Quadratics Cheat Sheet

Edexcel Pure Year 1

Similar to the quadratic expression, quadratic equation can be represented in the form $ax^2 + bx + c = 0$, where a, b and c are real number and $a \neq 0$.

Solving Quadratic Equations

- To solve quadratic equations, a given equation must be rewritten in the form or kept in form of $\underline{ax^2 + bx + c} = 0$.
- Once in the correct form, the left-hand side of the equation, underlined in red, must be factorised and the factors must be equated to 0. For example, if the factorisation came to $(x+c)(x+d) = 0$, to solve for 'x' you have to set the factor $(x+c) = 0$ and the factor $(x+d) = 0$ and find the values of x for each respective case.

Note that quadratic equations can only have one, two or no real solutions.

Example 1: Solve the following quadratic equation.

$$3x^2 - 2x - 8 = 0$$

Factorising
$$3x^2 - 6x + 4x - 8 = 0$$
$$3x(x-2) + 4(x-2) = 0$$

Solving for values of x
$$(3x+4)(x-2) = 0$$
$$\therefore x = -\frac{4}{3} \quad and \quad x = 2$$

You may come across quadratic equations that may seem impossible to solve through factorisation. In this scenario, we could utilise the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 2: Solve the following equation using the quadratic formula.

$$2x^2 - 8x + 3 = 0$$
$$a = 2, b = -8 \; and \; c = 3$$

- Substitute the values of a, b and c into the quadratic formula

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 2 \times 3}}{2 \times 2}$$

$$x = \frac{4 + \sqrt{10}}{2} \qquad x = \frac{4 - \sqrt{10}}{2}$$

Completing the square

Rewriting equations or expressions by completing the square, can be applied to many different applications in maths. Hence this would be regularly used in your further studies.

- $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$
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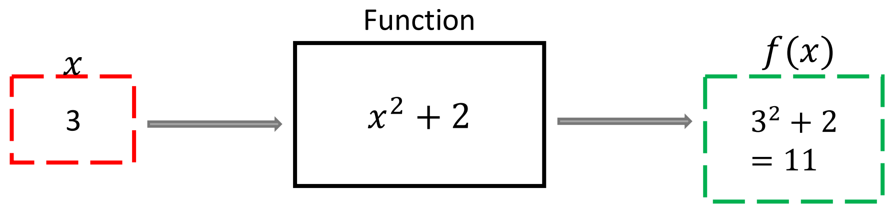
Example 3: Write the following expression in the form of $(x+a)^2 + b$

$$x^2 + 6x + 2$$
$$\left(x + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 2$$
$$(x+3)^2 - 7$$

Functions

A function can be seen as a machine that takes in an input, converts this value mathematically and gives an output. The input is most commonly denoted with the term 'x' and the output is most commonly represented as $f(x)$ or $g(x)$. For a given function, the set of possible inputs is called domain and the set of possible outputs is called range.



During your mathematics course you will come across the questions or statements which are related to 'finding the roots of a function'. The roots of a function are the values of the input x for which the output $f(x)$ is equal to 0 ($\therefore f(x) = 0$).

Example 3: Find the value $f(2)$ of the following function and find the roots of the function.

a) $f(x) = 2x^2 + 5x - 3$
$$f(2) = 2(2)^2 + 5(2) - 3 = 15$$

b) $f(x) = 2x^2 + 5x - 3$

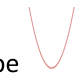
To find the root of a function we have to equate the output to 0.
$$f(x) = 0$$
$$2x^2 + 5x - 3 = 0$$
$$(2x-1)(x+3) = 0$$
$$2x - 1 = 0 \; or \; x + 3 = 0$$

Hence the roots of the function are:
$$x = \frac{1}{2} \; and \; x = -3$$

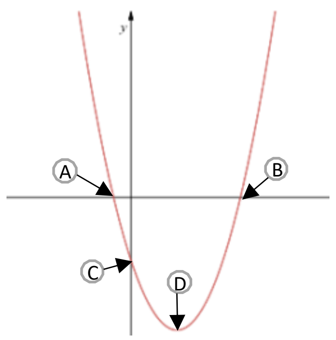
Quadratic Graphs

For functions which come in the form of a quadratic expression, the plot of $y = f(x)$ would be illustrated on a graph in the form of a shape called a parabola.

For a given quadratic function $f(x) = ax^2 + bx + c$, if

- a is positive the shape of the parabola would be 
- a is negative the shape of the parabola would be 

For a given quadratic graph, points A and B are the roots of the function as this is where the graph intercepts the $x$axis and at these two points the output $y = 0$.



At point C on the graph is where the $y = c$ as $x = 0$ at this point. In other words, $f(0) = c$.

Point D on the graph represents the 'turning' or 'stationery' point. The coordinates of the turning point of the graph can be found by completing the square

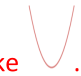
$$f(x) = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Hence the turning point would be at $\left(-\frac{b}{2}, -\left(\frac{b}{2}\right)^2 + c\right)$

Example 4: Sketch the graph of $f(x) = 2x^2 + 5x - 3$. Label all the intercepts and turning point.

Before we start sketching, we need key pieces of information about the characteristics of the graph, which can be obtained by doing some calculations using the methods you have learnt so far.

We can compare the function to a general function of $f(x) = ax^2 + bx + c$ to determine the shape of the parabola.

$a = 2$ and $2 > 0$ hence the shape of the graph would look like .

The next step would be to find the roots of the functions so we can determine where it would cross the x axis. To do this we need to solve the quadratic equation of
$$2x^2 + 5x - 3 = 0$$
$$(2x-1)(x+3) = 0$$
$$2x - 1 = 0 \; or \; x + 3 = 0$$

Hence the roots of the function are:
$$x = \frac{1}{2} \; and \; x = -3$$

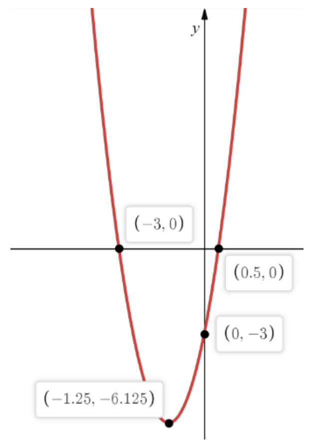
At coordinates $\left(\frac{1}{2}, 0\right)$ & $(-3, 0)$ is where the x intercepts will be located.

The last bit of information we need is the location of turning point for which we need to complete the square.

$$f(x) = 2x^2 + 5x - 3$$
$$f(x) = 2\left[x^2 + \frac{5}{2}x - \frac{3}{2}\right]$$
$$f(x) = 2\left[\left(x + \frac{5}{4}\right)^2 - \frac{25}{16} - \frac{3}{2}\right]$$
$$f(x) = 2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}$$

Hence the turning point would have a coordinate of $\left(-\frac{5}{4}, -\frac{49}{8}\right)$.

Using all the information that we just obtained we can sketch the graph.



The Discriminant

The expression $b^2 - 4ac$ is called the discriminant. The value obtained using the discriminant expression on a quadratic function will indicate how many roots a given function $f(x)$ has.

For the quadratic function $f(x) = ax^2 + bx + c$

- If the discriminant $b^2 - 4ac > 0$, then the function $f(x)$ has two distinct real roots.
- If the discriminant $b^2 - 4ac = 0$, then the function $f(x)$ has one repeated real root.
- If the discriminant $b^2 - 4ac < 0$, then the function $f(x)$ has no real roots.

Example 5: Find the range of values of k for which $x^2 + 2x + k = 0$ has two distinct real roots.

$$x^2 + 2x + k = 0$$
$$a = 1, b = 2 \text{ and } c = k$$
$$b^2 - 4ac > 0 \; for \; two \; distinct \; real \; roots$$
$$2^2 - 4 \times 1 \times k > 0$$
$$2^2 - 4k > 0$$
$$-4k > -4$$
$$k < 1$$