Basic Algebra Questions – Mainly Quadratics

3 (a) (i) Express $x^2 - 4x + 9$ in the form $(x-p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y = x^2 - 4x + 9$. *(2 marks)*

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4 The quadratic equation $x^2 + (m+4)x + (4m+1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

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2 (a) Express $x^2 + 8x + 19$ in the form $(x+p)^2 + q$, where p and q are integers. *(2 marks)*

(b) Hence, or otherwise, show that the equation $x^2 + 8x + 19 = 0$ has no real solutions. *(2 marks)*

(c) Sketch the graph of $y = x^2 + 8x + 19$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(d) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 8x + 19$. *(3 marks)*

(ii) Find the values of k for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

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7 The quadratic equation $(k+1)x^2 + 12x + (k-4) = 0$ has real roots.

(a) Show that $k^2 - 3k - 40 \leqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

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3 (a) (i) Express $x^2 + 10x + 19$ in the form $(x+p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Write down the coordinates of the vertex (minimum point) of the curve with equation $y = x^2 + 10x + 19$. *(2 marks)*

(iii) Write down the equation of the line of symmetry of the curve $y = x^2 + 10x + 19$. *(1 mark)*

(iv) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 10x + 19$. *(3 marks)*

(b) Determine the coordinates of the points of intersection of the line $y = x + 11$ and the curve $y = x^2 + 10x + 19$. *(4 marks)*

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7 The quadratic equation

$$(2k-3)x^2 + 2x + (k-1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leqslant 0$. *(3 marks)*

(b) (i) Factorise $2k^2 - 5k + 2$. *(1 mark)*

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leqslant 0$$ *(3 marks)*

Basic Algebra Answers – Mainly Quadratics

3(a)(i)	$(x-2)^2$ $+5$	B1 B1	2	$p = 2$ $q = 5$
(ii)	Minimum point (2, 5) or $x = 2$, $y = 5$	B2√	2	B1 for each coordinate correct or ft **Alt method** M1, A1 sketch, differentiation

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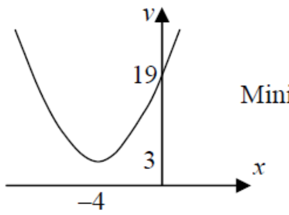
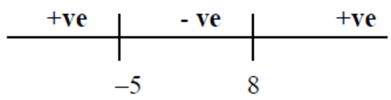
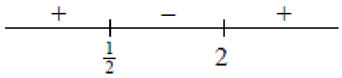
4(a)	$(m+4)^2 = m^2 + 8m + 16$	B1		Condone $4m + 4m$
	$b^2 - 4ac = (m+4)^2 - 4(4m+1) = 0$	M1		$b^2 - 4ac$ (attempted and involving m's and no x's) or $b^2 - 4ac = 0$ stated
	$m^2 + 8m + 16 - 16m - 4 = 0$ $\Rightarrow m^2 - 8m + 12 = 0$	A1	3	**AG** (be convinced – all working correct- $= 0$ appearing more than right at the end)
(b)	$(m-2)(m-6) = 0$	M1		Attempt at factors or quadratic formula
	$m = 2$, $m = 6$	A1	2	**SC B1** for 2 or 6 only without working
	Total		**5**	

2(a)	$(x+4)^2$ $+3$	B1 B1	2	$p = 4$ $q = 3$
(b)	$(x+4)^2 = -3$ or "their" $(x+p)^2 = -q$	M1		Or discriminant = 64 –76
	No real square root of –3	A1	2	Disc < 0 so no real roots (all correct figs)
(c)	 Minimum (– 4, 3)	B1√		ft their $-p$ and q (or correct)
	graph	B1		Parabola (vertex roughly as shown)
		B1	3	Crossing at $y = 19$ marked or (0, 19) stated
(d)	Translation (and no additional transf'n)	E1		Not shift, move, transformation, etc
	through $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$	M1 A1	3	One component correct eg 3 units up All correct – if not vector – must say 4 units in negative x- direction, to left etc
	Total		**10**	

(ii)	$4(k+1)^2 - 4(2k^2 - 7)$	M1		"$b^2 - 4ac$" in terms of k (either term correct)
	$4k^2 - 8k - 32 = 0$ or $k^2 - 2k - 8 = 0$	A1		$b^2 - 4ac = 0$ correct quadratic equation in k
	$(k-4)(k+2) = 0$	m1		Attempt to factorise, solve equation
	$k = -2$, $k = 4$	A1	4	SC B1, B1 for –2 , 4 (if M0 scored)

Q	Solution	Marks	Total	Comments
7(a)	$b^2 - 4ac \; = \; 144 - 4(k+1)(k-4)$	M1		Clear attempt at $b^2 - 4ac$ Condone slip in one term of expression
	Real roots when $b^2 - 4ac \geqslant 0$	B1		Not just a statement, must involve k
	$36 - (k^2 - 3k - 4) \geqslant 0$ $\Rightarrow k^2 - 3k - 40 \leqslant 0$	A1	3	**AG** (watch signs carefully)
(b)	$(k-8)(k+5)$	M1		Factors attempt or formula
	Critical points 8 and -5	A1		
	Sketch or sign diagram **correct,** must have 8 and -5	M1		
	$-5 \leqslant k \leqslant 8$	A1	4	
	A0 for $-5 < k < 8$ or two separate inequalities unless word AND used			
	Total		**7**	

Q	Solution	Marks	Total	Comments
3(a)(i)	$(x+5)^2$	B1		$p = 5$
	-6	B1	2	$q = -6$
(ii)	$x_{\text{vertex}} = -5$ (or their $-p$) $y_{\text{vertex}} = -6$ (or their q)	B1√ B1√	2	may differentiate but must have $x = -5$ and $y = -6$. Vertex $(-5, -6)$
(iii)	$x = -5$	B1	1	
(iv)	Translation (not shift, move etc)	E1		and NO other transformation stated
	through $\begin{bmatrix} -5 \\ -6 \end{bmatrix}$ (or 5 left, 6 down)	M1 A1	3	either component correct M1, A1 independent of E mark
(b)	$x + 11 = x^2 + 10x + 19$			quadratic with all terms on one side of equation
	$\Rightarrow x^2 + 9x + 8 = 0$ or $y^2 - 13y + 30 = 0$	M1		
	$(x+8)(x+1) = 0$ or $(y-3)(y-10) = 0$	m1		attempt at formula (1 slip) or to factorise
	$\left.\begin{array}{l} x = -1 \\ y = 10 \end{array}\right\}$ or $\left.\begin{array}{l} x = -8 \\ y = 3 \end{array}\right\}$	A1 A1	4	both x values correct both y values correct and linked **SC** $(-1, 10)$ B2, $(-8, 3)$ B2 no working
	Total		**12**	

7(a)	$b^2-4ac=4-4(k-1)(2k-3)$	M1		(or seen in formula) condone one slip
	Real roots when $b^2-4ac\geqslant 0$	E1		must involve $\mathrm{f}(k)\geqslant 0$ (usually M1 must be earned)
	$4-4\left(2k^2-5k+3\right)\geqslant 0$			
	$\Rightarrow -2k^2+5k-3+1\geqslant 0$			at least one step of working justifying $\leqslant 0$
	$\Rightarrow 2k^2-5k+2\leqslant 0$	A1	3	AG
(b)(i)	$(2k-1)(k-2)$	B1	1	
(ii)	(Critical values) $\frac{1}{2}$ and 2	B1√		ft their factors or correct values seen on diagram, sketch or inequality or stated
		M1		use of sketch / sign diagram
	$\Rightarrow 0.5\leqslant k\leqslant 2$	A1	3	M1A0 for $0.5<k<2$ or $k\geqslant 0.5,\ k\leqslant 2$
	Total		**7**	