

Questions**Q1.**

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(Total for question = 6 marks)

Q2.

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

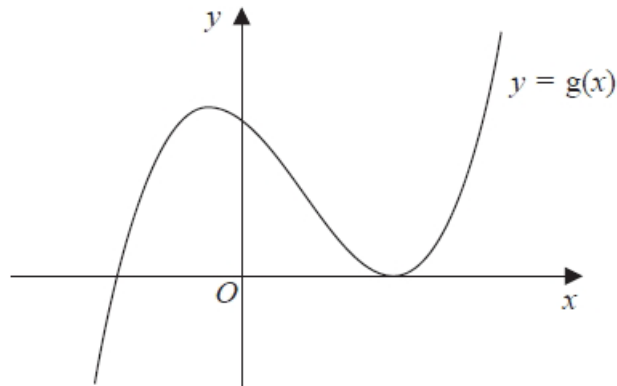


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

- (i) $g(x) \leq 0$
- (ii) $g(2x) = 0$

(3)

(Total for question = 9 marks)

Q3.

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

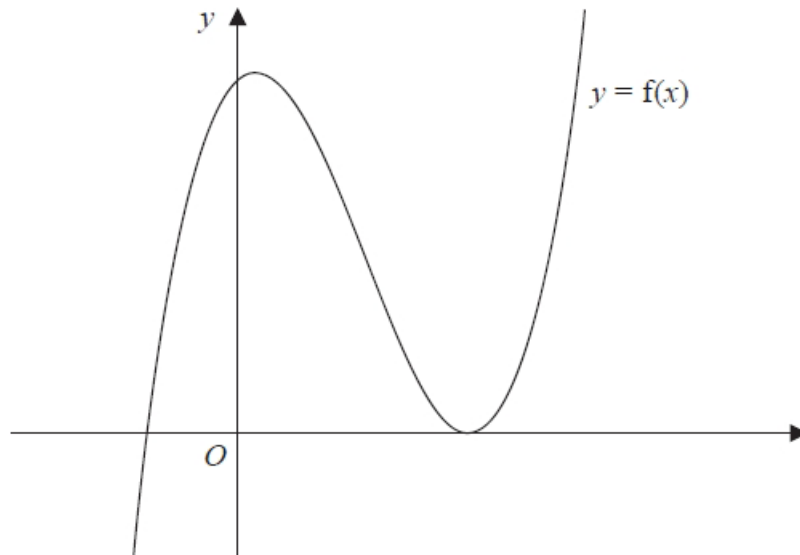


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k .

(2)

(Total for question = 10 marks)

Q4.

$$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$.

(2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x -axis, and lies below the x -axis.

(c) Find, using algebraic integration, the exact value of the area of R .

(4)

(Total for question = 10 marks)

Q5.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a .

(3)**(Total for question = 3 marks)**

Q6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate $f(2)$

(ii) Write $f(x)$ as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leq \theta < 10\pi$, to the equation

$$3 \tan^3 \theta - 8 \tan^2 \theta + 9 \tan \theta - 10 = 0$$

(1)

(Total for question = 6 marks)

Q7.

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a .

(Total for question = 3 marks)

Q8.

$$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a .

You must make your method clear.

(Total for question = 3 marks)

Mark Scheme

Q1.

| Question | Scheme | Marks | AOs |
|--|--|-------|------|
| (a) | States or uses $f(+3) = 0$ | M1 | 1.1b |
| | $4(3)^3 - 12(3)^2 + 2(3) - 6 = 108 - 108 + 6 - 6 = 0$ and so $(x - 3)$ is a factor | A1 | 1.1b |
| | | (2) | |
| (b) | Begins division or factorisation so $4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + \dots)$ | M1 | 2.1 |
| | $4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + 2)$ | A1 | 1.1b |
| | Considers the roots of their quadratic function using completion of square or discriminant | M1 | 2.1 |
| | $(4x^2 + 2) = 0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2 + 2 > 0$ for all x) So $x = 3$ is the only real root of $f(x) = 0$ * | A1* | 2.4 |
| | (4) | | |
| (6 marks) | | | |
| Notes | | | |
| (a) M1: States or uses $f(+3) = 0$ A1: See correct work evaluating and achieving zero, together with correct conclusion | | | |
| (b) M1: Needs to have $(x - 3)$ and first term of quadratic correct A1: Must be correct – may further factorise to $2(x - 3)(2x^2 + 1)$ M1: Considers their quadratic for no real roots by use of completion of the square or consideration of discriminant then A1*: a correct explanation. | | | |

Q2.

| Question | Scheme | Marks | AOs |
|-------------------------|---|------------|--------------|
| (a) | $(g(-2)) = 4 \times -8 - 12 \times 4 - 15 \times -2 + 50$ | M1 | 1.1b |
| | $g(-2) = 0 \Rightarrow (x + 2)$ is a factor | A1 | 2.4 |
| | | (2) | |
| (b) | $4x^3 - 12x^2 - 15x + 50 = (x + 2)(4x^2 - 20x + 25)$ | M1 A1 | 1.1b 1.1b |
| | $= (x + 2)(2x - 5)^2$ | M1 A1 | 1.1b 1.1b |
| | | (4) | |
| | (c) (i) $x \leq -2, x = 2.5$ | M1 A1ft | 1.1b 1.1b |
| (ii) $x = -1, x = 1.25$ | B1ft | 2.2a | |
| | (3) | | |
| (9 marks) | | | |

(a)**M1:** Attempts $g(-2)$ Some sight of (-2) embedded or calculation is required.So expect to see $4 \times (-2)^3 - 12 \times (-2)^2 - 15 \times (-2) + 50$ embeddedOr $-32 - 48 + 30 + 50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

A1: $g(-2) = 0 \Rightarrow (x+2)$ is a factor.Requires a correct statement and conclusion. Both " $g(-2) = 0$ " and " $(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $g(-2) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.Also accept, in one coherent line/sentence, explanations such as, 'as $g(x) = 0$ when $x = -2$, $(x+2)$ is a factor.'**(b)****M1:** Attempts to divide $g(x)$ by $(x+2)$ May be seen and awarded from part (a)If inspection is used expect to see $4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 \dots \dots \dots \pm 25)$ If algebraic / long division is used expect to see
$$x+2 \overline{) 4x^3 - 12x^2 - 15x + 50} \begin{array}{r} 4x^2 + 20x \\ \hline \end{array}$$
A1: Correct quadratic factor is $(4x^2 - 20x + 25)$ may be seen and awarded from part (a)**M1:** Attempts to factorise their $(4x^2 - 20x + 25)$ usual rule $(ax+b)(cx+d)$, $ac = \pm 4$, $bd = \pm 25$ **A1:** $(x+2)(2x-5)^2$ or seen on a single line. $(x+2)(-2x+5)^2$ is also correct.Allow recovery for all marks for $g(x) = (x+2)(x-2.5)^2 = (x+2)(2x-5)^2$ **(c)(i)****M1:** For identifying that the solution will be where the curve is on or below the axis. Award for either $x \leq -2$ or $x = 2.5$ Follow through on their $g(x) = (x+2)(ax+b)^2$ only where $ab < 0$ (that is a positive root). Condone $x < -2$ See SC below for $g(x) = (x+2)(2x+5)^2$

| | | | |
|--|--|------------|------|
| <p>A1ft: BOTH $x \leq -2$, $x = 2.5$ Follow through on their $-\frac{b}{a}$ of their $g(x) = (x+2)(ax+b)^2$ May see $\{x \leq -2 \cup x = 2.5\}$ which is fine.</p> <p>(c) (ii)</p> <p>B1ft: For deducing that the solutions of $g(2x) = 0$ will be where $x = -1$ and $x = 1.25$ Condone the coordinates appearing $(-1, 0)$ and $(1.25, 0)$ Follow through on their 1.25 of their $g(x) = (x+2)(ax+b)^2$</p> <hr/> <p>SC: If a candidate reaches $g(x) = (x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award</p> <p>In (i) M1 A0 for $x \leq -2$ or $x < -2$ In (ii) B1 for $x = -1$ and $x = -1.25$</p> <hr/> | | | |
| Alt (b) | $4x^3 - 12x^2 - 15x + 50 = (x+2)(ax+b)^2$ $= a^2x^3 + (2ba + 2a^2)x^2 + (b^2 + 4ab)x + 2b^2$ | | |
| | Compares terms to get either a or b | M1 | 1.1b |
| | Either $a = 2$ or $b = -5$ | A1 | 1.1b |
| | Multiplies out expression $(x+2)(\pm 2x \pm 5)^2$ and compares to $4x^3 - 12x^2 - 15x + 50$ | M1 | |
| | All terms must be compared or else expression must be multiplied out and establishes that $4x^3 - 12x^2 - 15x + 50 = (x+2)(2x-5)^2$ | A1 | 1.1b |
| | | (4) | |

Q3.

| Question | Scheme | Marks | AOs |
|-------------------|---|-------|------|
| (a) | Attempts $f(4) = 2 \times 4^3 - 13 \times 4^2 + 8 \times 4 + 48$ | M1 | 1.1b |
| | $f(4) = 0 \Rightarrow (x - 4)$ is a factor | A1 | 1.1b |
| | | (2) | |
| (b) | $2x^3 - 13x^2 + 8x + 48 = (x - 4)(2x^2 \dots x - 12)$ | M1 | 2.1 |
| | $= (x - 4)(2x^2 - 5x - 12)$ | A1 | 1.1b |
| | Attempts to factorise quadratic factor or solve quadratic eqn | dM1 | 1.1b |
| | $f(x) = (x - 4)^2(2x + 3) \Rightarrow f(x) = 0$ has only two roots, 4 and -1.5 | A1 | 2.4 |
| | | (4) | |
| (c) | Deduces either three roots or deduces that $f(x)$ is moved down two units | M1 | 2.2a |
| | States three roots, as when $f(x)$ is moved down two units there will be three points of intersection (with the x - axis) | A1 | 2.4 |
| | | (2) | |
| (d) | For sight of $k = \pm 4, \pm \frac{3}{2}$ | M1 | 1.1b |
| | $k = 4, -\frac{3}{2}$ | A1ft | 1.1b |
| | | (2) | |
| (10 marks) | | | |

Notes

(a)

M1: Attempts to calculate $f(4)$.Do not accept $f(4) = 0$ without sight of embedded values or calculations.If values are not embedded look for two correct terms from $f(4) = 128 - 208 + 32 + 48$ Alternatively attempts to divide by $(x - 4)$. Accept via long division or inspection.

See below for awarding these marks.

A1: Correct reason with conclusion. Accept $f(4) = 0$, hence factor as long as M1 has been scored.This should really be stated on one line after having performed a correct calculation. It could appear as a preamble if the candidate states "If $f(4) = 0$, then $(x - 4)$ is a factor before doing the calculation and then writing hence proven or ✓ oe.If division/inspection is attempted it must be correct and there must be some attempt to explain why they have shown that $(x - 4)$ is a factor. Eg Via division they must state that there is no remainder, hence factor

(b)

M1: Attempts to find the quadratic factor by inspection (correct first and last terms) or by division (correct first two terms)So for inspection award for $2x^3 - 13x^2 + 8x + 48 = (x - 4)(2x^2 \dots x \pm 12)$

$$\begin{array}{r} 2x^2 - 5x \\ x-4 \overline{) 2x^3 - 13x^2 + 8x + 48} \end{array}$$

For division look for

$$\begin{array}{r} 2x^3 - 8x^2 \\ \underline{-5x^2} \end{array}$$

A1: Correct quadratic factor $(2x^2 - 5x - 12)$ For division award for sight of this "in the correct place" You don't have to see it paired with the $(x - 4)$ for this mark.**If a student has used division in part (a) they can score the M1 A1 in (b) as soon as they start attempting to factorise their $(2x^2 - 5x - 12)$.****dM1:** Correct attempt to solve or factorise their $(2x^2 - 5x - 12)$ including use of formulaApply the usual rules $(2x^2 - 5x - 12) = (ax + b)(cx + d)$ where $ac = \pm 2$ and $bd = \pm 12$ Allow the candidate to move from $(x - 4)(2x^2 - 5x - 12)$ to $(x - 4)^2(2x + 3)$ for this mark.

M1: Correct attempt to solve or factorise their $(2x^2 - 5x - 12)$ including use of formula
 Apply the usual rules $(2x^2 - 5x - 12) = (ax + b)(cx + d)$ where $ac = \pm 2$ and $bd = \pm 12$
 Allow the candidate to move from $(x - 4)(2x^2 - 5x - 12)$ to $(x - 4)^2(2x + 3)$ for this mark.

A1: Via factorisation

Factorises twice to $f(x) = (x - 4)(2x + 3)(x - 4)$ or $f(x) = (x - 4)^2(2x + 3)$ or

$f(x) = 2(x - 4)^2\left(x + \frac{3}{2}\right)$ followed by a valid explanation why there are only two roots.

The explanation can be as simple as

- hence $x = 4$ and $-\frac{3}{2}$ (only). The roots must be correct
- only two distinct roots as 4 is a repeated root

There must be some understanding between roots and factors.

E.g. $f(x) = (x - 4)^2(2x + 3)$

only two distinct roots is insufficient.

This would require two distinct factors, so there are two distinct roots.

Via solving.

Factorises to $(x - 4)(2x^2 - 5x - 12)$ and solves $2x^2 - 5x - 12 = 0 \Rightarrow x = 4, -\frac{3}{2}$ followed

by an explanation that the roots are $4, 4, -\frac{3}{2}$ so only two distinct roots.

Note that this question asks the candidate to use algebra so you cannot accept any attempt to use their calculators to produce the answers.

(c)

M1: For a valid **deduction**.

Accept **either** there are 3 roots **or** state that it is a solution of $f(x) = 2$ or $f(x) - 2 = 0$

A1: Fully explains:

Eg. States three roots, as $f(x)$ is moved down by **two** units (giving three points of intersection with the x - axis)

Eg. States three roots, as it is where $f(x) = 2$ (You may see $y = 2$ drawn on the diagram)

(d)

M1: For sight of ± 4 **and** $\pm \frac{3}{2}$ Follow through on \pm their roots.

A1ft: $k = 4, -\frac{3}{2}$ Follow through on their roots. Accept $4, -\frac{3}{2}$ but not $x = 4, -\frac{3}{2}$

Q4.

| Question | Scheme | Marks | AOs |
|------------|--|----------|--------------|
| (a) | $g(5) = 2 \times 5^3 + 5^2 - 41 \times 5 - 70 = \dots$ | M1 | 1.1a |
| | $g(5) = 0 \Rightarrow (x-5)$ is a factor, hence $g(x)$ is divisible by $(x-5)$. | A1 | 2.4 |
| | | (2) | |
| (b) | $2x^3 + x^2 - 41x - 70 = (x-5)(2x^2 \dots x \pm 14)$ | M1 | 1.1b |
| | $= (x-5)(2x^2 + 11x + 14)$ | A1 | 1.1b |
| | Attempts to factorise quadratic factor | dM1 | 1.1b |
| | $(g(x)) = (x-5)(2x+7)(x+2)$ | A1 | 1.1b |
| | | (4) | |
| (c) | $\int 2x^3 + x^2 - 41x - 70 \, dx = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$ | M1 A1 | 1.1b 1.1b |
| | Deduces the need to use $\int_{-2}^5 g(x) \, dx$ | M1 | 2.2a |
| | $\frac{1525}{3} - \frac{190}{3}$ | | |
| | Area = $571\frac{2}{3}$ | A1 | 2.1 |
| | | (4) | |
| (10 marks) | | | |

Notes

(a)

M1: Attempts to calculate $g(5)$ Attempted division by $(x-5)$ is M0

Look for evidence of embedded values or two correct terms of

$$g(5) = 250 + 25 - 205 - 70 = \dots$$

A1: Correct calculation, reason and conclusion. It must follow M1. Accept, for example,

$$g(5) = 0 \Rightarrow (x-5) \text{ is a factor, hence divisible by } (x-5)$$

$$g(5) = 0 \Rightarrow (x-5) \text{ is a factor } \checkmark$$

Do not allow if candidate states

$$f(5) = 0 \Rightarrow (x-5) \text{ is a factor, hence divisible by } (x-5) \quad \text{(It is not f)}$$

$$g(x) = 0 \Rightarrow (x-5) \text{ is a factor} \quad \text{(It is not } g(x) \text{ and there is no conclusion)}$$

This may be seen in a preamble before finding $g(5) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to find the quadratic factor by inspection (correct coefficients of first term and \pm last term) or by division (correct coefficients of first term and \pm second term). Allow this to be scored from division in part (a)**A1:** $(2x^2 + 11x + 14)$ You may not see the $(x-5)$ which can be condoned**dM1:** Correct attempt to factorise their $(2x^2 + 11x + 14)$

A1: $(g(x) = (x-5)(2x+7)(x+2)$ or $(g(x) = (x-5)(x+3.5)(2x+4)$

It is for the product of factors and not just a statement of the three factors

Attempts with calculators via the three roots are likely to score 0 marks. The question was "Hence" so the two M's must be awarded.

(c)

M1: For $x^n \rightarrow x^{n+1}$ for any of the terms in x for $g(x)$ so
 $2x^3 \rightarrow \dots x^4$, $x^2 \rightarrow \dots x^3$, $-41x \rightarrow \dots x^2$, $-70 \rightarrow \dots x$

A1: $\int 2x^3 + x^2 - 41x - 70 \, dx = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$ which may be left unsimplified (ignore any reference to $+C$)

M1: Deduces the need to use $\int_{-2}^5 g(x) \, dx$.

This may be awarded from the limits on their integral (either way round) or from embedded values which can be subtracted either way round.

A1: For clear work showing all algebraic steps leading to $\text{area} = 571\frac{2}{3}$ oe

So allow $\int_{-2}^5 2x^3 + x^2 - 41x - 70 \, dx = \left[\frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x \right]_{-2}^5 = -\frac{1715}{3} \Rightarrow \text{area} = \frac{1715}{3}$

for 4 marks

Condone spurious notation, as long as the algebraic steps are correct. If they find $\int_{-2}^5 g(x) \, dx$

then withhold the final mark if they just write a positive value to this integral since

$$\int_{-2}^5 g(x) \, dx = -\frac{1715}{3}$$

Note $\int_{-2}^5 2x^3 + x^2 - 41x - 70 \, dx \Rightarrow \frac{1715}{3}$ with no algebraic integration seen scores M0A0M1A0

Q5.

| Question | Scheme | Marks | AOs |
|---|--|-------|------|
| | Sets $f(-2) = 0 \Rightarrow 2 \times (-2)^3 - 5 \times (-2)^2 + a \times -2 + a = 0$ | M1 | 3.1a |
| | Solves linear equation $2a - a = -36 \Rightarrow a =$ | dM1 | 1.1b |
| | $\Rightarrow a = -36$ | A1 | 1.1b |
| (3 marks) | | | |
| Notes: | | | |
| <p>M1: Selects a suitable method given that $(x + 2)$ is a factor of $f(x)$ Accept either setting $f(-2) = 0$ or attempted division of $f(x)$ by $(x + 2)$</p> <p>dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a = .. \Rightarrow a =$</p> <p>A1: $a = -36$</p> | | | |

Q6.

| Question | Scheme | Marks | AOs |
|------------------|--|-------|------|
| | (a) $f(x) = -3x^3 + 8x^2 - 9x + 10, x \in \mathbb{R}$ | | |
| (a) | (i) $\{f(2) = -24 + 32 - 18 + 10 \Rightarrow\} f(2) = 0$ | B1 | 1.1b |
| | (ii) $\{f(x) = \} (x-2)(-3x^2 + 2x - 5)$ or $(2-x)(3x^2 - 2x + 5)$ | M1 | 2.2a |
| | | A1 | 1.1b |
| | | (3) | |
| (b) | $-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$ | | |
| | Gives a partial explanation by <ul style="list-style-type: none"> explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only} | M1 | 2.4 |
| | Complete proof that the given equation has exactly two {real} solutions | A1 | 2.1 |
| | | (2) | |
| (c) | $3 \tan^3 \theta - 8 \tan^2 \theta + 9 \tan \theta - 10 = 0; 7\pi \leq \theta < 10\pi$ | | |
| | {Deduces that} there are 3 solutions | B1 | 2.2a |
| | | (1) | |
| (6 marks) | | | |

| Notes for Question | |
|--------------------|---|
| (a)(i) | |
| B1: | $f(2) = 0$ or 0 stated by itself in part (a)(i) |
| (a)(ii) | |
| M1: | Deduces that $(x-2)$ or $(2-x)$ is a factor and attempts to find the other quadratic factor by <ul style="list-style-type: none"> using long division to obtain either $\pm 3x^2 \pm kx + \dots$, $k = \text{value} \neq 0$ or $\pm 3x^2 \pm \alpha x + \beta$, $\beta = \text{value} \neq 0$, α can be 0 factorising to obtain their quadratic factor in the form $(\pm 3x^2 \pm kx \pm c)$, $k = \text{value} \neq 0$, c can be 0, or in the form $(\pm 3x^2 \pm \alpha x \pm \beta)$, $\beta = \text{value} \neq 0$, α can be 0 |
| A1: | $(x-2)(-3x^2+2x-5)$, $(2-x)(3x^2-2x+5)$ or $-(x-2)(3x^2-2x+5)$ stated together as a product |
| (b) | |
| M1: | See scheme |
| A1: | See scheme. Proof must be correct <i>with no errors</i> , e.g. giving an incorrect discriminant value |
| Note: | Correct calculation e.g. $(2)^2 - 4(-3)(-5)$, $4 - 60$ or -56 must be given for the first explanation |
| Note: | Note that M1 can be allowed for <ul style="list-style-type: none"> a correct follow through calculation for the discriminant of their "$-3y^4 + 2y^2 - 5$" which would lead to a value < 0 together with an explanation that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions or for the omission of < 0 |
| Note: | < 0 must also be stated in a discriminant method for A1 |
| Note: | Do not allow A1 for incorrect working, e.g. $(2)^2 - 4(-3)(-5) = -54 < 0$ |
| Note: | $y^2 = 2 \Rightarrow y = \pm 2$, so 2 solutions is not allowed for A1, but can be condoned for M1 |
| Note: | Using the formula on $-3y^4 + 2y^2 - 5 = 0$ or $-3x^2 + 2x - 5 = 0$ gives y^2 or $x = \frac{-2 \pm \sqrt{-56}}{-6}$ or $\frac{-1 \pm \sqrt{-14}}{-3}$ |
| Note: | Completing the square on $-3x^2 + 2x - 5 = 0$ gives $x^2 - \frac{2}{3}x + \frac{5}{3} = 0 \Rightarrow \left(x - \frac{1}{3}\right)^2 - \frac{1}{9} + \frac{5}{3} = 0 \Rightarrow x = \frac{1}{3} \pm \sqrt{\frac{-14}{9}}$ |
| Note: | Do not recover work for part (b) in part (c) |
| (c) | |
| B1: | See scheme |
| Note: | Give B0 for stating $\theta = \text{awrt } 23.1, \text{awrt } 26.2, \text{awrt } 29.4$ without reference to 3 solutions |

Q7.

| Part | Working or answer an examiner might expect to see | Mark | Notes |
|------|---|------|---|
| | $f(-3) = 3 \times (-3)^2 + 2a \times (-3)^2 - 4 \times -3 + 5a = 0$ | M1 | This mark is given for a method to set $f(-3) = 0$ |
| | $f(-3) = 23a - 69 = 0$ $23a = 69$ | M1 | This mark is given for finding an equation to solve for a |
| | $a = 3$ | A1 | This mark is given for finding the correct value of a |

Q8.

| Question | Scheme | Marks | AOs |
|-----------|---|-------|------|
| | $f(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$ | M1 | 3.1a |
| | $6 - 2a = 0 \Rightarrow a = \dots$ | M1 | 1.1b |
| | $a = 3$ | A1 | 1.1b |
| | | (3) | |
| (3 marks) | | | |
| Notes | | | |

Main method seen:

M1: Attempts $f(1) = 0$ to set up an equation in a It is implied by $a + 10 - 3a - 4 = 0$ Condone a slip but attempting $f(-1) = 0$ is M0M1: Solves a linear equation in a .Using the main method it is dependent upon having set $f(\pm 1) = 0$ It is implied by a solution of $\pm a \pm 10 \pm 3a \pm 4 = 0$.

Don't be concerned about the mechanics of the solution.

A1: $a = 3$ (following correct work)

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 Answers without working scores 0 marks. The method must be made clear. Candidates cannot guess.

However if a candidate states for example, when $a = 3$, $f(x) = 3x^3 + 10x^2 - 9x - 4$ and shows that $(x - 1)$ is a factor of this $f(x)$ by an allowable method, they should be awarded M1 M1 A1

E.g. 1: $3x^3 + 10x^2 - 9x - 4 = (x - 1)(3x^2 + 13x + 4)$ Hence $a = 3$

E.g. 2: $f(x) = 3x^3 + 10x^2 - 9x - 4$, $f(1) = 3 + 10 - 9 - 4 = 0$ Hence $a = 3$

The solutions via this method must end with the value for a to score the A1

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Other methods are available. They are more difficult to determine what the candidate is doing. Please send to review if you are uncertain

It is important that a correct method is attempted so look at how the two M's are scored

Amongst others are:

| | | | |
|------|---------|-------------|------|
| | ax^2 | $(10+a)x$ | 4 |
| x | ax^3 | $(10+a)x^2$ | $4x$ |
| -1 | $-ax^2$ | $-(10+a)x$ | -4 |

Alt (1) by inspection which may be seen in a table

$$ax^3 + 10x^2 - 3ax - 4 = (x-1)(ax^2 + (10+a)x + 4) \quad \text{and sets terms in } x \text{ equal}$$

$$-3a = -(10+a) + 4 \Rightarrow 2a = 6 \Rightarrow a = 3$$

M1: This method is implied by a correct equation, usually $-3a = -(10+a) + 4$

M1: Attempts to find the quadratic factor which must be of the form $ax^2 + g(a)x + 4$ and then forms and solves a linear equation formed by linking the coefficients or terms in x

Alt (2) By division:

$$\begin{array}{r}
 ax^2 + (\pm 10 \pm a)x + (10 - 2a) \\
 x-1 \overline{) ax^3 + 10x^2 - 3ax - 4} \\
 \underline{ax^3 - ax^2} \\
 (10+a)x^2 - 3ax \\
 \underline{(10+a)x^2 - (10+a)x} \\
 (-2a+10)x - 4
 \end{array}$$

M1: This method is implied by a correct equation, usually $-10 + 2a = -4$

M1: Attempts to divide with quotient of $ax^2 + (\pm 10 \pm a)x + h(a)$ and then forms and solves a linear equation in a formed by setting the remainder = 0.