Oxford Cambridge and RSA

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Wednesday 13 June 2018 – Morning

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

Model Solutions

* 7 0 0 9 6 0 5 4 0 4 *

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a\,(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \quad (|x|<1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}, \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1 - \tfrac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k+\tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B|A) = \mathrm{P}(B)\mathrm{P}(A|B)$ **or** $\mathrm{P}(A|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \textbf{ or } \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 **(i)** Express $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$. **[4]**

1.i) $2x^2 - 12x + 23 = 2\left[x^2 - 6x + \frac{23}{2}\right]$

$= 2\left[(x-3)^2 - 9 + \frac{23}{2}\right]$

$= 2\left[(x-3)^2 + \frac{5}{2}\right]$

$= 2(x-3)^2 + 5$

(ii) Use your result to show that the equation $2x^2 - 12x + 23 = 0$ has no real roots. **[1]**

ii $2(x-3)^2$ is always positive

So $2(x-3)^2 + 5 \geqslant 5$

Therefore $2(x-3)^2 + 5$ cannot $= 0$

so it has no real roots.

(iii) Given that the equation $2x^2 - 12x + k = 0$ has repeated roots, find the value of the constant k. **[2]**

iii for repeated roots, the discriminant equals zero

$12^2 - 4(2)(k) = 0$

$144 - 8k = 0$

$8k = 144$

$k = 18$

So $2(x-3)^2 + 5 \geqslant 5$

Therefore $2(x-3)^2 + 5$ cannot $= 0$

so it has no real roots.

2 The points A and B have position vectors $\begin{pmatrix}1\\-2\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\-1\\2\end{pmatrix}$ respectively.

(i) Find the exact length of AB. **[2]**

2 i) $\overrightarrow{AB} = B - A = \begin{pmatrix}-3\\-1\\2\end{pmatrix} - \begin{pmatrix}1\\-2\\5\end{pmatrix} = \begin{pmatrix}-4\\1\\-3\end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{(-4)^2 + 1^2 + (-3)^2}$
$= \sqrt{16+1+9}$
$= \underline{\sqrt{26}}$

(ii) Find the position vector of the midpoint of AB. **[1]**

ii) $\frac{1}{2}\begin{pmatrix}1-3\\-2-1\\5+2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}-2\\-3\\7\end{pmatrix} = \begin{pmatrix}-1\\-1.5\\3.5\end{pmatrix}$

The points P and Q have position vectors $\begin{pmatrix}1\\2\\0\end{pmatrix}$ and $\begin{pmatrix}5\\1\\3\end{pmatrix}$ respectively.

(iii) Show that $ABPQ$ is a parallelogram. **[3]**

iii $\overrightarrow{QP} = P - Q = \begin{pmatrix}-4\\1\\-3\end{pmatrix}$

So QP is parallel to AB

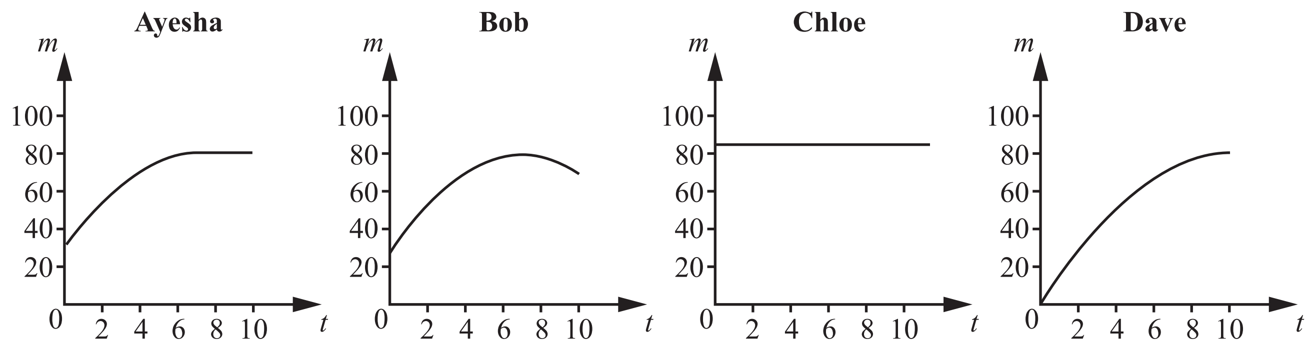
$\overrightarrow{AQ} = Q - A = \begin{pmatrix}4\\3\\-2\end{pmatrix}$

$\overrightarrow{BP} = P - B = \begin{pmatrix}4\\3\\-2\end{pmatrix}$

AQ and BP are parallel

So we have two sets of parallel lines, meaning ABPQ is a parallelogram

3 Ayesha, Bob, Chloe and Dave are discussing the relationship between the time, t hours, they might spend revising for an examination, and the mark, m, they would expect to gain. Each of them draws a graph to model this relationship for himself or herself.



(i) Assuming Ayesha's model is correct, how long would you recommend that she spends revising? **[1]**

3 i) 6 hours, after this time she does not gain anymore marks

(ii) State one feature of Dave's model that is likely to be unrealistic. **[1]**

ii) He will get no marks if he does no revision

(iii) Suggest a reason for the shape of Bob's graph as compared with Ayesha's graph. **[1]**

iii) Bob suggests that too much revision leads to a drop in number of marks, for example from tiredness

(iv) What does Chloe's model suggest about her attitude to revision? **[1]**

iv) The amount of revision she does will not affect the number of marks

4 Prove that $\sin^2(\theta + 45)° - \cos^2(\theta + 45)° \equiv \sin 2\theta°$. **[4]**

4. $\sin^2(\theta + 45) - \cos^2(\theta + 45) \equiv \sin 2\theta$

LHS : using $\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2(\theta + 45) - \cos^2(\theta + 45) = -\cos(2(\theta + 45))$
$= -\cos(2\theta + 90)$

using $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$-\cos(2\theta + 90) = -\cancel{\cos 2\theta \cos 90} + \sin 2\theta \sin 90$

$= \sin 2\theta$

$= RHS$

Therefore $\sin^2(\theta + 45) - \cos^2(\theta + 45) \equiv \sin 2\theta$

5 Charlie claims to have proved the following statement.

"The sum of a square number and a prime number cannot be a square number."

(i) Give an example to show that Charlie's statement is not true. **[1]**

5. i) eg. $1 + 3 = 4$

Charlie's attempt at a proof is below.

Assume that the statement is not true.
⇒ There exist integers n and m and a prime p such that $n^2 + p = m^2$.
⇒ $p = m^2 - n^2$
⇒ $p = (m - n)(m + n)$
⇒ p is the product of two integers.
⇒ p is not prime, which is a contradiction.
⇒ Charlie's statement is true.

(ii) Explain the error that Charlie has made. **[1]**

ii) If $m - n = 1$ then p will be an integer multiplied by 1. This means that p will not be prime in this case

(iii) Given that 853 is a prime number, find the square number S such that $S + 853$ is also a square number. **[4]**

iii) let ~~8~~ $S = n^2$

The other squared number is $(n+1)^2$

$853 + n^2 = (n+1)^2$

Find the area of the shaded region. **[7]**

$853 + \cancel{n^2} = \cancel{n^2} + 2n + 1$

$852 = 2n$

$n = 426$

$\Rightarrow \quad S = 426^2 = 181476$

6 In this question you must show detailed reasoning.

A curve has equation $y = \frac{\ln x}{x}$.

(i) Find the x-coordinate of the point where the curve crosses the x axis. **[2]**

6. i) $y = \frac{\ln x}{x}$

when $y = 0$: $0 = \frac{\ln x}{x}$

$0 = \ln x$

$x = 1$

(ii) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x-axis. **[2]**

ii) when $x = 2$, $y = \frac{\ln 2}{2}$

when $x = 4$, $y = \frac{\ln 4}{4} = \frac{\ln 2^2}{4}$

$= \frac{2\ln 2}{4}$

$= \frac{\ln 2}{2}$

y coordinates are the same so the line is parallel to the x axis and gradient $= 0$.

(iii) Find the coordinates of the turning point on the curve. **[4]**

iii) $\frac{dy}{dx} = uv' + vu' = \frac{x\left(\frac{1}{x}\right) - \ln x(1)}{x^2}$

$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$

Stationary point occurs when $\frac{dy}{dx} = 0$:

$0 = \frac{1 - \ln x}{x^2}$

$0 = 1 - \ln x$

$\ln x = 1$

$x = e \Rightarrow y = \frac{\ln e}{e} = \frac{1}{e}$

Coordinates : $(e, \frac{1}{e})$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

(iv) Determine whether this turning point is a maximum or a minimum. **[5]**

iv) $\frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (2x)(1 - \ln x)}{x^4}$

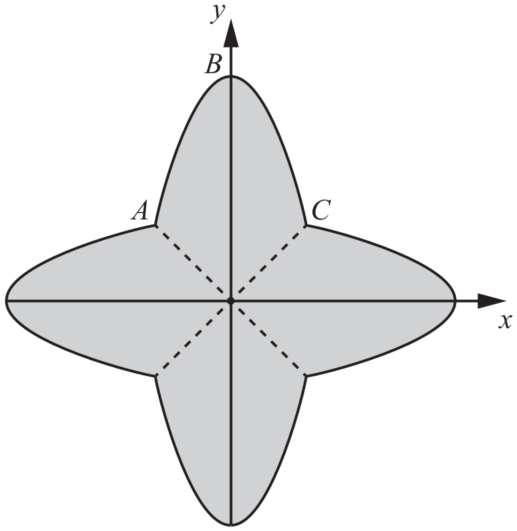
$\frac{d^2y}{dx^2} = \frac{-x - 2x + 2x\ln x}{x^4}$

$\frac{d^2y}{dx^2} = \frac{-3 + 2\ln x}{x^3}$

when $x = e$: $\frac{d^2y}{dx^2} = \frac{-3 + 2\ln e}{e^3} = -\frac{1}{e^3}$

$-\frac{1}{e^3} < 0$ hence $x = e$ is a maximum

7 The diagram shows a part ABC of the curve $y = 3 - 2x^2$, together with its reflections in the lines $y = x$, $y = -x$ and $y = 0$.



Find the area of the shaded region. **[7]**

7. First find where the curve intersects the lines $y = x$ and $y = -x$, points C and A

$x = 3 - 2x^2$ at C

$2x^2 + x - 3 = 0$

$(2x + 3)(x - 1) = 0$

x is positive at C so the x coordinate of C is $x = 1$, and at A it is $x = -1$ due to symmetry

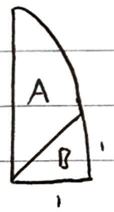
Now integrate:

$\int_0^1 3 - 2x^2 \, dx = \left[3x - \frac{2}{3}x^2\right]_0^1$

$= 3 - \frac{2}{3}$

$= \frac{7}{3}$

The total shaded area is :



$8 \times$ Area of A

$\frac{7}{3} = A + B$ is what we know

we have worked out the area of A+B but we actually want the area of A, as 8A = total shaded area

So if $A + B = \frac{7}{3}$, and $B = \frac{1}{2}(1 \times 1)$ then

$A = \frac{7}{3} - \frac{1}{2} = \frac{11}{6}$

$8 \times A = 8 \times \frac{11}{6} = \frac{44}{3}$

Section B: Statistics

Answer **all** the questions.

8 **(i)** The variable X has the distribution N(20, 9).

(a) Find P($X > 25$). **[1]**

8 i)a) $X \sim N(20, 9)$

$P(X > 25) = P\left(\frac{X-20}{3} > \frac{25-20}{3}\right)$

$= P(Z > 1.67)$

$= 1 - 0.9525$

$= 0.0475$

(b) Given that $P(X > a) = 0.2$, find a. **[1]**

b) $P(X > a) = 0.2$

$P\left(Z > \frac{a-20}{3}\right) = 0.2$

$\frac{a-20}{3} = 0.84$

$a = 2.52 + 20$

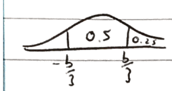
$a = 22.5$

(c) Find b such that $P(20 - b < X < 20 + b) = 0.5$. **[3]**

c) $P(20 - b < X < 20 + b) = 0.5$

$= P\left(\frac{20-b-20}{3} < \frac{X-20}{3} < \frac{20+b-20}{3}\right) = 0.5$

$= P\left(\frac{-b}{3} < Z < \frac{b}{3}\right) = 0.5$



$P\left(Z < \frac{b}{3}\right) = 0.75$

$\frac{b}{3} = 0.67$

$b = 2.01$

(ii) The variable Y has the distribution $N(\mu, \frac{\mu^2}{9})$. Find $P(Y > 1.5\mu)$. **[3]**

ii. $Y \sim N\left(\mu, \frac{\mu^2}{3}\right)$

$P(Y > 1.5\mu) = P\left(\frac{Y-\mu}{\frac{\mu}{3}} > \frac{1.5\mu - \mu}{\mu/3}\right)$

$= P\left(Z > \frac{0.5}{\frac{1}{3}}\right)$

$= P(Z > 1.5)$

$= 1 - 0.9332$

$= 0.0668$

9 Briony suspects that a particular 6-sided dice is biased in favour of 2. She plans to throw the dice 35 times and note the number of times that it shows a 2. She will then carry out a test at the 4% significance level. Find the rejection region for the test. **[7]**

9 $H_0 : p = \frac{1}{6}$
$H_1 : p > \frac{1}{6}$

under the null hypothesis, let $X \sim B(35, \frac{1}{6})$ be the number of 2s thrown

$P(X \geq 10) = 1 - P(X \leq 9)$
$= 1 - 0.945$
$= 0.055$

$P(X \geq 11) = ~~+~~ 1 - P(X \leq 10)$
$= 1 - 0.9768$
$= 0.0232$

$0.0232 < 0.04$ so the rejection region is $X \geq 11$

10 A certain forest contains only trees of a particular species. Dipak wished to take a random sample of 5 trees from the forest. He numbered the trees from 1 to 784. Then, using his calculator, he generated the random digits 14 781 049. Using these digits, Dipak formed 5 three-digit numbers. He took the first, second and third digits, followed by the second, third and fourth digits and so on. In this way he obtained the following list of numbers for his sample.

147 478 781 104 49

(i) Explain why Dipak omitted the number 810 from his list. **[1]**

10 i) There are only 784 so there is no tree numbered 810

(ii) Explain why Dipak's sample is not random. **[1]**

ii) Each of the random numbers is dependent on the previous number, so they are not independent, meaning they are not random

(iii) Carry out the test at the 2% significance level.

iii. $H_0 : \mu = 4.2$

$H_1 : \mu < 4.2$

$\bar{X} \sim N(4.2, \frac{0.8^2}{50})$

$P(\bar{x} < 4.0) = P\left(\frac{\bar{x} - 4.2}{\frac{0.8}{\sqrt{50}}} < \frac{4.0 - 4.2}{\frac{0.8}{\sqrt{50}}}\right)$

$= P(Z < -1.77)$

$= 1 - 0.9616$

$= 0.0384$

0.0384 > 0.02 so do not reject H_0

Insufficient evidence to suggest that the mean height of the trees is less than 4.2m

11 Christa used Pearson's product-moment correlation coefficient, *r*, to compare the use of public transport with the use of private vehicles for travel to work in the UK.

(i) Using the pre-release data set for all 348 UK Local Authorities, she considered the following four variables.

Number of employees using public transport	x
Number of employees using private vehicles	y
Proportion of employees using public transport	a
Proportion of employees using private vehicles	b

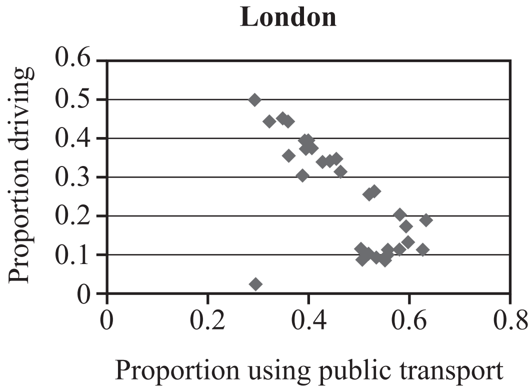
(a) Explain, in context, why you would expect strong, positive correlation between x and y. **[1]**

11 i) a) Both the number of employees using public transport and the number using private transport will depend on the population of the LA

(b) Explain, in context, what kind of correlation you would expect between a and b. **[2]**

b) Negative. The more people who use public transport, the less people there are using private transport, and vice versa

(ii) Christa also considered the data for the 33 London boroughs alone and she generated the following scatter diagram.



One London Borough is represented by an outlier in the diagram.

(a) Suggest what effect this outlier is likely to have on the value of r for the 32 London Boroughs. **[1]**

ii. a) It will increase r (will become less negative)

(b) Suggest what effect this outlier is likely to have on the value of r for the whole country. **[1]**

b) It will have very little effect because the population of this LA is very small compared to the whole population

(c) What can you deduce about the area of the London Borough represented by the outlier? Explain your answer. **[1]**

c) People work close by where they can cycle or walk in so the area is relatively small.

12 The discrete random variable X takes values 1, 2, 3, 4 and 5, and its probability distribution is defined as follows.

$$P(X=x)=\begin{cases} a & x=1, \\ \frac{1}{2}P(X=x-1) & x=2,3,4,5, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a=\frac{16}{31}$. **[2]**

12 i) total probability = 1

$a + \frac{1}{2}a + \frac{1}{4}a + \frac{1}{8}a + \frac{1}{16}a = 1$

$\frac{31}{16}a = 1$

$a = \frac{16}{31}$

The discrete probability distribution for X is given in the table.

x	1	2	3	4	5
$P(X=x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

(ii) Find the probability that X is odd.. **[1]**

ii) $= P(X=1,3,5) = \frac{16}{31} + \frac{4}{31} + \frac{1}{31} = \frac{21}{31}$

Two independent values of X are chosen, and their sum S is found.

(iii) Find the probability that S is odd. **[2]**

iii) To get an odd sum you need one even number, and one odd number

$P(\text{odd sum}) = P(O)P(E) + P(E)P(O)$

$= \frac{21}{31}\left(1-\frac{21}{31}\right) + \left(1-\frac{21}{31}\right)\frac{21}{31}$

$= 2 \times \frac{21}{31} \times \frac{10}{31}$

$= \frac{420}{961}$

(iv) Find the probability that S is greater than 8, given that S is odd. **[3]**

iv) The only combination which is odd and greater than 8 is with 4 and 5, in any order

$$P(S > 8 \mid S \text{ is odd}) = \frac{P(S > 8 \text{ and } S \text{ is odd})}{P(S \text{ is odd})}$$

$$= \frac{P(4)P(5) + P(5)P(4)}{\frac{420}{961}}$$

$$= \frac{2 \times \frac{2}{31} \times \frac{1}{31}}{\frac{420}{961}}$$

$$= \frac{1}{105}$$

Sheila sometimes needs several attempts to start her car in the morning. She models the number of attempts she needs by the discrete random variable Y defined as follows.

$$P(Y = y+1) = \frac{1}{2}P(Y = y) \quad \text{for all positive integers } y.$$

(v) Find $P(Y = 1)$. **[2]**

v) $S_\infty = 1$

$$\frac{a}{1 - \frac{1}{2}} = 1$$

$a = 0.5$

(vi) Give a reason why one of the variables, X or Y, might be more appropriate as a model for the number of attempts that Sheila needs to start her car. **[1]**

vi) X cannot be more than 5 but Y can take all values. X may be more appropiate as it probably won't take her more than 5 goes

13 In this question you must show detailed reasoning.

The probability that Paul's train to work is late on any day is 0.15, independently of other days.

(i) The number of days on which Paul's train to work is late during a 450-day period is denoted by the

13 i) $Y \sim B(450, 0.15)$

Use a normal approximation

$np = 450 \times 0.15 = 67.5$

$np(1-p) = 450 \times 0.15 \times 0.85 = 57.375$

$Y \sim N(67.5, 57.375)$

$P(Y > a) = \frac{1}{6}$

$$P\left(\frac{Y - 67.5}{\sqrt{57.375}} > \frac{a - 67.5}{\sqrt{57.375}}\right) = 0.17$$

$$\frac{a - 67.5}{\sqrt{57.375}} = 0.97$$

$a = 67.5 + 0.97\sqrt{57.375}$

$a = 74.8$

So $a = 75$ days

(ii) Show that $\dfrac{T_r}{T_{r+1}} = \dfrac{17(r+1)}{3(50-r)}$. **[3]**

ii) $$\frac{T_r}{T_{r+1}} = \frac{\binom{50}{r} \times 0.15^r \times 0.85^{50-r}}{\binom{50}{r+1} \times 0.15^{r+1} \times 0.85^{50-(r+1)}}$$

$$= \frac{\frac{\cancel{50!}}{r!(50-r)!} \times 0.15^{\cancel{r}} \times 0.85^{50-\cancel{r}}}{\frac{\cancel{50!}}{(r+1)!(50-r-1)!} \times 0.15^{\cancel{r}+1} \times 0.85^{49-\cancel{r}}}$$

$$= \frac{\frac{1}{50-r} \times 0.85}{\frac{1}{r+1} \times 0.15}$$

$$= \frac{0.85(r+1)}{0.15(50-r)}$$

$$= \frac{17(r+1)}{3(50-r)}$$

(iii) The number of days on which Paul's train to work is late during a 50-day period is modelled by the random variable X.

(a) Find the values of r for which $P(X=r) \leqslant P(X=r+1)$. **[4]**

iii. a) $X \sim B(50, 0.15)$ ie. $(0.15 + 0.85)^{50}$

$$P(X=r) \leqslant P(X=r+1)$$

$$T_r \leqslant T_{r+1}$$

$$\frac{T_r}{T_{r+1}} \leqslant 1$$

$$\frac{17(r+1)}{3(50-r)} \leqslant 1$$

$$17(r+1) \leqslant 3(50-r)$$

$$17r + 17 \leq 150 - 3r$$

$$20r \leq 133$$

$$r \leq 6.65$$

r is an integer so $r \leq 6$

(b) Hence find the most likely number of days on which the train will be late during a 50-day period. **[2]**

b) Most likely value is $r = 6$ or 7

$$\frac{P(X=6)}{P(X=7)} = \frac{T_6}{T_7} = \frac{17(6+1)}{3(50-6)} = 0.902$$

$0.902 < 1$ so $P(X=7)$ must be larger than $P(X=6)$

Therefore the most likely value is 7

END OF QUESTION PAPER

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