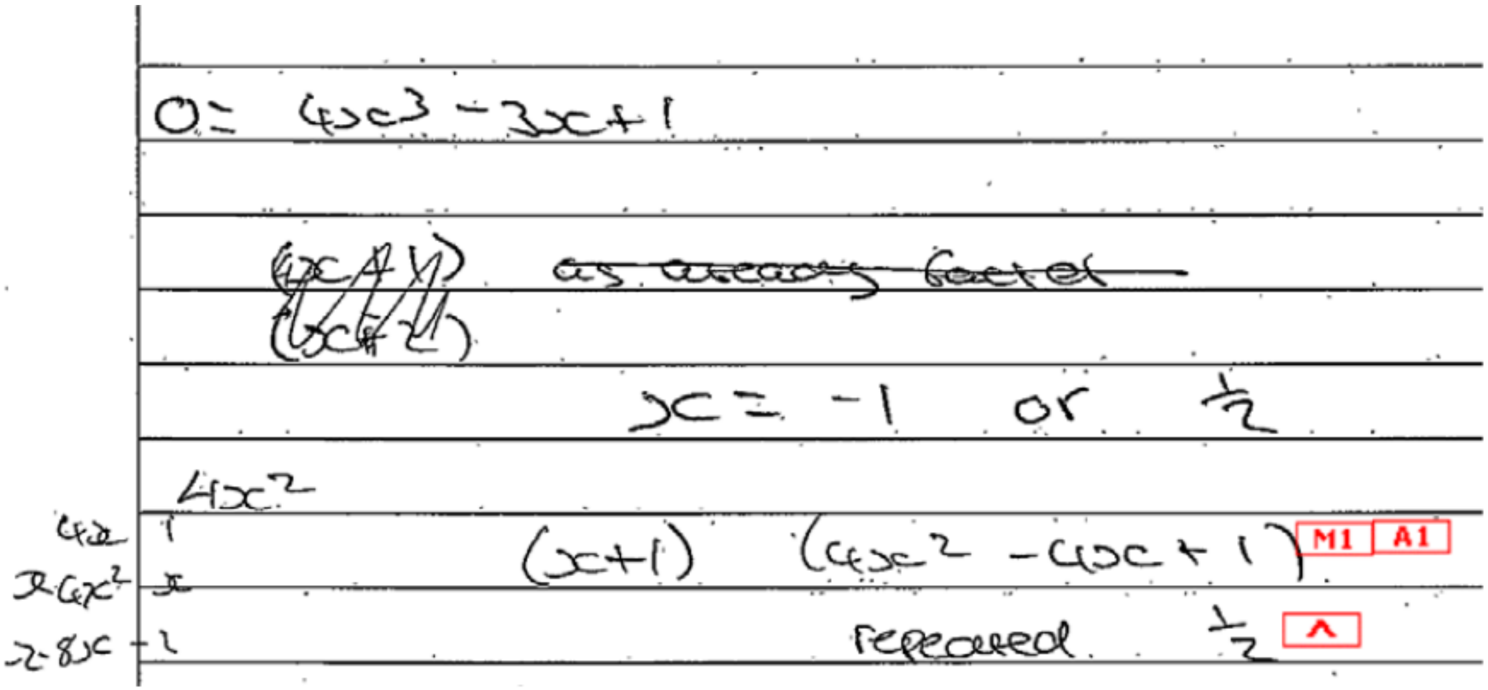
			hence no more linear factors]		if last M1 not earned, allow SC1 for stating that the only factors of 5 are 1 and 5 and reasoning eg that $(x-1)(x-5)$ and $(x+1)(x+5)$ do not give x^2-2x+5 [hence x^2-2x+5 does not factorise] **Examiner's Comments** Only about a third of the candidates found the correct quadratic factor. Those who found the quadratic usually gave sensible arguments based on the discriminant to show that only two real roots existed for the quartic. Some tried to use b^2-4ac on the cubic x^3+x+10. Several candidates went back to square one and attempted to factorise the quartic rather than linking the earlier parts to the problem. Some candidates who had not progressed far in the first two parts sometimes made no attempt at this part.	no real roots [of $(x-1)^2+4=0$] [and hence no linear factors] or similar conclusion from min pt
			Total	10		
2			$4+2k+c=0$ or $2^2+2k+c=0$	B1	may be rearranged	
			$9-3k+c=35$	B1	may be rearranged; the $(-3)^2$ must be evaluated / used as 9	condone -3^2 seen if used as 9
			correct method to eliminate one variable from their eqns	M1	eg subtraction or substitution for c; condone one error	M0 for addition of eqns unless also multiplied appropriately
			$k=-6$, $c=8$	A1	from fully correct method, allowing recovery from slips	if no errors and no method seen, allow correct answers to imply M1 provided B1B1 has been earned

			or $[x^2 + kx + c =] (x - 2)(x - a)$ $-5 \times (-3 - a) = 35$ oe $a = 4$ $k = -6$, $c = 8$	M1 M1 A1 A1	or $(x - 2)(x + b)$ **Examiner's Comments** Most candidates were able to make a start and substitute 2 and −3 into f(x), although not all used the information given to write the results as equations. Errors in handling $(-3)^2$ or the 35 were common. Having obtained equations, many did not then go on to use standard methods to solve the simultaneous equations, or made errors in doing so. This meant that the full 4 marks available were given less often than examiners had hoped, although many picked up 2 or 3 marks.	Polynomials
			Total	4		
3		a	$[384 - 400 + 8 + 8 =] 0$	B1(AO1.1) [1]		
		b	Long division or equating coefficients $6x^2 - x - 2$ seen $(x - 4)(3x \pm 2)(2x \pm 1)$ $(x - 4)(3x - 2)(2x + 1)$	M1(AO2.1) A1(AO1.1) M1(AO1.1) A1(AO1.1) [4]	DR	
			Total	5		

4		a	$f(3) = 6\times27 - 17\times9 - 5\times3 + 6 = 0$	B1(AO1.1b) [1]	use of Factor theorem with convincing detail	Polynomials
		b	$(x-3)(6x^2 + x - 2)$ valid attempt to factorise trinomial $(x-3)(2x-1)(3x+2)$	M1(AO1.1) A1 (AO 1.1b) M1(AO1.1b) A1(AO1.1b) [4]	allow sign errors in trinomial or attempt at long division	
			Total	1		
5			EITHER f (2) 3×2^3 8×2^2 $+ 3 \times 2 + 2 + 24 - 32 = 6 + 2 = 0$ Therefore by the factor theorem $(x - 2)$ is a factor OR $f(x) = (x-2)(3x^2 - 2x - 1)$	M1 (AO 1.1a) A1 (AO 1.1b) E1 (AO 2.2a) [3] M1	AG Function notation need not be used Zero must be seen Reason required Using algebraic division as far as $3x^2$ Correct quotient	

			No remainder so $(x-2)$is a factor	A1 E1 [3]	Reason required **Examiner's Comments** There were many good answers and candidates could choose whether to use the factor theorem or to divide. Some candidates lost the mark as they presented the evidence but did not write that f(2)= 0 implied that $(x-2)$ is a factor, or that no remainder implied that $(x-2)$ is a factor. Exemplar 1  This exemplar shows why some candidates were only credited 2 of 3 marks.	Polynomials
			Total	3		
6		a	$f(-1) = 4 \times (-1)^3 - 3(-1) + 1 = -4 + 3 + 1 = 0$	M1 (AO2.1)	DR Use of f(–1) must be seen. Do not allow for algebraic divison. Allow without conclusion if preceded by "If f(–1) = 0 then $(x+1)$ will be a factor" or similar	

			Therefore $(x+1)$ is a factor	A1 (AO2.2a) [2]	<table><tr><td>Clear conclusion must be made</td><td></td></tr></table> **Examiner's Comments** This question specified a method that was to be tested, so no marks were obtained by candidates who used algebraic division here – marks for this skill were credited in part (b). Most candidates were able to evaluate f(–1) = 0 but this did not obtain full marks without the detailed reasoning that this meant that $(x+1)$ was a factor of $f(x)$.	Polynomials
		b	$f(x) = (x+1)(4x^2 - 4x + 1) = 0$ $= (x+1)(2x-1)^2 = 0$ $x = -1, \frac{1}{2}$ [repeated]	M1 (AO1.1a) A1 (AO1.1b) A1 (AO1.1b) [3]	<table><tr><td>DR Attempt to divide or to factorise by inspection with $4x^2$ correct quadratic factor seen or implied by correct linear factors Both roots seen derived from 3 correct linear factors or use of quadratic formula</td><td>Allow full credit for $(x+1)(4x-2)(x-0.5)$ No marks for solving the cubic on the calculator</td></tr></table> **Examiner's Comments** There were many good answers seen but some candidates who correctly divided then did not state what the roots of the equation were. Some candidates used their calculators to find the roots of the equation but were not able to give the correct linear factors of $f(x)$ that were needed for full marks as in this exemplar. Exemplar 2	

						Polynomials
			Total	5		
7			Attempt to find Quotient Quotient is $x^2 - 2x + 2$ Remainder is -1	M1(AO 1.1a) A1(AO 1.1) B1(AO 1.1) [3]	Allow **M1** for $x^2 \pm 2x$ seen in quotient and $\pm 2x^2 \pm 2x$ seen in long division.	
			Total	3		
8			-2 5, -5	B1(AO 1.1a)		

				B1(AO 1.1) [2]	<table><tr><td>Both needed</td><td></td></tr></table>	Polynomials
			Total	2		