DO NOT WRITE IN THIS AREA

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

1. (a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(a) If $(x-3)$ is a factor of $f(x)$, then $f(3)=0$

$f(3) = 4(3)^3 - 12(3)^2 + 2(3) - 6$

$= 108 - 108 + 6 - 6$

$\underline{= 0}$

$\therefore (x-3)$ is a factor of $f(x)$.

(b) $4x^3 - 12x^2 + 2x - 6 = (x-3)(4x^2+2)$

For $4x^2+2=0$, $a=4, b=0, c=2$

Discriminant, $b^2 - 4ac = 0^2 - (4)(4)(2)$

$\underline{= -32}$

Since the discriminant is less than 0, $4x^2+2=0$ has no real roots.

$\therefore$ 3 is the only real root of the equation $f(x)=0$

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

2. (a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

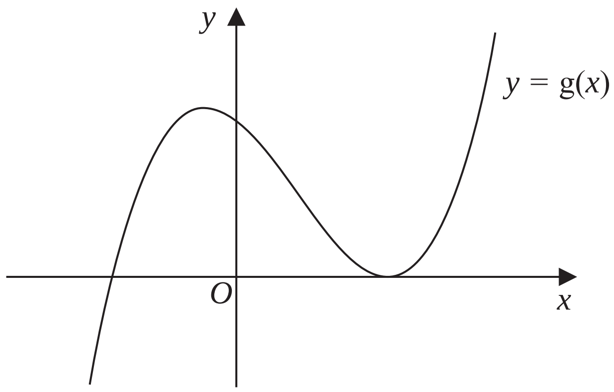

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

a) $(x+2) = 0 \quad x = -2$

$g(-2) = 4(-2)^3 - 12(-2)^2 - 15(-2) + 50$

$= 0$

$\therefore x+2$ is a factor

b)
$$\begin{array}{r|l} & 4x^2 - 20x + 25 \\ x+2 & 4x^3 - 12x^2 - 15x + 50 \\ & 4x^3 + 8x^2 \\ & \quad -20x^2 - 15x \\ & \quad -20x^2 - 40x \\ & \qquad\quad 25x + 50 \\ & \qquad\quad 25x + 50 \end{array}$$

$g(x) = (x+2)(4x^2 - 20x + 25)$

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c i) $(x+2)(4x^2-20x+25)=0$

$x=-2$ $(2x-5)^2=0$

$x = \pm 2.5$

$x<-2$ $x=2.5$

ii) $x=-1$ $x=1.25$

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DO NOT WRITE IN THIS AREA

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

3. (a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

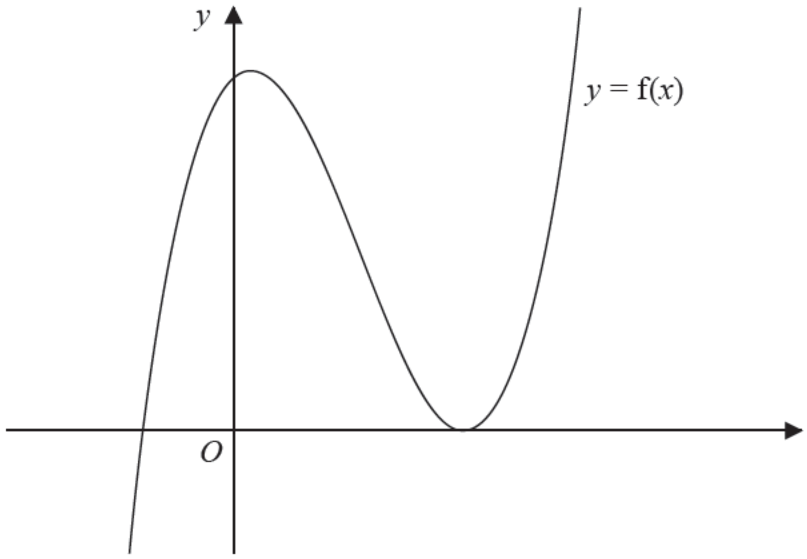

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

a) $f(4) = 2(4)^3 - 13(4)^2 + 8(4) + 48$

$= 0$ // $\therefore (x-4)$ is a factor

b) long division :

$$\begin{array}{r} 2x^2 - 5x - 12 \\ x-4 \overline{)2x^3 - 13x^2 + 8x + 48} \\ 2x^3 - 8x^2 \\ \hline 0 \quad -5x^2 + 8x \\ -5x^2 + 20x \end{array}$$

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$$\begin{array}{r} 0 \quad -12x + 48 \\ -12x + 48 \\ \hline 0 \quad 0 \end{array}$$ //

so $f(x) = (x-4)(2x^2-5x-12)$

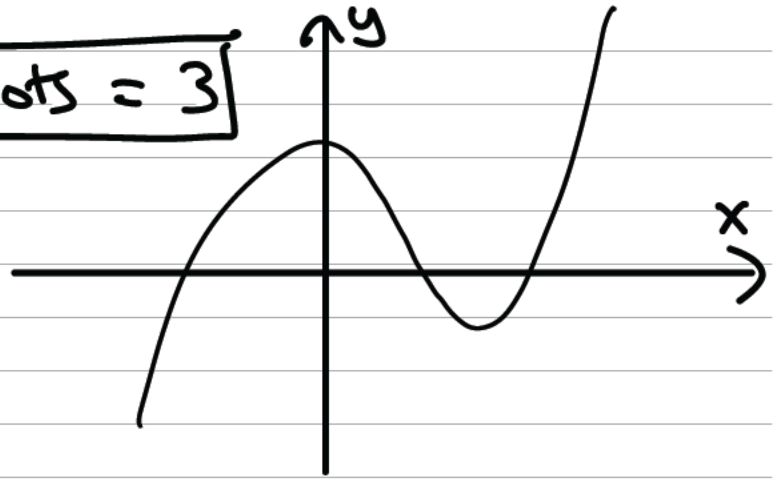
$= (x-4)(2x+3)(x-4)$

$= (x-4)^2(2x+3)$ //

hence f(x) has just 2 distinct roots ; $x=4$
$x=-3/2$

c) The curve is shifted down by 2 units

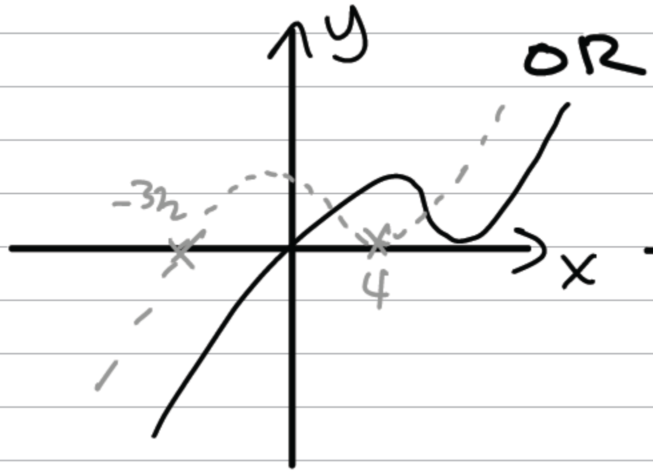
so $\boxed{\# \text{ of roots} = 3}$



Turn over

ORIGINAL

d) we could have :



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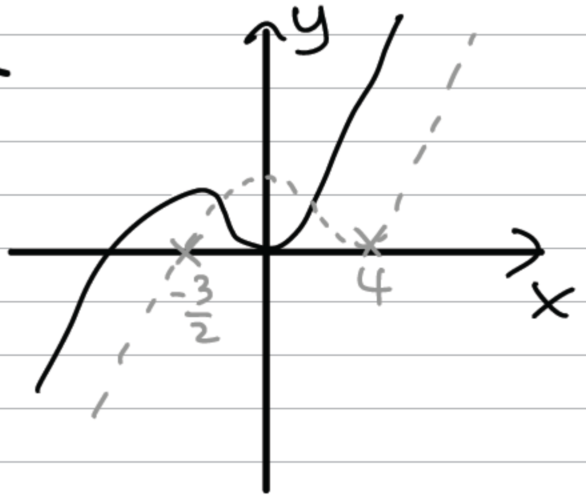


f(x) shifted to the right by $3/2$

ie $f(x - 3/2)$

so $\boxed{k = -\frac{3}{2}}$

f(x) shifted to the left by 4

ie $f(x+4)$

so $\boxed{k = 4}$

$$g(x) = 2x^3 + x^2 - 41x - 70$$

4. (a) Use the factor theorem to show that g(x) is divisible by (x – 5).

(2)

(b) Hence, showing all your working, write g(x) as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R.

(4)

a) factor theorem: if $f(x)$ is divisible by $(x-a)$, then $f(a)=0$

$g(5) = 2\times5^3 + 5^2 - 41\times5 - 70 = 250 + 25 - 205 - 70$

$= \underline{0}$

$g(5)=0 \Rightarrow (x-5)$ is a factor, so $g(x)$ is divisible by $(x-5)$

b) divide $g(x)$ by $(x-5)$ to get a quadratic

$$\begin{array}{r l}
 & 2x^2 + 11x + 14 \\
(x-5) & \overline{)\,2x^3 + x^2 - 41x - 70} \\
 & \underline{-(2x^3 - 10x^2)} \\
 & 11x^2 - 41x - 70 \\
 & \underline{-(11x^2 - 55x)} \\
 & 14x - 70
\end{array}$$

$g(x) = (x-5)(2x^2+11x+14)$

$= (x-5)(2x+7)(x+2)$

c) g's roots: $-\frac{7}{2}, -2, 5$

so R bound by $x=-2, x=5$

Sketch to help you find R

$$\int_{-2}^{5} g(x)\,dx = \int_{-2}^{5} (2x^3+x^2-41x-70)\,dx$$

$$= \left[\frac{2}{4}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x\right]_{-2}^{5}$$

$$= \frac{1}{2}(625) + \frac{1}{3}(125) - \frac{41}{2}(25) - 70(5)$$

$$- \frac{1}{2}(16) - \frac{1}{3}(-8) + \frac{41}{2}(4) - 70(-2)$$

$$= -\frac{1525}{3} - \frac{190}{3}$$

area $= 571\frac{2}{3}$

$$f(x) = 2x^3 - 5x^2 + ax + a$$

5. Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

If $(x+a)$ is a factor, $f(-a)=0$

$(x+2)$ is a factor, $f(-2):0$

$2(-2)^3 - 5(-2)^2 + a(-2) + a = 0$ — 1

$-16 - 20 - 2a + a = 0$

$-36 - a = 0$ — 1

$a = -36$ — 1

$$f(x) = 3x^3 + 8x^2 - 9x + 10 \quad x \in \mathbb{R}$$

6. (a) (i) Calculate f(2)

(ii) Write f(x) as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

ai) $f(2) = -3(2)^3 + 8(2)^2 - 9(2) + 10 = \underline{\underline{0}}$ ✓1

ii) If $x = 2$ gives $f(2) = 0$, then $(x-2)$ is a factor by the factor theorem.

$f(x) = (x-2)(ax^2 + bx + c) = -3x^3 + 8x^2 - 9x + 10$

$\underline{x^3:}$ $a = -3$

$\therefore a = -3$

$\underline{x^2:}$ $b - 2a = 8$

$b = 8 + 2a$

$b = 8 + 2(-3) = 2$

$\underline{\text{const.}:}$ $-2c = 10$

$c = -5$

$f(x) = \underline{(x-2)(-3x^2 + 2x - 5)}$ ✓1 ✓1

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

b) $f(x) = -3x^3 + 8x^2 - 9x + 10$
$= (x-2)(-3x^2 + 2x - 5)$

$-3y^6 + 8y^4 - 9y^2 + 10 = 0$

Use the sub : $x = y^2$

$-3x^3 + 8x^2 - 9x + 10 = 0$

$(x-2)(-3x^2 + 2x - 5) = 0$

$x = 2$
$y^2 = 2$
$y = \pm\sqrt{2}$ ✓ 1

↓

only two real solutions.

$b^2 - 4ac < 0$

$(2)^2 - 4(-3)(-5) < 0$

$4 - 60 < 0$
$-56 < 0$

∴ there are no real solutions to this quadratic. ✓ 1

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

c) $f(x) = -3x^3 + 8x^2 - 9x + 10$

Use the sub.: $x = \tan(\theta)$

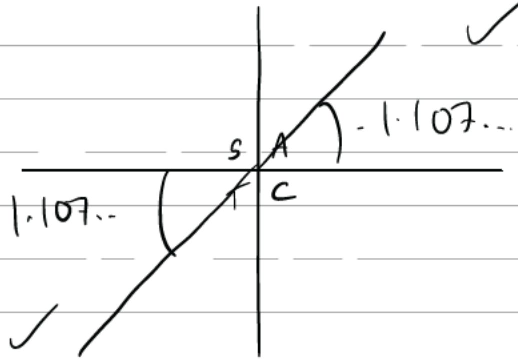
$3x^3 - 8x^2 + 9x - 10 = 0$

$-3x^3 + 8x^2 - 9x + 10 = 0$

From earlier parts: $x = 2$ is the only solution

$\tan(\theta) = 2$

$\theta = 1.107...$



No. of solutions in $7\pi \leq \theta \leq 10\pi$

is the same in $0 \leq \theta \leq 3\pi$

$\theta = 1.107...,\ 1.107... + \pi,\ 1.107... + 2\pi$

$\Rightarrow \therefore$ there are 3 solutions for θ.

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

7. Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

when $f(k) = 0$ then $(x-k)$ is a factor of $f(x)$

$f(-3) = 0$ $\quad (x - -3)$

$f(-3) = 3(-3)^3 + 2a(-3)^2 - 4(-3) + 5a = 0$ ①

$3(-27) + 2a(9) + 12 + 5a = 0$

$-81 + 18a + 12 + 5a = 0$

$-69 + 23a = 0$ $\quad$ ∴ $a = 3$ ①

$+69 \quad +69$

$23a = 69$ ① $\quad a = 69/23 = 3$

$\div 23 \quad \div 23$

$$f(x) = ax^3 + 10x^2 - 3ax - 4$$

8. Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a.

You must make your method clear.

(3)

If $(x-1)$ is a factor of $f(x)$, then $f(1) = 0$

$f(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4$ 1

$0 = a + 10 - 3a - 4$

$0 = -2a + 6$ 1

$2a = 6$

$a = 3$ 1