Equations and Inequalities Cheat Sheet

This chapter covers both linear and quadratic simultaneous equations and how to solve them algebraically. You should also be able to interpret solutions of a given equation graphically. It also cover both linear and quadratic inequalities.

Linear Simultaneous Equations

Linear simultaneous equations have two same unknowns in their respective equation and has one set of values between them which makes both the equations valid.

① $2x + y = 6$

② $6x + 2y = 24$

① $2 \times 6 + (-6) = 6$

② $6 \times 6 + 2 \times -6 = 24$

In the two equations 1 and 2, x and y are two unknowns. For the equations to be valid, the x value in equation 1 has to be the same x value in equation 2; the same would apply for the unknown y. The only set of values for which this would be true is if the unknowns have the values of $x = 6$ & $y = -6$

In order to solve these simultaneous equations and find the set of values which make the given equations valid, we can use either the method of elimination or substitution.

Example 1: Solve the following simultaneous equation.

$$1)\ 2x + y = 6$$
$$2)\ 6x + 2y = 24$$

We can use the method of elimination to solve this. To eliminate one unknown, we need the coefficient of a single unknown to be the same for both the equations. In order to achieve this, we can multiply the first equation by 2. This would give use a third equation which is equivalent to equation 1:

$$3)\ 4x + 2y = 12$$

Now we can eliminate the unknown y by taking away equation 3) from 2).

② $6x + 2y = 24$
③ $4x + 2y = 12$ −

$$2x + 0 = 12$$

$$2x = 12$$
$$x = 6$$
$$1)\ \ 2 \times 6 + y = 6$$
$$y = -6$$

Quadratic Simultaneous Equations

You may come across simultaneous equations where one equation is quadratic, and one equation is linear. For this scenario you will need to use the method of substitution. As a quadratic equation is involved there can be up to two sets of values/solutions for the simultaneous equation.

Example 2: Solve the simultaneous equation.

$$1)\ y^2 + 2x = 10$$
$$2)\ 2x + y + 2 = 0$$

Equation 2 can be rewritten as:

$$2)\ x = -1 - \frac{y}{2}$$

We can substitute this rewritten equation into equation 1).

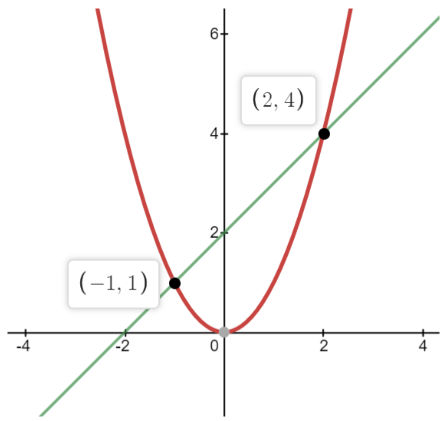
$$1)\ y^2 + 2\left(-1 - \frac{y}{2}\right) = 10$$
$$y^2 - 2 - y = 10$$
$$y^2 - y - 12 = 0$$
$$(y + 3)(y - 4) = 0$$
$$y = -3 \text{ and } y = 4$$

Substitute these two y values into equation 2) to obtain the x values.

$$x = -1 - \frac{-3}{2} = \frac{1}{2} \text{ and } x = -1 - \frac{4}{2} = -3$$

Simultaneous Equations on graphs

The solutions of a set of simultaneous equation can be represented on a graph. Simultaneous equations share the same set of values for the unknowns, hence if two given simultaneous equations were illustrated on a graph then at some point on their respective plot, they would share the same coordinate and hence intersect. Hence, the intersection point on a graph of two lines would be the solution or at least one of the solutions for the curves' or lines' simultaneous equation.



The graph on the left shows the plot of $y = x^2$ and $y - x - 2 = 0$.
If we take the point (2,4), we can check if this set of values are valid for both the equations.

$y = x^2$ $\quad (4) = (2)^2$ ✔
$y - x - 2 = 0$ $\quad (4) - (2) - 2 = 0$ ✔

e can also check if the point $(-1,1)$ agrees th the two equations.

$y = x^2$ $\quad (1) = (-1)^2$ ✔
$y - x - 2 = 0$ $\quad (1) - (-1) - 2 = 0$ ✔

Hence from this we can see that we can use graphs to solve simultaneous equations by plotting and observing any intersection points.

- Note that when solving simultaneous equations that come in the form of a quadratic equation $ax^2 + bx + c = 0$, the discriminant of the equation after substituting can be used to determine the number of solutions that the simultaneous equations have. Hence, on a graph it can also indicate the number of intersection points.

Linear Inequalities

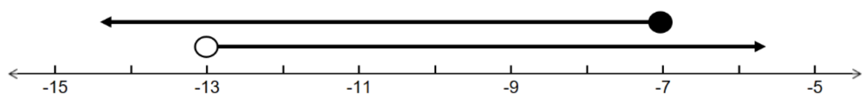
Similar to the methods we have learnt to solve linear equations, we can also solve linear inequality problems using the same approach. When you solve an inequality, you find the set of all real numbers that make the inequality valid.

Example 3: Find the set of values for which

$$2x - 5 < 3x + 8 \quad and \quad 3x + 9 \le x - 5$$

$2x - 5 < 3x + 8$
$-13 < x$
$x > -13$

$3x + 9 \le x - 5$
$2x \le -14$
$x \le -7$



We can plot this inequality on a number line to find the set of values which agree with both the inequalities. The area which overlaps on the number line is between -13 and -7. Hence the solution for this inequality would be $-13 < x \le -7$.

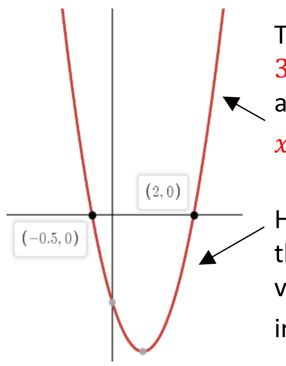
Quadratic Inequalities

Similar to the method we used to solve quadratic equations, we can also solve quadratic inequalities.

Let us look at the inequality $2x^2 - 3x - 2 > 0$. To solve this, we need to first find the critical (similar to roots of a function) values of this inequality by solving the quadratic equation on the left-hand side.

$$2x^2 - 3x - 2 > 0$$
$$(2x + 1)(x - 2) > 0$$

Hence one critical point is at $x = -\frac{1}{2}$ and the other one at $x = 2$. Now we can plot the graph of $2x^2 - 3x - 2$.



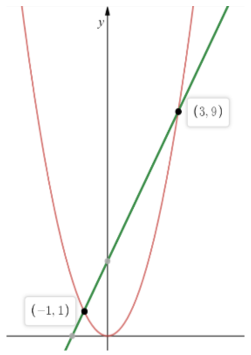
The set of values which corresponds to the inequality $2x^2 - 3x - 2 > 0$, are the x values of the plot which are above the x axis. Hence, the solution to this inequality would be $x > 2$ and $x < -\frac{1}{2}$.

However, if the inequality were to be $2x^2 - 3x - 2 < 0$, then the set of values which would correspond to it, would be the x values that are below the x axis. Hence, the solution to this inequality would be $-\frac{1}{2} < x < 2$

Inequalities on graphs and Regions

You may come across question where you are asked to find the solutions to the inequality by interpreting the functions graphically.

Example 4: The graph shows the plot of $y = x^2$ and $y = 2x + 3$. Determine the solutions to the inequality $2x + 3 > x^2$.



First you will need to find the points of intersection, which can be done by equating the two equations and solving it.

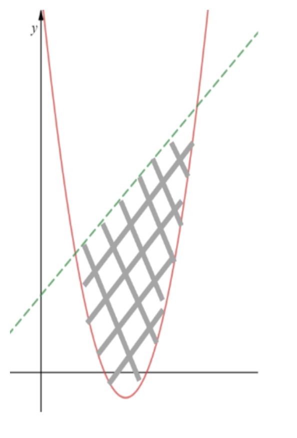
$$x^2 = 2x + 3$$
$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$
$$x = 3 \ and \ x = -1$$

Hence the two points of intersection are (3,9) and $(-1,1)$. The set of values that validates to the inequality $2x + 3 > x^2$ is the line $y = 2x + 3$ is above the curve $y = x^2$. Hence, the set of values lie between the intersection points $-1 < x < 3$

Regions on graphs can be shaded to identify the areas that satisfies given linear or quadratic inequalities.

Example 5: Shade the regions which satisfy the inequalities.

$$x^2 - 8x + 15 \le y$$
$$y - x < 3$$



The graph of $f(x) = x^2 - 8x + 15$ and $f(x) = x + 3$ is plotted.

- If $y > f(x)$, then this would represent the region above the curve or line.
- If $y < f(x)$, then this would represent the region below the curve or line.

Therefore, for the inequality $y - x < 3$, the region satisfied is represented by the area below the green dotted line and for the inequality $x^2 - 8x + 15 \le y$, the area above the red curve. Hence the region satisfied for both the inequalities is illustrated by the grey shaded area.