

1. A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$\underline{2^{2x} + 2^4} - 9(2^x) = 0$$

$$\text{Let } 2^x = y$$

$$y^2 - 9y + \underline{8} = 0$$

$$(y - 8)(y - 1) = 0$$

$$y = 8 \text{ or } y = 1$$

$$\text{So } x = 3 \text{ or } x = 0$$

- (a) Identify the two errors made by the student.

(2)

- (b) Find the exact solution to the equation.

(2)

$$(a) \quad 2^{2x+4} = 2^{2x} \cdot 2^4 \quad \text{not} \quad 2^{2x} + 2^4$$

$$\text{Also, } 2^4 = 16 \quad \text{not } 8$$

$$(b) \quad 2^{2x+4} - 9(2^x) = 0$$

$$2^{2x} \cdot 2^4 - 9(2^x) = 0$$

$$16(2^{2x}) - 9(2^x) = 0$$

$$\text{Let } 2^x = y, \text{ then } 16y^2 - 9y = 0$$

$$y(16y - 9) = 0$$

$$\text{Either } y = 0 \text{ or } 16y - 9 = 0$$

$$y = 0 \text{ or } y = \frac{9}{16}$$

So, $2^x = 0$ or $2^x = \frac{9}{16}$

$2^x = 0$ is Invalid

Now, $2^x = \frac{9}{16}$

$$x = \log_2 \left(\frac{9}{16} \right)$$

OR $x = \frac{\log \left(\frac{9}{16} \right)}{\log 2}$

2. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

i) $16a^2 - 2\sqrt{a} = 0$

$$a^{\frac{1}{2}} [16a^{\frac{3}{2}} - 2] = 0$$

so $a^{\frac{1}{2}} = 0$ or $16a^{\frac{3}{2}} - 2 = 0$

$$\Rightarrow \boxed{a = 0}$$

$$a^{\frac{3}{2}} = \frac{1}{8}$$

$$a^{\frac{1}{2}} = \frac{1}{2}$$

$$\Rightarrow \boxed{a = \frac{1}{4}}$$

ii) let $b^2 = y$,

$$y^2 + 7y - 18 = 0$$

$$(y + 9)(y - 2) = 0$$

$$\Rightarrow y = -9$$

$$b^2 = -9$$

$$\boxed{\text{no real solutions}}$$

$$y = 2$$

$$b^2 = 2$$

$$\boxed{b = \pm\sqrt{2}}$$

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In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

3. (i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

i. rearrange to make x the subject:

$$x\sqrt{2} - x = \sqrt{18}$$

$$x(\sqrt{2} - 1) = \sqrt{18}$$

$$x = \frac{\sqrt{18}}{\sqrt{2} - 1}$$

surd in denominator $\Rightarrow$ rationalise: $x = \frac{\sqrt{18}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$$= \frac{\sqrt{18}(\sqrt{2} + 1)}{2 - 1}$$

$$= \frac{\sqrt{36} + \sqrt{18}}{1}$$

$$= 6 + \sqrt{9 \times 2}$$

$$= 6 + 3\sqrt{2}$$



ii. LHS has a power of 2, RHS has power of 2 $\Rightarrow$ Manipulate so

2 is 'base' of each side: $(2^2)^{3x-2} = \frac{1}{2 \times 2^{\frac{1}{2}}}$

$$\hookrightarrow 2^{6x-4} = \frac{1}{2^{\frac{3}{2}}} = 2^{-\frac{3}{2}}$$

$$\therefore 6x-4 = -\frac{3}{2}$$

$$6x = 2.5 = \frac{5}{2}$$

$$x = \frac{5}{12}$$



In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

4. Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x , writing your answer in simplest form.

(3)

$$\frac{9^{x-1}}{3^{y+2}} = 81 \quad \rightarrow \text{find indices with a common base}$$

$$\frac{3^{2(x-1)}}{3^{y+2}} = 3^4 \quad (1)$$

$$3^{(2x-2)-(y+2)} = 3^4 \quad \rightarrow \text{you can then eliminate the common base}$$

$$2x - 2 - y - 2 = 4 \quad (1)$$

$$2x = y + 8$$

$$y = 2x - 8 \quad (1)$$



5. Find

$$\int \frac{3x^4 - 4}{2x^3} dx$$

writing your answer in simplest form.

(4)

$$\int \frac{3x^4 - 4}{2x^3} dx$$

$$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3} dx \quad (1)$$

$$\int \frac{3}{2} x - 2x^{-3} dx \quad (1)$$

$$= \frac{3}{4} x^2 + x^{-2} + c \quad \# \quad (1)$$

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6. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x .

(3)

$$2^x \cdot 4^y = \frac{1}{2\sqrt{2}}$$

$$2^x \cdot 2^{2y} = \frac{1}{2 \cdot 2^{1/2}} \quad (1)$$

$$\sqrt{2} = 2^{1/2}$$

$$1 + \frac{1}{2} = \frac{3}{2}$$

Indices laws:

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{1}{a^m} = a^{-m}$$

$$a^m = a^n \quad \text{then} \quad m = n$$

$$\Rightarrow 2^{x+2y} = \frac{1}{2^{3/2}}$$

$$\Rightarrow 2^{x-2y} = 2^{-3/2} \quad (1)$$

$$\Rightarrow x - 2y = -\frac{3}{2}$$

$$\Rightarrow 2y = \frac{3}{2} + x \quad \Rightarrow \quad y = \frac{3}{4} + \frac{x}{2} \quad (1)$$