Questions

Q1.

A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$2^{2x+4} - 9(2^x) = 0$
$2^{2x} + 2^4 - 9(2^x) = 0$
Let $2^x = y$
$y^2 - 9y + 8 = 0$
$(y - 8)(y - 1) = 0$
$y = 8$ or $y = 1$
So $x = 3$ or $x = 0$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

(Total for question = 4 marks)

Q2.

Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

(Total for question = 8 marks)

Q3.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

(Total for question = 6 marks)

Q4.

In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x, writing your answer in simplest form.

(Total for question = 3 marks)

Q5.

Find

$$\int \frac{3x^4 - 4}{2x^3} \, dx$$

writing your answer in simplest form.

(Total for question = 4 marks)

Q6.

Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express *y* as a function of *x*.

(Total for question = 3 marks)

Mark Scheme

Q1.

Question	Scheme		Marks	AOs
(a)	$2^{2x}+2^4$ is wrong in line 2 - it should be $2^{2x}\times 2^4$		B1	2.3
	In line 4, 2^4 has been replaced by 8 instead of by 16		B1	2.3
			(2)	
(b)	Way 1 $2^{2x+4}-9(2^x)=0$ $2^{2x}\times 2^4-9(2^x)=0$ Let $2^x=y$ $16y^2-9y=0$	Way 2 $(2x+4)\log 2-\log 9-x\log 2=0$	M1	2.1
	$y=\frac{9}{16}$ or $y=0$ So $x=\log_2\left(\frac{9}{16}\right)$ or $\dfrac{\log\left(\frac{9}{16}\right)}{\log 2}$ o.e. with no second answer.	$x=\dfrac{\log 9}{\log 2}-4$ o.e.	A1	1.1b
			(2)	
				(4 marks)

Notes

(a) B1: Lists error in line 2 (as above)
B1 : Lists error in line 4 (as above)

(b) M1: Correct work with powers reaching this equation
A1 : Correct answer here – there are many exact equivalents

Q2.

Question	Scheme		Marks	AOs
(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow (b^2 + 9)(b^2 - 2) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}$, $-\sqrt{2}$ only		A1	1.1b
			(4)	
				(8 marks)

Notes
(i) **M1:** Combines the two algebraic terms to reach $a^{\pm\frac{3}{2}} = C$ or equivalent such as $\left(\sqrt{a}\right)^3 = C$ $(C \neq 0)$ An alternative is via squaring and combining the algebraic terms to reach $a^{\pm 3} = k, k > 0$ E.g. $...a^4 = ...a \Rightarrow a^{\pm 3} = k$ or $...a^4 = ...a \Rightarrow ...a^4 - ...a = 0 \Rightarrow ...a\left(a^3 - ...\right) = 0 \Rightarrow a^3 = ...$ Allow for slips on coefficients. **M1:** Undoes the indices correctly for their $a^{\frac{m}{n}} = C$ (So M0 M1 A0 is possible) You may even see logs used. **A1:** $a = \frac{1}{4}$ and no other solutions apart from 0 Accept exact equivalents Eg 0.25 **B1:** Deduces that $a = 0$ is a solution. (ii) **M1:** Attempts to solve as a quadratic equation in b^2 Accept $\left(b^2 + m\right)\left(b^2 + n\right) = 0$ with $mn = \pm 18$ or solutions via the use of the quadratic formula Also allow candidates to substitute in another variable, say $u = b^2$ and solve for u **A1:** Correct solution. Allow for $b^2 = 2$ or $u = 2$ with no incorrect solution given. Candidates can choose to omit the solution $b^2 = -9$ or $u = -9$ and so may not be seen **dM1:** Finds at least one solution from their $b^2 = k \Rightarrow b = \sqrt{k}, k > 0$. Allow $b = 1.414$ **A1:** $b = \sqrt{2}$, $-\sqrt{2}$ only. The solution asks for real values so if $3i$ is given then score A0

Answers with minimal or no working:

In part **(i)**

- no working, just answer(s) with they can score the B1
- If they square and proceed to the quartic equation $256a^4 = 4a$ oe, and then write down the answers they can have access to all marks.

In part (ii)

- Accept for 4 marks $b^2 = 2 \Rightarrow b = \pm\sqrt{2}$
- No working, no marks.

Q3.

Question	Scheme	Marks	AOs
(i)	$x\sqrt{2}-\sqrt{18}=x \Rightarrow x\left(\sqrt{2}-1\right)=\sqrt{18} \Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}$	M1	1.1b
	$\Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$	dM1	3.1a
	$x=\dfrac{\sqrt{18}\left(\sqrt{2}+1\right)}{1}=6+3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 2^{6x-4}=2^{-\frac{3}{2}}$	M1	2.5
	$6x-4=-\dfrac{3}{2} \Rightarrow x=...$	dM1	1.1b
	$x=\dfrac{5}{12}$	A1	1.1b
		(3)	
			(6 marks)

Notes

(i)

M1: Combines the terms in x, factorises and divides to find x. Condone sign slips and ignore any attempts to simplify $\sqrt{18}$

Alternatively squares both sides $x\sqrt{2}-\sqrt{18}=x \Rightarrow 2x^2-12x+18=x^2$

dM1: Scored for a complete method to find x. In the main scheme it is for making x the subject and then multiplying both numerator and denominator by $\sqrt{2}+1$

In the alternative it is for squaring both sides to produce a 3TQ and then factorising their quadratic equation to find x. (usual rules apply for solving quadratics)

A1: $x=6+3\sqrt{2}$ only following a correct intermediate line. Allow $\dfrac{6+3\sqrt{2}}{1}$ as an intermediate line.

In the alternative method the $6-3\sqrt{2}$ must be discarded.

(ii)

M1: Uses correct mathematical notation and attempts to set both sides as powers of 2 or 4.

Eg $2^{ax+b}=2^{c}$ or $4^{dx+e}=4^{f}$ is sufficient for this mark.

Alternatively uses logs (base 2 or 4) to get a linear equation in x.

$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow \log_2 4^{3x-2}=\log_2\dfrac{1}{2\sqrt{2}} \Rightarrow 2(3x-2)=\log_2\dfrac{1}{2\sqrt{2}}$.

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 3x-2=\log_4\dfrac{1}{2\sqrt{2}}$

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 4^{3x}=4\sqrt{2} \Rightarrow 3x=\log_4 4\sqrt{2}$

dM1: Scored for a complete method to find x.
Scored for setting the indices of 2 or 4 equal to each other and then solving to find x.
There must be an attempt on both sides.

You can condone slips for this mark Eg bracketing errors $4^{3x-2} = 2^{2\times 3x-2}$ or $\frac{1}{2\sqrt{2}} = 2^{-1+\frac{1}{2}}$

In the alternative method candidates cannot just write down the answer to the rhs.

So expect some justification. E.g. $\log_2 \frac{1}{2\sqrt{2}} = \log_2 2^{-\frac{3}{2}} = -\frac{3}{2}$

or $\log_4 \frac{1}{2\sqrt{2}} = \log_4 2^{-\frac{3}{2}} = -\frac{3}{2}\times\frac{1}{2}$ condoning slips as per main scheme

or $3x = \log_4 4\sqrt{2} \Rightarrow 3x = 1 + \frac{1}{4}$

A1: $x = \frac{5}{12}$ with correct intermediate work

Q4.

Question	Scheme	Marks	AOs
	$\frac{9^{x-1}}{3^{y+2}} = 81 \Rightarrow \frac{3^{2x-2}}{3^{y+2}} = 3^4$ or $\frac{9^{x-1}}{3^{y+2}} = 81 \Rightarrow \frac{9^{x-1}}{9^{\frac{1}{2}(y+2)}} = 9^2$	M1	1.1b
	$\Rightarrow 2x-2-y-2=4 \Rightarrow y =$ or $\Rightarrow x-1-\frac{1}{2}y-1=2 \Rightarrow y =$	dM1	1.1b
	$\Rightarrow y = 2x-8$	A1	1.1b
		(3)	
Alt	Eg. $\log_3\left(\frac{9^{x-1}}{3^{y+2}}\right) = \log_3 81$	M1	1.1b
	$\Rightarrow (x-1)\log_3(9^{x-1}) - (y+2)\log_3(3^{y+2}) = 4$ $\Rightarrow 2(x-1)-y-2=4 \Rightarrow y =$	dM1	1.1b
	$\Rightarrow y = 2x-8$	A1	1.1b
			(3 marks)

Notes

M1: Attempts to set 9^{x-1} and 81 as powers of 3. Condone $9^{x-1} = 3^{2x-1}$ and $9^{x-1} = 3^{3x-3}$.
Alternatively attempts to write each term as a logarithm of base 3 or 9. You must see the base written to award this mark.

dM1: Attempts to use the addition (or subtraction) index law, or laws or logarithms, correctly and rearranges the equation to reach y in terms of x. Condone slips in their rearrangement.

A1: $y = 2x-8$

Q5.

Question	Scheme	Marks	AOs
	$\int \frac{3x^4-4}{2x^3}\,dx = \int \frac{3}{2}x - 2x^{-3}\,dx$	M1 A1	1.1b 1.1b
	$= \frac{3}{2}\times\frac{x^2}{2} - 2\times\frac{x^{-2}}{-2} \quad (+c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e	A1	1.1b
		(4)	
			(4 marks)

Notes:

(i)

M1: Attempts to divide to form a sum of terms. Implied by two terms with one correct index.

$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$ scores this mark.

A1: $\int \frac{3}{2}x - 2x^{-3}\,dx$ o.e such as $\frac{1}{2}\int\left(3x - 4x^{-3}\right)dx$. The indices must have been processed on both terms. Condone spurious notation or lack of the integral sign for this mark.

dM1: For the full strategy to integrate the expression. It requires two terms with one correct index.

Look for $= ax^p + bx^q$ where $p = 2$ or $q = -2$

A1: Correct answer $\frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e. such as $\frac{3}{4}x^2 + x^{-2} + c$

Q6.

Part	Working or answer an examiner might expect to see	Mark	Notes
	$2^x \times (2^2)^y = 2^{-\frac{3}{2}} \Rightarrow 2^{x+2y} = 2^{-\frac{3}{2}}$	M1	This mark is given for writing all terms in the same base and applying an index law
	$x + 2y = -\frac{3}{2}$	M1	This mark is given for writing an equation to link x and y
	$y = -\frac{1}{2}x - \frac{3}{4}$	A1	This mark is given for rearranging to find a correct expression of y as a function of x
			(Total 3 marks)