Oxford Cambridge and RSA

Wednesday 15 May 2019 – Morning

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

7661869198

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \text{ or } \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 It is given that $f(x) = 3x - \frac{5}{x^3}$.

Find

(a) $f'(x)$, **[3]**

(b) $f''(x)$, **[2]**

(c) $\int f(x)\,dx$. **[3]**

a) $f(x) = 3x - 5x^{-3}$

$f'(x) = 3(1)x^0 - 5(-3)x^{-4}$

$= 3 + 15x^{-4}$

b) $f''(x) \Rightarrow \frac{dy}{dx}$ of $3+15x^{-4}$

$\Rightarrow 15(-4)x^{-5} \Rightarrow -60x^{-5}$

c) $\int f(x)\,dx \Rightarrow \int (3x - 5x^{-3})\,dx$

$\Rightarrow \frac{3x^2}{2} - \frac{5x^{-2}}{-2} + C \Rightarrow \frac{3}{2}x^2 + \frac{5}{2}x^{-2} + c$

2 The circle $x^2 + y^2 - 4x + ky + 12 = 0$ has radius 1.

Find the two possible values of the constant k. **[4]**

Completing the square

$x^2 - 4x + y^2 + ky + 12 = 0$.

$(x-2)^2 - (2)^2 + (y + k/2)^2 - (k/2)^2 + 12 = 0$

$(x-2)^2 - 4 + (y + k/2)^2 - \frac{k^2}{4} + 12 = 0$.

$(x-2)^2 + (y + k/2)^2 + 8 - \frac{k^2}{4} = 0$

$(x-2)^2 + (y + k/2)^2 = \frac{k^2}{4} - 8$.

From the formula

$(x-a)^2 + (y-b)^2 = r^2$ where (a, b) is the centre of the circle and r is the radius

$\frac{k^2}{4} - 8 = (1)^2 \Rightarrow \frac{k^2}{4} - 8 = 1$

$\frac{k^2}{4} = 9 \quad k^2 = 36 \quad k = \pm\sqrt{36} = \underline{\underline{\pm 6}}$

3 In this question you must show detailed reasoning.

(a) The polynomial $f(x)$ is defined by $f(x) = 2x^3 + 3x^2 - 8x + 3$.

(i) Show that $f(1) = 0$. **[1]**

(ii) Solve the equation $f(x) = 0$. **[4]**

(b) Hence solve the equation $2\sin^3\theta + 3\sin^2\theta - 8\sin\theta + 3 = 0$ for $0° \leqslant \theta < 360°$. **[5]**

ai) $2(1)^3 + 3(1)^2 - 8(1) + 3 \Rightarrow 2 + 3 - 8 + 3$

$\Rightarrow 8 - 8 = 0$ as required.

ii) From (ai) we know that $(x-1)$ is a factor of the polynomial

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline x-1 & 2x^3 + 3x^2 - 8x + 3 \\ & -\;2x^3 - 2x^2 \\ \hline & \quad 5x^2 - 8x \\ & -\;5x^2 - 5x \\ \hline & \quad -3x + 3 \\ & -\;-3x + 3 \\ \hline & \quad 0. \end{array}$$

$(x-1)(2x^2 + 5x - 3)$.

Factorising this gives

$(x-1)(x+3)(2x-1)$

$\therefore f(x) = 0 \Rightarrow (x-1)(x+3)(2x-1) = 0$

$x = 1, -3, \frac{1}{2}$

b) let $\sin\theta = x$.

$\sin\theta = 1, -3, \frac{1}{2}$.

① $\sin\theta = 1$

S ✓ | A ✓
T | C

$\theta = \sin^{-1}(1)$

$\theta = \underline{90°}$

② $\sin\theta = -3$

No real roots as $-1 \leq \sin\theta \leq 1$

$\therefore$ outside the range

③ $\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}(\frac{1}{2})$

$\theta = 30, 180-30$

$\therefore \theta = 30°, 150°$

$\therefore \theta = 30°, 90°, 150°$

4 **(a)** Find the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x$. **[4]**

(b) The equation $x^3 - 6x^2 + 9x + k = 0$ has exactly one real root.

Using your answers from part **(a)** or otherwise, find the range of possible values of k. **[2]**

a) Stationary points occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 3x^2 - 12x + 9$.

$\frac{3x^2 - 12x + 9}{3} = \frac{0}{3} \Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow (x-3)(x-1) = 0 \qquad x = 3 \quad x = 1$

when $x = 1$

$y = (1)^3 - 6(1)^2 + 9(1)$

$= 1 - 6 + 9$

$= 4$

when $x = 3$

$y = (3)^3 - 6(3)^2 + 9$

$y = 0$

$\therefore \Rightarrow (1,4) \quad (3,0)$

b) $f(1) = (1)^3 - 6(1)^2 + 9(1) + k \Rightarrow 1 - 6 + 9 + k$

$\Rightarrow 4 + k$

$f(1) = 0 \quad \Rightarrow \quad 4 + k = 0 \quad \underline{k = -4}$

$f(3) = (3)^3 - 6(3)^2 + 9(3) + k = 0 \quad \Rightarrow \underline{k = 0}$

$\therefore \quad k < -4 \quad$ or $\quad k > 0$

5 **(a)** Prove that the following statement is **not** true.

m is an odd number greater than 1 $\Rightarrow m^2 + 4$ is prime. **[1]**

(b) By considering separately the case when n is odd and the case when n is even, prove that the following statement **is** true.

n is a positive integer $\Rightarrow n^2 + 1$ is not a multiple of 4. **[4]**

a) Let $m = 9 \Rightarrow (9)^2 + 4 = 85$

$5 \times 17 = 85$

$\therefore$ 85 is a multiple of 5 $\therefore$ statement isn't true (as 85 isn't prime)

b) Even numbers

$(2k)^2 + 1 = 4k^2 + 1 \longrightarrow$ This is clearly not a multiple of 4, as it's in the form $(4 \times m) + 1$

even no.

where k is an integer.

Odd numbers

$(2k+1)^2 + 1 = 4k^2 + 4k + 1 + 1 = 4k^2 + 4k + 2$

$\Rightarrow 4(k^2 + k) + 2$

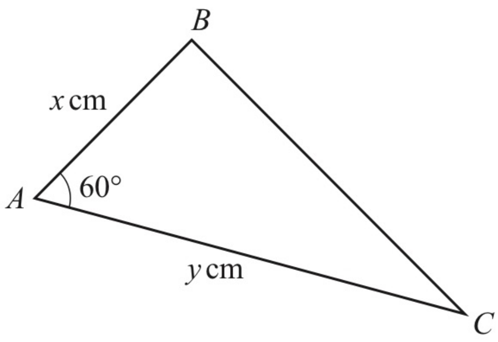
odd no.

where k is an integer

This is in the form $(4 \times m) + 2$ which is not a multiple of 4.

$\therefore$ proved for both odd and even numbers.

6



The diagram shows triangle ABC, with $AB = x$ cm, $AC = y$ cm and angle $BAC = 60°$. It is given that the area of the triangle is $(x+y)\sqrt{3}\ \text{cm}^2$.

(a) Show that $4x + 4y = xy$. **[2]**

When the vertices of the triangle are placed on the circumference of a circle, AC is a diameter of the circle.

(b) Determine the value of x and the value of y. **[4]**

a) Formula for Area of triangle = $\frac{1}{2}ab\sin\theta$.

$\frac{1}{2} \times x \times y \times \sin 60 = (x+y)\sqrt{3}$

$\sin 60 = \frac{\sqrt{3}}{2}$

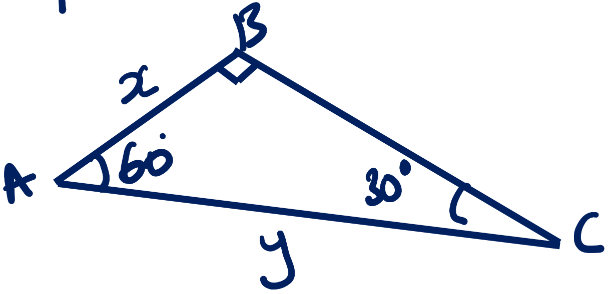
$\frac{1}{2} \times xy \times \frac{\sqrt{3}}{2} = (x+y)\sqrt{3}$

$\frac{xy\sqrt{3}}{4\sqrt{3}} = \frac{(x+y)\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{xy}{4} = x+y$

$\Rightarrow xy = 4(x+y) \Rightarrow xy = 4x+4y$

$\Rightarrow 4x+4y = xy$ as required

b) If AC is the diameter, then $A\hat{B}C = 90°$.



$A\hat{C}B$

$180 - (90 + 60)$
$= \underline{30°}$

Using trig

SOH CAH TOA

$\cos 60 = \frac{x}{y}$ $\qquad \frac{1}{2} = \frac{x}{y}$ $\qquad y = 2x \quad -(1)$

From part (a) $\quad 4x + 4y = xy \quad -(2)$

Using substition

$4x + 4(2x) = x(2x)$

$4x + 8x = 2x^2$

$12x = 2x^2 \Rightarrow x^2 = 6x \Rightarrow x^2 - 6x = 0$

$x(x-6) = 0$

$x = 6, 0$

$\underline{x = 6}$

if $x = 6$, $\quad y = 2(6) = \underline{12}$

$x = 6, \quad y = 12$

7 **(a)** Write down an expression for the gradient of the curve $y = e^{kx}$. **[1]**

(b) The line L is a tangent to the curve $y = e^{\frac{1}{2}x}$ at the point where $x = 2$.

Show that L passes through the point (0, 0). **[4]**

(c) Find the coordinates of the point of intersection of the curves $y = 3e^{x}$ and $y = 1 - 2e^{\frac{1}{2}x}$. **[6]**

a) $\frac{dy}{dx} = ke^{kx}$

b) $\frac{dy}{dx} = \frac{1}{2}e^{1/2 x}$

if $x = 2$

$\frac{dy}{dx} = \frac{1}{2}e^{1/2(2)}$

$= \frac{1}{2}e \rightarrow m.$

$y - y_0 = m(x - x_0) \rightarrow$ equation of a line.

when $x = 2$ $y = e^{1/2(2)} \Rightarrow e$

$y - e = \frac{1}{2}e(x - 2) \Rightarrow y = \frac{1}{2}ex - e + e$

$y = \frac{1}{2}ex$

when $x = 0$

$y = \frac{1}{2}e(0) = 0$ $\therefore$ passes through $(0,0)$ as required.

c) POI is where the 2 curves meet.

$3e^{x} = 1 - 2e^{1/2 x}$

let $e^{\frac{1}{2}x} = u$

$3u^2 = 1 - 2u$

$3u^2 + 2u - 1 = 0$

$(u+1)(3u-1) = 0$

$u = -1 \qquad u = \frac{1}{3}$

① $e^{\frac{1}{2}x} = -1$

no real solutions

② $e^{\frac{1}{2}x} = \frac{1}{3}$

$\frac{1}{2}x = \ln(\frac{1}{3})$

$x = 2\ln(\frac{1}{3})$

$y = 3e^{x}$

$= 3e^{2\ln(\frac{1}{3})} = \frac{1}{3}$

$\therefore$ POI $= \left(2\ln(\frac{1}{3}), \frac{1}{3}\right)$

Section B: Statistics

Answer **all** the questions.

8 **(a)** Joseph drew a histogram to show information about one Local Authority. He used data from the "Age structure by LA 2011" tab in the large data set. The table shows an extract from the data that he used.

Age group	0 to 4
Frequency	2143

Joseph used a scale of 1 cm = 1000 units on the frequency density axis.

Calculate the height of the histogram block for the 0 to 4 class. **[2]**

(b) Magdalene wishes to draw a statistical diagram to illustrate some of the data from the "Method of travel by LA 2011" tab in the large data set.

State why she cannot draw a histogram. **[1]**

a) $\frac{2143}{5} = 428.6 = 429$ (3sf)

$= 0.429$ cm

b) This is non-numerical data.

9 The table shows information about the number of days absent last year by students in class 2A at a certain school.

Number of days absent	0	1	2 to 4	5 to 10	11 to 20	21 to 30	More than 30	
Number of students	7	12	9	1	0	1	0	$= 30$

(a) Calculate an estimate of the mean for these data. **[2]**

(b) Find the median of these data. **[1]**

The headteacher is writing a report on the numbers of absences at her school. She wishes to include a figure for the average number of absences in class 2A. A governor suggests that she should quote the mean. The class teacher suggests that she should quote the median, because it is lower than the mean.

(c) Give another reason for using the median rather than the mean for the average number of absences in class 2A. **[1]**

a) Formula for mean $= \frac{\Sigma fx}{\Sigma f}$.

$\Rightarrow (0\times7) + (1\times12) + (\frac{2+4}{2}\times9) + (\frac{5+10}{2}\times1)$

$+ (\frac{11+20}{2}\times0) + (\frac{21+30}{2}\times1)$

$= \frac{72}{30} = 2.4$

b) $= 1$ (as value 15 occurs at days absent $= 1$)

c) The median is less influenced than the mean by the one student in the (21-30) class.

10 The table shows extracts from the "Method of travel by LA" tabs for 2001 and 2011 in the large data set.

Local authority (LA)	All people in employment	Underground, metro, light rail, tram	Train	Bus, minibus or coach	Motorcycle, scooter or moped	Driving a car or van
LA1 2001	79 226	14 369	5235	20 575	1227	16 052
LA1 2011	118 556	22 486	8336	30 541	1220	12 445
LA2 2001	203 614	190	1062	15 327	1256	121 690
LA2 2011	227 894	323	1865	13 732	1038	146 644
LA3 2001	42 993	35	482	4363	274	24 105
LA3 2011	49 014	33	828	3380	191	28 981
LA4 2001	101 697	65	693	21 758	846	45 407
LA4 2011	123 218	2495	1315	24 275	763	54 020

(a) In one of these four LAs a new tram system was opened in 2004.

Suggest, with a reason taken from the data, which LA this could have been. **[2]**

(b) Julian suggests that the figures for "Bus, minibus or coach" for LA1 show that some new bus routes were probably introduced in this LA between 2001 and 2011.

Use data from the table to comment on this suggestion. **[2]**

(c) In one of these four LAs a congestion charge on vehicles was introduced in 2003.

Suggest, with a reason taken from the data, which LA this could have been. **[2]**

a) ⇒ LA4, because there is a large increase in numbers travelling by tram.

b) The ratio for the bus is approximately the same as ratio for all people.
∴ no new reason to suggest new bus routes.

c) LA1 because there is a decrease in the number driving cars despite the increase in total number of people.

11 It is known that, under the standard treatment for a certain disease, 9.7% of patients with the disease experience side effects within one year.

In a trial of a new treatment, a random sample of 450 patients with this disease was selected and the number X who experienced side effects within one year was noted.

(a) State one assumption needed in order to use a binomial model for X. **[1]**

It was found that 51 of the 450 patients experienced side effects within one year.

(b) Test, at the 10% significance level, whether the proportion of patients experiencing side effects within one year is greater under the new treatment than under the standard treatment. **[7]**

a) The probability of side effects is constant for each patient.

b) $H_0: p = 0.097$. $\quad$ where p is the proportion of patients that experience side effects.

$H_1: p > 0.097$.

$X \sim B(450, 0.097)$

$p(X \geq 51) = 1 - p(X \leq 50)$

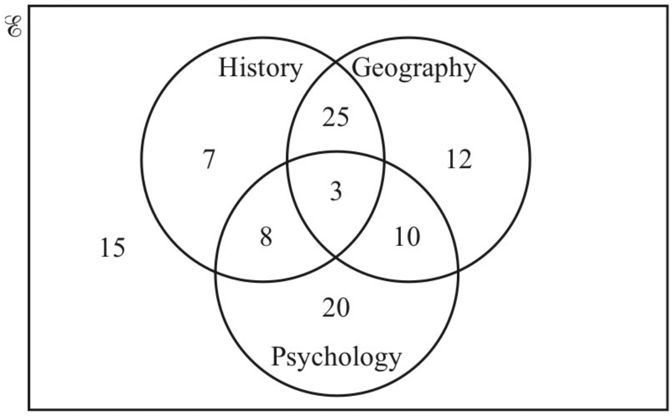
using the calc. you get 0.862

$1 - 0.862 = \underline{\underline{0.138}}$ (3sf)

$0.138 > 0.1$ $\therefore$ insufficient evidence to reject H_0.

∴ no evidence to suggest that proportion of people experiencing side effects in one year under new treatment is greater than standard treatment.

12 The Venn diagram shows the numbers of students studying various subjects, in a year group of 100 students.



A student is chosen at random from the 100 students. Then another student is chosen from the remaining students.

Find the probability that the first student studies History and the second student studies Geography but not Psychology. **[4]**

p(1st H G P' & 2nd G P' or 1st H G P & 2nd G P'
or 1st H G' & 2nd G P')

$$\Rightarrow \left(\frac{25}{100} \times \frac{36}{99}\right) + \left(\frac{3}{100} \times \frac{37}{99}\right) + \left(\frac{15}{100} \times \frac{37}{99}\right)$$

$$= \frac{87}{550} = 0.158 \text{ (3sf)}$$