GCE A LEVEL MARKING SCHEME

SUMMER 2019

A LEVEL (NEW)
FURTHER MATHEMATICS
UNIT 5 FURTHER STATISTICS B
1305U50-1

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE FURTHER MATHEMATICS

A2 UNIT 5 FURTHER STATISTICS B

SUMMER 2019 MARK SCHEME

Qu. No.	Solution	Mark	Notes
1 (a)	$\Sigma x = 249.6$ $\quad \Sigma x^2 = 7792.26$		
	$\bar{x} = 31.2$	B1	
	$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right)$		
	$= \frac{237}{350} = 0.677\ldots\ldots$	B1	
	DF = 7	B1	
	t value = 2.365	B1	FT their DOF
	Standard error = $\sqrt{\frac{0.677\ldots}{8}}$	B1	si
	CL = $31.2 \pm 2.365 \times \sqrt{\frac{0.677\ldots}{8}}$	M1	FT their $\bar{x}$, *t* value and s.e.
	95%CI is [30.5,31.9]	A1	cao
(b)	Appropriate explanation. e.g. The Central Limit Theorem is not required because the underlying distribution is normal. e.g. The Central Limit Theorem is not used because n is small.	E1 **Total [8]**	
2 (a)	E(X) = θ + 2	B1	
	Var (X) = 3	B1	
(b)	E($\bar{X}$) = θ + 2 OR E($\bar{X} - 2$) = θ	M1	FT their linear E(X) for M1A1
	$\bar{X}$ – 2 is an unbiased estimator for θ	A1	
	SE($\bar{X}$ – 2) = SE($\bar{X}$)	M1	Used FT their Var(X)
	$= \sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{3}$ oe	A1 **Total [6]**	

Qu. No.	Solution	Mark	Notes
3 (a)	For maximum acceptable standard deviation distribution must be symmetrical about Mean = 159.45 159.45 + 2.5758σ = 163 OR 159.45 – 2.5758σ = 155.9 σ = 1.378…g	 B1 M1 A1	 si or 2.576 from tables.
(b)	Let the random variable X be the weights of cricket balls. Let the random variable Y be the weights of the tennis balls. Consider W = 3Y – X E(W) = 16 Var(W) = 3^2 Var(Y) + Var(X) = 16.65 P(W < 0) = 0.00004	 M1 A1 M1 A1 M1 A1 **Total [9]**	
4 (a)	(SE of difference of means) $= \sqrt{\frac{40^2}{12} + \frac{40^2}{10}}$ = 17.1(2697677) 98% CI $30 \pm 2.3263\sqrt{\frac{40^2}{12} + \frac{40^2}{10}}$ [-9.84, 69.84]	 M1 A1 M1 A1	 Award M1 for Var = $\frac{40^2}{12} + \frac{40^2}{10}$ Or 2.326 from tables. cao
(b)	We cannot conclude that either protein powder is better than the other in promoting weight gain. Because the confidence interval contains 0	E1 E1	FT their CI.
(c)	$30 - k\sqrt{\frac{40^2}{12} + \frac{40^2}{10}} > 0$ k < 1.7516… Probability from calculator = 0.96008 Or 0.95994 from tables Confidence level 92%	M1 A1 A1 A1	Condone = FT their SE from (a) and their difference in means for possible M1A1A1A1
(d)	Valid assumption e.g. Rest of the diet is the same. They exercise the same amount. They follow the same program for muscle gain.	 E1 **Total [11]**	

<table>
<tr><th>Qu. No.</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>5(a)</td><td>Valid explanation.
e.g. It is a small sample.
e.g.There is no reason to suppose that there is an underlying normal distribution.
e.g. The data is paired.</td><td>E1
E1</td><td>Do not allow contradicting statements.</td></tr>
<tr><td>(b)(i)</td><td>H_0: There is on average no difference between scores of the trainee and experienced examiner.
H_1: The trainee and experienced examiners give different scores on average.
OR $H_0: \eta_1 = \eta_2$ $H_1: \eta_1 \neq \eta_2$

<table><tr><td>Student</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>Difference</td><td>6</td><td>7</td><td>3</td><td>-2</td><td>-8</td><td>-4</td><td>-1</td><td>-10</td></tr></table>

Ranks
<table><tr><td>Student</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>Ranks</td><td>5</td><td>6</td><td>3</td><td>2</td><td>7</td><td>4</td><td>1</td><td>8</td></tr></table>

W^- = Sum of negative ranks OR W^+ = Sum of positive ranks
= 2 + 7 + 4 + 1 + 8 = 22   = 5 + 6 + 3 = 14

Upper CV = 32   Lower CV = 4

Because 22 < 32 (OR 14 > 4) there is insufficient evidence to reject H_0.</td><td>B1

B1

M1
A1

M1
A1

B1

B1</td><td>Both (alternative: H_0: The scores of the trainee and experienced examiner have the same distribution. H_1: The scores of the trainee and experienced examiner don't have the same distribution.)
Accept ranks with opposite signs.
M1 either attempt at ranks. FT one slip in difference for A1</td></tr>
<tr><td>(ii)</td><td>The trainee examiner is suitable to qualify.</td><td>E1

Total [11]</td><td></td></tr>
</table>

Qu. No.	Solution	Mark	Notes
6 (a)	Valid reason. e.g. A consumer, (Hopcyn), would only be concerned with whether the company was overstating and therefore only wish to use a lower tail test. Valid reason. e.g. The company would not wish to overstate the distance the car could travel because they would be liable to have claims of false advertising brought against them, nor understate the distance the car could travel because they would like to claim the greatest mileage possible.	E1 E1	Reasonable explanations.
(b)(i)	H_0: μ = 123 H_1: μ < 123	B1	
(ii)	$\bar{x} = \frac{11007}{90}$ = 122.3 $s^2 = \frac{1}{89} \times \left(1361913 - \frac{11007^2}{90}\right)$ $s = 13.3 \ldots$ p-value = P($\bar{X}$ < 122.3) p-value = 0.30886 Since p > 0.05 there is insufficient evidence to reject H_0. Insufficient evidence to reject the manufacturer's claim that a one hour charge gives 123 miles of travel.	B1 M1 A1 M1 A1 B1 B1 **Total [10]**	*Alternative p-value method* M1 for Test statistic = $\frac{122.3-123}{13.30578\ldots/\sqrt{90}}$ if standardising. A1 p-value from tables = 0.30854 *Alternative CV method* M1$\frac{c-123}{13.30578\ldots/\sqrt{90}} = -1.6449$ A1 c = 120.7 B1 since 120.7<122.3
7	H_0: The median number of sheep sheared by shearers from Wales and New Zealand is the same. H_1: The median number of sheep sheared by shearers from Wales is **more** than the median number of sheep sheared by shearers from New Zealand. Upper critical value is 48 OR Lower CV is 8 The critical region is (U ≥ 48) Critical region is (U ≤ 8) Use of the formula U = $\sum\sum z_{ij}$ U = 7 + 7 + 7 + 6 + 6 + 6 + 5 + 5 U = 0 + 0 + 0 + 1 + 1 + 1 + 2 + 2 = 49 U = 7 49 is in the critical region OR 7 is in the critical region. There is sufficient evidence to reject H_0. There is sufficient evidence to suggest shearers from Wales can shear more sheep, on average, in a given time than shearers from New Zealand.	 B1 B1 B1 M1 A1 B1 B1 **Total [7]**	Accept $H_0: \eta_1 = \eta_2$ $H_1: \eta_1 > \eta_2$

Qu. No.	Solution	Mark	Notes
8 (a)(i)	E(X) = $\int x\left(1+\frac{3\lambda x}{2}\right)dx$	M1	Attempt to integrate xf(x) at least one increase in power
	E(X) = $\int_{\frac{-1}{2}}^{\frac{1}{2}}\left(x+\frac{3\lambda x^2}{2}\right)dx$		
	E(X) = $\left[\frac{x^2}{2}+\frac{\lambda x^3}{2}\right]_{\frac{-1}{2}}^{\frac{1}{2}}$	A1	Correct integration
	= $\frac{\lambda}{8}$	A1	cao
(ii)	$E(X^2)=\int_{\frac{-1}{2}}^{\frac{1}{2}}\left(x^2+\frac{3\lambda x^3}{2}\right)dx$	M1	Attempt to integrate x^2f(x) at least one increase in power
	$Var(X)=\int_{\frac{-1}{2}}^{\frac{1}{2}}\left(x^2+\frac{3\lambda x^3}{2}\right)dx-\left(\frac{\lambda}{8}\right)^2$	m1	subtracting 'their $(E(X))^2$' from 'their $E(X^2)$'
	$Var(X)=\left[\frac{x^3}{3}+\frac{3\lambda x^4}{8}\right]_{\frac{-1}{2}}^{\frac{1}{2}}-\frac{\lambda^2}{64}$		
	$Var(X)=\left(\frac{1}{24}+\frac{3\lambda}{128}\right)-\left(\frac{-1}{24}+\frac{3\lambda}{128}\right)\quad-\frac{\lambda^2}{64}$		
	$Var(X)=\frac{1}{12}-\frac{\lambda^2}{64}$	A1	Show substitution of limits and arrive at $\frac{1}{12}-\frac{\lambda^2}{64}$ convincing
	$Var(X)=\frac{16-3\lambda^2}{192}$ *ag		
(b)	P(X > 0) = $\int_0^{\frac{1}{2}}\left(1+\frac{3\lambda x}{2}\right)dx$	M1	Attempt to integrate f(x) at least one increase in power. With correct limits
	= $\left[x+\frac{3\lambda x^2}{4}\right]_0^{\frac{1}{2}}$		
	= $\frac{1}{2}+\frac{3\lambda}{16}$	A1	Convincing
	= $\frac{8+3\lambda}{16}$ *ag		

Qu. No.	Solution	Mark	Notes
8(c)(i)	Binomial. Y~B(n, $\frac{8+3\lambda}{16}$)	B1	
(ii)	E(T_1) = E$\left(\frac{16Y}{3n}-\frac{8}{3}\right)$		
	= $\frac{16E(Y)}{3n}-\frac{8}{3}$	M1	Use of $\frac{16E(Y)}{3n}-\frac{8}{3}$
	= $\frac{16n\left(\frac{8+3\lambda}{16}\right)}{3n}-\frac{8}{3}$	B1	Use of E(Y)=np
	= $\frac{8}{3}+\lambda-\frac{8}{3}$		
	= λ (therefore unbiased)	A1	
(d) (i)	Var (T_1) = $Var\left(\frac{16Y}{3n}-\frac{8}{3}\right)$		
	$=\frac{256n}{9n^2}\left(\frac{8+3\lambda}{16}\right)\left(1-\frac{8+3\lambda}{16}\right)$	M2	M1 for coeff² M1 for use of npq
	$=\frac{256}{9n}\left(\frac{8+3\lambda}{16}\right)\left(\frac{8-3\lambda}{16}\right)$		
	$=\frac{256}{9n}\left(\frac{64-9\lambda^2}{256}\right)$	A1	
	= $\frac{64-9\lambda^2}{9n}$ *ag		convincing
(ii)	Var (T_2) = $Var(8\overline{X})$		
	= $8^2\left(\frac{16-3\lambda^2}{192n}\right)$	M1	
	= $\frac{1024-192\lambda^2}{192n}=\frac{16-3\lambda^2}{3n}$ oe	A1	
(iii)	Var (T_2) = $\frac{48-9\lambda^2}{9n}<\frac{64-9\lambda^2}{9n}$ ∴T_2 is better because it has a smaller variance.	B1 **Total [18]**	Must include attempt to compare variances.