Questions

Q1.

(a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^3 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the *x*-axis.

(2)

The point with coordinates (–3, 0) lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where *a* is a constant.

(c) Find the two possible values of *a*.

(3)

(Total for question = 7 marks)

Q2.

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

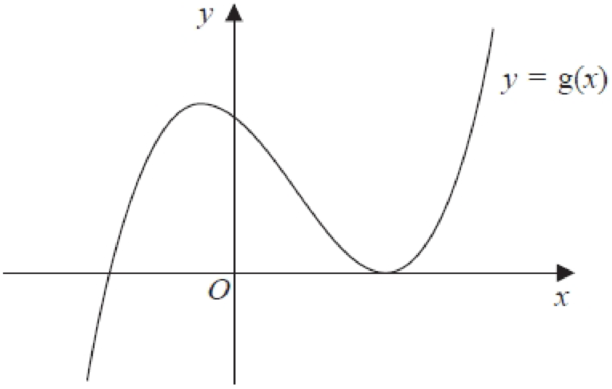


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leq 0$
(ii) $g(2x) = 0$

(3)

(Total for question = 9 marks)

Q3.

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

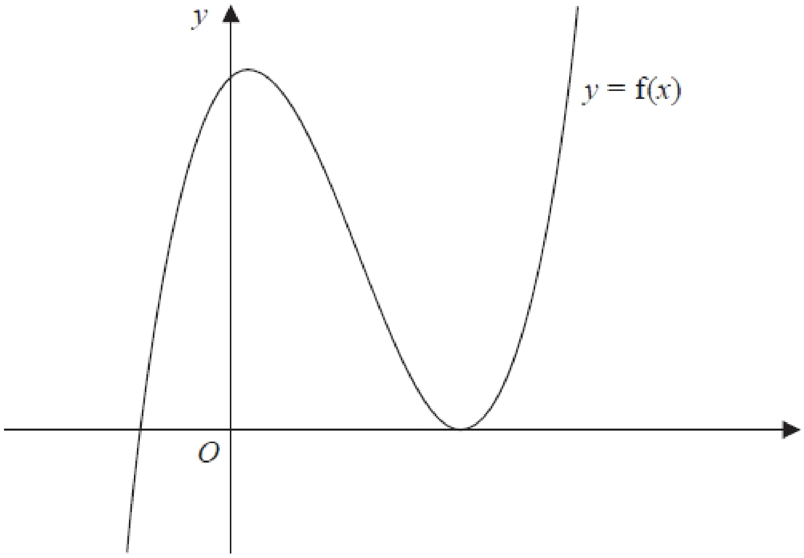


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

(Total for question = 10 marks)

Q4.

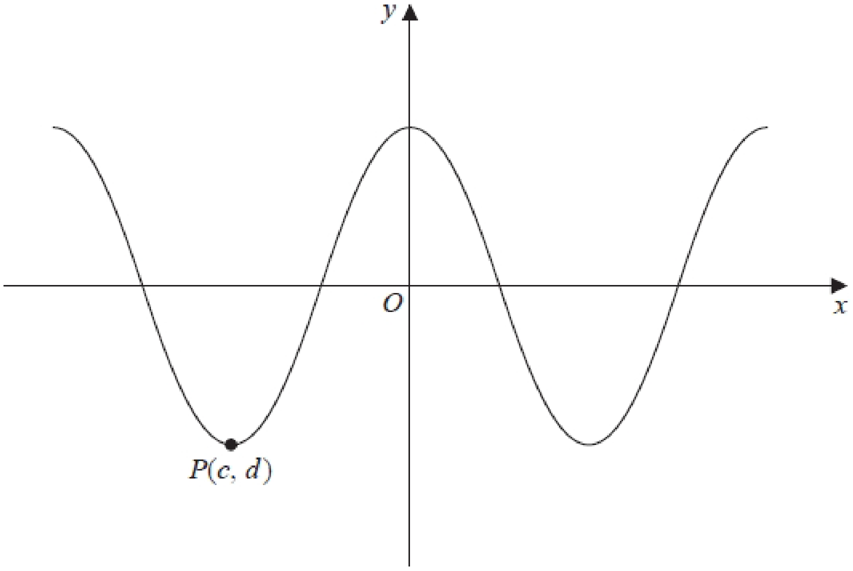


Figure 3

Figure 3 shows part of the curve with equation $y = 3 \cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which
a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which

transforms the curve with equation $y = 3 \cos x°$ to the curve with equation

(i) $y = 3 \cos \left(\frac{x°}{4}\right)$
(ii) $y = 3 \cos (x - 36)°$

(2)

(c) Solve, for $450° \leq \theta < 720°$

$$3 \cos \theta = 8 \tan \theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

(Total for question = 8 marks)

Q5.

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation y = f(x) showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation y = f(x) onto the curve with equation y = g(x) where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

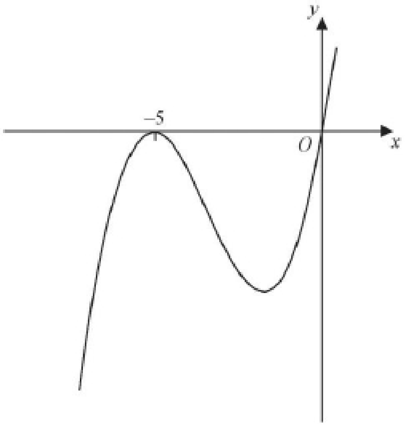
$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

(Total for question = 10 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$x^3+10x^2+25x=x(x^2+10x+25)$	M1	1.1b
	$=x(x+5)^2$	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	M1	1.1b
	Curve passes through the origin (0, 0) and touches at (−5, 0) (see note below for ft)	A1ft	1.1b
		(2)	
(c)	Curve has been translated *a* to the left	M1	3.1a
	$a = -2$	A1ft	3.2a
	$a=3$	A1ft	1.1b
		(3)	
			(7 marks)

Notes

(a) M1: Takes out factor x

A1: Correct factorisation - allow $x(x+5)(x+5)$

(b) M1: Correct shape

A1ft: Curve passes through the origin (0, 0) and touches at (−5, 0) – allow follow through from incorrect factorisation

(c) M1: May be implied by one of the correct answers for a or by a statement

A1ft: ft from their cubic as long as it meets the x-axis only twice.

A1ft : ft from their cubic as long as it meets the x-axis only twice.

Q2.

Question	Scheme	Marks	AOs
(a)	$(g(-2)) = 4\times -8 - 12\times 4 - 15\times -2 + 50$	M1	1.1b
	$g(-2) = 0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 - 20x + 25)$	M1 A1	1.1b 1.1b
	$= (x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x = 2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x = -1,\ x = 1.25$	B1ft	2.2a
		(3)	
			(9 marks)

(a)

M1: Attempts $g(-2)$ Some sight of (-2) embedded or calculation is required.

So expect to see $4\times(-2)^3 - 12\times(-2)^2 - 15\times(-2) + 50$ embedded

Or $-32 - 48 + 30 + 50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

A1: $g(-2) = 0 \Rightarrow (x+2)$ is a factor.

Requires a correct statement and conclusion. Both "$g(-2) = 0$" and "$(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $g(-2) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

Also accept, in one coherent line/sentence, explanations such as, 'as g (x) =0 when $x = -2$, $(x+2)$ is a factor.'

(b)

M1: Attempts to divide $g(x)$ by $(x+2)$ May be seen and awarded from part (a)

If inspection is used expect to see $4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 \text{...........} \pm 25)$

If algebraic / long division is used expect to see

$$\begin{array}{r} 4x^2 \pm 20x \\ x+2\overline{\big)\ 4x^3 - 12x^2 - 15x + 50} \end{array}$$

A1: Correct quadratic factor is $(4x^2 - 20x + 25)$ may be seen and awarded from part (a)

M1: Attempts to factorise their $(4x^2 - 20x + 25)$ usual rule $(ax+b)(cx+d), ac = \pm 4, bd = \pm 25$

A1: $(x+2)(2x-5)^2$ oe seen on a single line. $(x+2)(-2x+5)^2$ is also correct.

Allow recovery for all marks for $g(x) = (x+2)(x-2.5)^2 = (x+2)(2x-5)^2$

(c)(i)

M1: For identifying that the solution will be where the curve is on or below the axis. Award for either $x \leqslant -2$ or $x = 2.5$ Follow through on their $g(x) = (x+2)(ax+b)^2$ only where $ab < 0$ (that is a positive root). Condone $x < -2$ See SC below for $g(x) = (x+2)(2x+5)^2$

A1ft: BOTH $x \leqslant -2$, $x = 2.5$ Follow through on their $-\frac{b}{a}$ of their $g(x) = (x+2)(ax+b)^2$

May see $\{x \leqslant -2 \cup x = 2.5\}$ which is fine.

(c) (ii)

B1ft: For deducing that the solutions of $g(2x) = 0$ will be where $x = -1$ and $x = 1.25$

Condone the coordinates appearing $(-1, 0)$ and $(1.25, 0)$

Follow through on their 1.25 of their $g(x) = (x+2)(ax+b)^2$

……

SC: If a candidate reaches $g(x) = (x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award

In (i) M1 A0 for $x \leqslant -2$ or $x < -2$

In (ii) B1 for $x = -1$ and $x = -1.25$

……

Alt (b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(ax+b)^2$ $= a^2x^3 + (2ba + 2a^2)x^2 + (b^2 + 4ab)x + 2b^2$		
	Compares terms to get either *a* or *b*	M1	1.1b
	Either $a = 2$ or $b = -5$	A1	1.1b
	Multiplies out expression $(x+2)(\pm 2x \pm 5)^2$ and compares to $4x^3 - 12x^2 - 15x + 50$	M1	
	All terms must be compared or else expression must be multiplied out and establishes that $4x^3 - 12x^2 - 15x + 50 = (x+2)(2x-5)^2$	A1	1.1b
		(4)	

Q3.

Question	Scheme	Marks	AOs
(a)	Attempts $f(4) = 2\times4^3 - 13\times4^2 + 8\times4 + 48$	M1	1.1b
	$f(4) = 0 \Rightarrow (x-4)$ is a factor	A1	1.1b
		(2)	
(b)	$2x^3 - 13x^2 + 8x + 48 = (x-4)(2x^2 ... x - 12)$	M1	2.1
	$= (x-4)(2x^2 - 5x - 12)$	A1	1.1b
	Attempts to factorise quadratic factor or solve quadratic eqn	dM1	1.1b
	$f(x) = (x-4)^2(2x+3) \Rightarrow f(x) = 0$ has only two roots, 4 and -1.5	A1	2.4
		(4)	
(c)	Deduces either three roots or deduces that $f(x)$ is moved down two units	M1	2.2a
	States three roots, as when $f(x)$ is moved down two units there will be three points of intersection (with the x - axis)	A1	2.4
		(2)	
(d)	For sight of $k = \pm4, \pm\frac{3}{2}$	M1	1.1b
	$k = 4, -\frac{3}{2}$	A1ft	1.1b
		(2)	
			(10 marks)

Notes

(a)

M1: Attempts to calculate $\mathrm{f}(4)$.

Do not accept $\mathrm{f}(4)=0$ without sight of embedded values or calculations.

If values are not embedded look for two correct terms from $\mathrm{f}(4)=128-208+32+48$

Alternatively attempts to divide by $(x-4)$. Accept via long division or inspection.

See below for awarding these marks.

A1: Correct reason with conclusion. Accept $\mathrm{f}(4)=0$, hence factor as long as M1 has been scored.

This should really be stated on one line after having performed a correct calculation. It could appear as a preamble if the candidate states "If $\mathrm{f}(4)=0$, then $(x-4)$ is a factor before doing the calculation and then writing hence proven or ✓ oe.

If division/inspection is attempted it must be correct and there must be some attempt to explain why they have shown that $(x-4)$ is a factor. Eg Via division they must state that there is no remainder, hence factor

(b)

M1: Attempts to find the quadratic factor by inspection (correct first and last terms) or by division (correct first two terms)

So for inspection award for $2x^3-13x^2+8x+48=(x-4)(2x^2 ... x \pm 12)$

For division look for

$$\begin{array}{r|l} & 2x^2-5x \\ \hline x-4 & 2x^3-13x^2+8x+48 \\ & \underline{2x^3-8x^2} \\ & \quad -5x^2 \end{array}$$

A1: Correct quadratic factor $(2x^2-5x-12)$ For division award for sight of this "in the correct place" You don't have to see it paired with the $(x-4)$ for this mark.

If a student has used division in part (a) they can score the M1 A1 in (b) as soon as they start attempting to factorise their $(2x^2-5x-12)$.

dM1: Correct attempt to solve or factorise their $(2x^2-5x-12)$ including use of formula

Apply the usual rules $(2x^2-5x-12)=(ax+b)(cx+d)$ where $ac=\pm2$ and $bd=\pm12$

Allow the candidate to move from $(x-4)(2x^2-5x-12)$ to $(x-4)^2(2x+3)$ for this mark.

dM1: Correct attempt to solve or factorise their $\left(2x^2-5x-12\right)$ including use of formula

Apply the usual rules $\left(2x^2-5x-12\right)=(ax+b)(cx+d)$ where $ac=\pm2$ and $bd=\pm12$

Allow the candidate to move from $(x-4)\left(2x^2-5x-12\right)$ to $(x-4)^2(2x+3)$ for this mark.

A1: Via factorisation

Factorises twice to $\mathrm{f}(x)=(x-4)(2x+3)(x-4)$ or $\mathrm{f}(x)=(x-4)^2(2x+3)$ or

$\mathrm{f}(x)=2(x-4)^2\left(x+\frac{3}{2}\right)$ followed by a valid explanation why there are only two roots.

The explanation can be as simple as

- hence $x=4$ and $-\frac{3}{2}$ (only). The roots must be correct
- only two distinct roots as 4 is a repeated root

There must be some understanding between roots and factors.

E.g. $\mathrm{f}(x)=\underset{\nwarrow}{(x-4)^2}\underset{\uparrow}{(2x+3)}$

only two distinct roots is insufficient.

This would require two distinct factors, so there are two distinct roots.

Via solving.

Factorsises to $(x-4)\left(2x^2-5x-12\right)$ and solves $2x^2-5x-12=0\Rightarrow x=4,-\frac{3}{2}$ followed by an explanation that the roots are $4,4,-\frac{3}{2}$ so only two distinct roots.

Note that this question asks the candidate to use algebra so you cannot accept any attempt to use their calculators to produce the answers.

(c)

M1: For a valid **deduction.**

Accept **either** there are 3 roots **or** state that it is a solution of $\mathrm{f}(x)=2$ or $\mathrm{f}(x)-2=0$

A1: Fully explains:

Eg. States three roots, as $\mathrm{f}(x)$ is moved down by **two** units (giving three points of intersection with the x - axis)

Eg. States three roots, as it is where $\mathrm{f}(x)=2$ (You may see $y=2$ drawn on the diagram)

(d)

M1: For sight of ±4 **and** $\pm\frac{3}{2}$ Follow through on ± their roots.

A1ft: $k=4,-\frac{3}{2}$ Follow through on their roots. Accept $4,-\frac{3}{2}$ but not $x=4,-\frac{3}{2}$

Q4.

Question	Scheme	Marks	AOs
(a)	$(-180°, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720°, -3)$	B1ft	2.2a
	(ii) $(-144°, -3)$	B1 ft	2.2a
		(2)	
(c)	Attempts to use both $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$ and solves a quadratic equation in $\sin\theta$ to find at least one value of θ	M1	3.1a
	$3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$	B1	1.1b
	$3\sin^2\theta + 8\sin\theta - 3 = 0$ $(3\sin\theta - 1)(\sin\theta + 3) = 0$	M1	1.1b
	$\sin\theta = \frac{1}{3}$	A1	2.2a
	awrt $520.5°$ only	A1	2.1
		(5)	
			(8 marks)

(a)

B1: Deduces that $P(-180°, -3)$ or $c = -180^{(°)}, d = -3$

(b)(i)

B1ft: Deduces that $P'(-720°, -3)$ Follow through on their $(c, d) \to (4c, d)$ where d is negative

(b)(ii)

B1ft: Deduces that $P'(-144°, -3)$ Follow through on their $(c, d) \to (c + 36°, d)$ where d is negative

(c)

M1: An overall problem solving mark, condoning slips, for an attempt to

- use $\tan\theta = \frac{\sin\theta}{\cos\theta}$,
- use $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$
- find at least one value of θ from a quadratic equation in $\sin\theta$

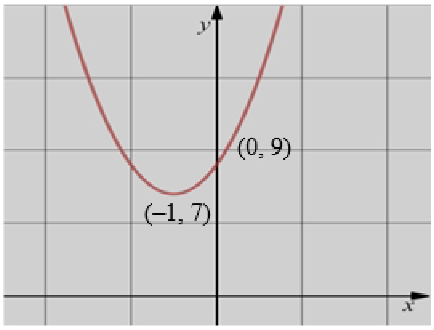
B1: Uses the correct identity and multiplies across to give $3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$ oe

M1: Uses the correct identity $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\sin\theta$ which they attempt to solve using an appropriate method. It is OK to use a calculator to solve this

A1: $\sin\theta = \frac{1}{3}$ Accept sight of $\frac{1}{3}$. Ignore any reference to the other root even if it is "used"

A1: Full method with all identities correct leading to the answer of awrt $520.5°$ and no other values.

Q5.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2x^2 + 4x + 9 = 2(x + b)^2 + c$	B1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + c$	M1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$ and $b = 1$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + 7$	A1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$, $b = 1$ and $c = 7$
(b)		B1	This mark is given for a U shaped curve in any position
		B1	This mark is given for a y-intercept shown at (0, 9)
		B1	This mark is given for a minimum shown at (–1, 7)
(c)(i)	$g(x) = 2(x - 2)^2 + 4(x - 2) + 5$	M1	This mark is given for writing g(x) in the form $a(x + b)^2 + c$ and comparing to f(x)
	Translation of $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	A1	This mark is given for deducing the translation of $y = f(x)$ to $y = g(x)$
(c)(ii)	$h(x) = \dfrac{21}{2(x+1)^2+7}$ Maximum value = $\dfrac{21}{7}$ (when $x = -1$)	M1	This mark is given for writing h(x) in the form $\dfrac{21}{a(x+b)^2+c}$ and finding its maximum value
	$0 < h(x) \leq 3$	A1	This mark is given for finding the correct range of the function h(x)