OCR
Oxford Cambridge and RSA

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Model Solutions

Wednesday 16 May 2018 – Morning

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

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INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \text{ or } \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

(i) Express $3^{\frac{7}{2}}$ in the form $a\sqrt{b}$, where a is an integer and b is a prime number. **[2]**

$$\frac{7}{2} = 3 + \frac{1}{2} \Rightarrow 3^{7/2} = 3^{3+1/2} = 3^3 \times \sqrt{3} = \underline{27}\sqrt{3} = a\sqrt{b}$$

Where $\underline{a = 27}$ and $\underline{b = 3}$.

(ii) Express $\dfrac{\sqrt{2}}{1-\sqrt{2}}$ in the form $c + d\sqrt{e}$, where c and d are integers and e is a prime number. **[3]**

$$\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -2-\sqrt{2} = c + d\sqrt{e}$$

Where $c = \underline{-2}$, $d = \underline{-1}$ and $e = \underline{2}$

$(1-\sqrt{2})(1+\sqrt{2}) = 1-2 = -1$

2 **(i)** The equation $x^2 + 3x + k = 0$ has repeated roots. Find the value of the constant k. **[2]**

Repeated Roots $\Rightarrow b^2 - 4ac = 0$

$x^2 + 3x + k = 0 \Rightarrow a = 1,\ b = 3$ and $c = k$

$\Rightarrow b^2 - 4ac = 0 \Rightarrow (3)^2 - 4(1)(k) = 0$

$\Rightarrow 9 - 4k = 0 \Rightarrow k = \underline{\underline{\frac{9}{4}}}$

(ii) Solve the inequality $6 + x - x^2 > 0$. **[2]**

$6 + x - x^2 > 0 \iff x^2 - x - 6 < 0$

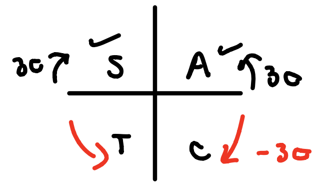
$(x-3)(x+2) < 0 \Rightarrow x < 3$ and $x < -2$

$\Rightarrow -2 < \underline{x} < 3$

3 (i) Solve the equation $\sin^2\theta = 0.25$ for $0° \leqslant \theta < 360°$. [3]

$\sin^2\theta = 0.25$ Square Root

$\sin\theta = \pm 0.5$



$\theta = \sin^{-1}(0.5) = 30°$

$\Rightarrow$ For $+0.5$, $\theta = 30°$ and $\theta = 180 - 30 = 150°$

$\Rightarrow$ For -0.5, $\theta = 180 + 30 = 210°$ and $\theta = 330°$

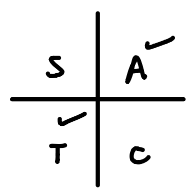
$\Rightarrow$ $\theta = \underline{\underline{30°}}$, $\theta = \underline{\underline{150°}}$, $\theta = \underline{\underline{210°}}$ and $\theta = \underline{\underline{330°}}$

(ii) **In this question you must show detailed reasoning.**

Solve the equation $\tan 3\phi = \sqrt{3}$ for $0° \leqslant \phi < 90°$. [3]

$\tan(3\phi) = \sqrt{3}$

$3\phi = \tan^{-1}(\sqrt{3})$



$3\phi = 60$

$\phi = 20$

$\Rightarrow$ $\phi = 20°$ and $\phi = 180 + 20 = 200°$ but $200 \notin (0°, 90°)$

$\Rightarrow$ exclude $\underline{200°}$

Then solutions will occur every $60°$ which means $60 + 20 = 80°$.

$\Rightarrow$ Our solutions in $0° \leq \phi \leq 90°$ are $\phi = \underline{\underline{20°}}$ and $\phi = \underline{\underline{80°}}$

4 **(i)** It is given that $y = x^2 + 3x$.

(a) Find $\frac{dy}{dx}$. **[2]**

$\frac{dy}{dx} = 2x + 3$ by differentiating.

(b) Find the values of x for which y is increasing. **[2]**

The values of x for which y is increasing occur when $\frac{dy}{dx} > 0$.

$\Rightarrow \quad 2x + 3 > 0 \quad \Rightarrow \quad \underline{\underline{x > -\frac{3}{2}}}$

(ii) Find $\int (3 - 4\sqrt{x})\,dx$. **[5]**

We integrate each term individually but to do this we note that $4\sqrt{x}$ is equivalent to $4x^{1/2}$.

multiply by $\frac{2}{3}$ is equivalent to dividing by $\frac{3}{2}$.

$\Rightarrow \int 3 - 4x^{1/2}\,dx = 3x - 4x^{3/2} \times \frac{2}{3} + C = \underline{\underline{3x - \frac{8}{3}x^{3/2} + C}}$

5 N is an integer that is not divisible by 3. Prove that N^2 is of the form $3p + 1$, where p is an integer. **[5]**

Since N is not divisible by 3, we can let $N = 3k + 1$ where k is an integer.

Then $N^2 = (3k+1)^2 = 9k^2 + 6k + 1$.

We can factorise this as follows:

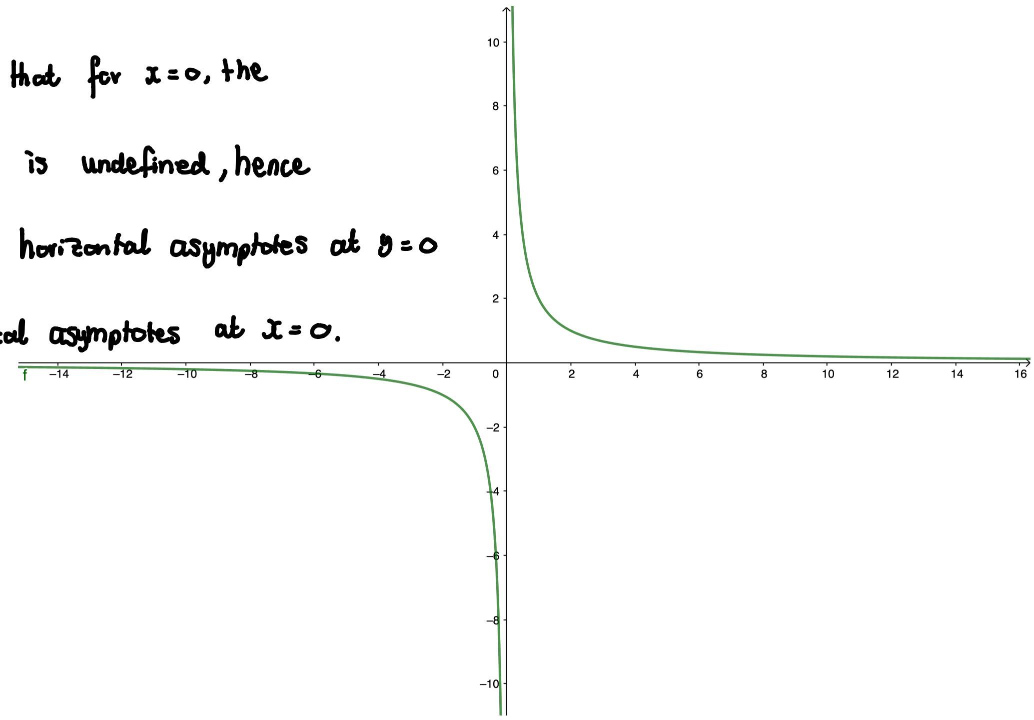
$9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1 = 3p + 1$, where $p = 3k^2 + 2k$,

which is an integer.

6 Sketch the following curves.

(i) $y = \dfrac{2}{x}$ **[2]**

We know that for $x = 0$, the expression is undefined, hence we have horizontal asymptotes at $y = 0$ and Vertical asymptotes at $x = 0$.



(ii) $y = x^3 - 6x^2 + 9x$ **[5]**

Find x roots: $y = x(x^2 - 6x + 9) = x(x-3)(x-3) \Rightarrow x = 0$ and $x = 3$.

Find y roots: $x = 0 \Rightarrow y = 0$

$\Rightarrow (3,0)$ and $(0,0)$ are points the curve passes through.

$\dfrac{dy}{dx} = 3x^2 - 12x + 9 = 0$

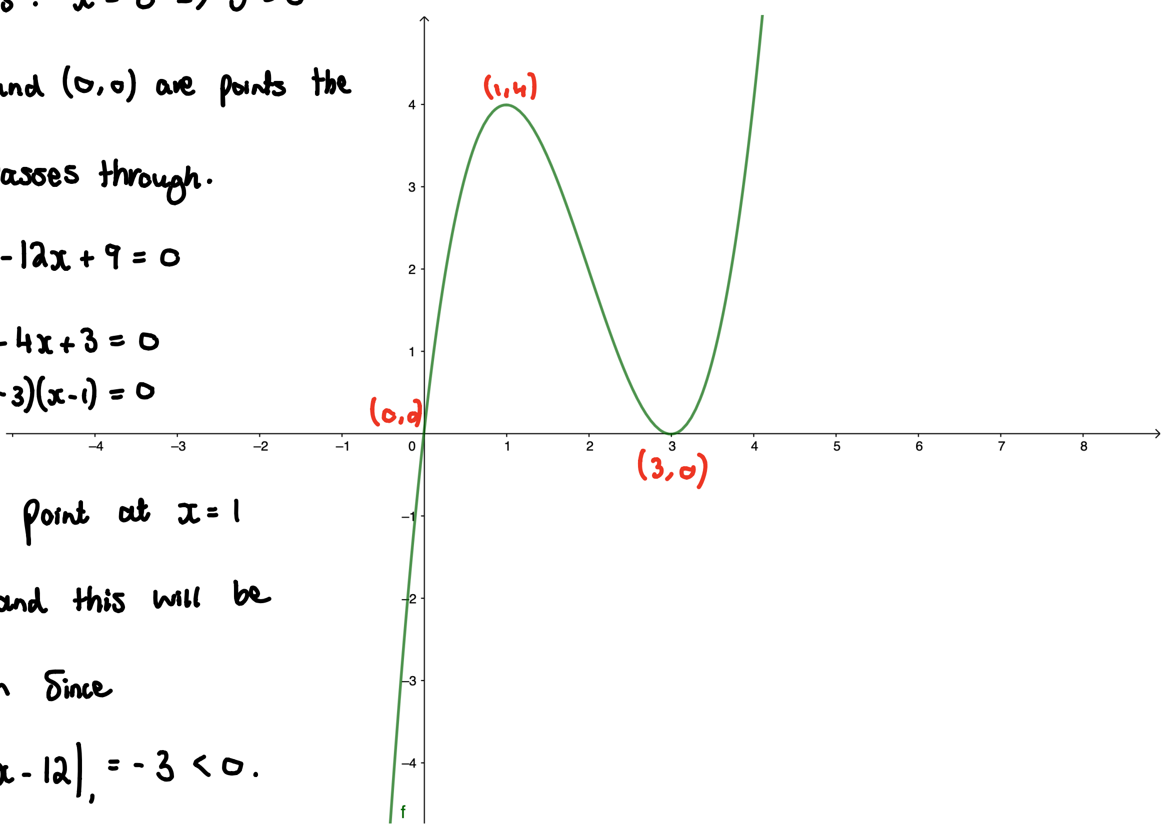
$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow (x-3)(x-1) = 0$

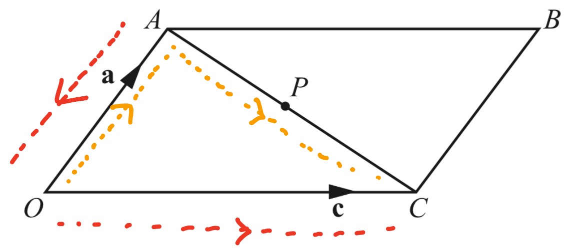
$\Rightarrow$ Turning point at $x = 1$

$\Rightarrow (1,4)$ and this will be a maximum since

$\dfrac{d^2y}{dx^2}\Big|_1 = 9x - 12\Big|_1 = -3 < 0.$



7 $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. P is the midpoint of AC.



(i) Find the following in terms of **a** and **c**, simplifying your answers.

(a) $\overrightarrow{AC}$ **[1]**

(b) $\overrightarrow{OP}$ **[2]**

a) * $\overrightarrow{AC} = -a + c \Rightarrow \overrightarrow{AC} = \underline{c - a}$

b) * $\overrightarrow{OP} = a + \dfrac{\overrightarrow{AC}}{2} = a + \dfrac{c-a}{2} = \underline{\tfrac{1}{2}(a+c)}$

divide by 2 since P is the midpoint of $\overrightarrow{AC}$.

(ii) Hence prove that the diagonals of a parallelogram bisect one another. **[4]**

The diagonals of the parallelogram will be $\overrightarrow{AC}$ and $\overrightarrow{OB} = 2\overrightarrow{OP}$

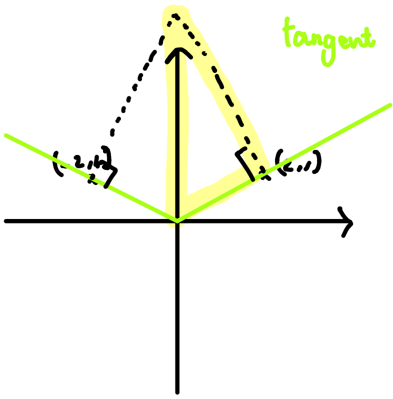
$\Rightarrow \overrightarrow{AC} = c - a$ and $2\overrightarrow{OP} = \overrightarrow{OB} = a + c$

Then we can say that P is the midpoint of $\overrightarrow{OB}$ and since we already knew P is the midpoint of $\overrightarrow{AC}$, we conclude the diagonals bisect one another.

8 **In this question you must show detailed reasoning.**

The lines $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$ are tangents to a circle at (2, 1) and (−2, 1) respectively. Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. **[6]**

The line adjacent to $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$ will pass through the center of the circle.



tangent lines, dotted lines are adjacent to tangent lines, so lets find the equation for (2,1) adjacent tangent line, as its coordinates will be the center of our circle.

We know the gradient will be $-\frac{1}{1/2} = -2$

$\Rightarrow y - 1 = -2(x-2) \Rightarrow y = 2x+5$

$\Rightarrow$ y intercept is 5 and our centre will be (0,5).

Then the radius will be $r = \sqrt{2^2 + 4^2} = \sqrt{20}$

$0 \to 2$ $1 \to 5$

$\Rightarrow$ equation of circle : $(x-0)^2 + (y-5)^2 = 20$ and expanding gives

$\Rightarrow x^2 + y^2 - 10y + 5 = 0$

$\underline{a = 0}$, $\underline{b = -10}$, $\underline{c = 5}$. in the form $x^2 + y^2 + ax + by + c = 0$.

Section B: Statistics

Answer **all** the questions.

9 Jo is investigating the popularity of a certain band amongst students at her school. She decides to survey a sample of 100 students.

(i) State an advantage of using a stratified sample rather than a simple random sample. **[1]**

Recall that Stratified Sampling involves the division of a population into smaller groups and then random samples are taken proportionally from each smaller group.
An advantage of this is that it uses the right/correct proportion of students, which prevents sampling too many students from a certain year group.

(ii) Explain whether it would be reasonable for Jo to use her results to draw conclusions about all students in the UK. **[1]**

This sample is not representitive of the whole UK since the school may be biased towards the band / type of music etc.

10 The probability distribution of a random variable X is given in the table.

x	0	2	4	6
$P(X=x)$	$\frac{3}{8}$	$\frac{5}{16}$	$4p$	p

(i) Find the value of p. **[2]**

$\Rightarrow \quad \frac{3}{8} + \frac{5}{16} + 4p + p = 1 \Rightarrow 5p = 1 - \frac{11}{16}$

$\Rightarrow p = \underline{\underline{\frac{1}{16}}}$

(ii) Two values of X are chosen at random. Find the probability that the product of these values is 0. **[3]**

The product of values will equal 0 when at least one $x = 0$.

We can find this by squaring (we square since two values are chosen) 1 minus the sum of probabilities.

$$\Rightarrow 1 - \left(\frac{5}{16} + \frac{4}{16} + \frac{1}{16}\right)^2 = \frac{39}{64}$$

11 The probability that Janice sees a kingfisher on any particular day is 0.3. She notes the number, X, of days in a week on which she sees a kingfisher.

(i) State one necessary condition for X to have a binomial distribution. **[1]**

- The probability of success is constant, i.e. each day the probability of seeing a kingfisher remains the same.
- Each time there are two outcomes, seeing a kingfisher (success) or not seeing a kingfisher (failure).

Assume now that X has a binomial distribution.

(ii) Find the probability that, in a week, Janice sees a kingfisher on exactly 2 days. **[1]**

1 week = 7 days : $X \sim B(7, 0.3)$

$\Rightarrow n = 7,\ p = 0.3$ and $x = 2$.

$\Rightarrow P(X = 2) = \binom{7}{2}(0.3)^2(1-0.3)^{7-2} = 0.31765...$

$P(X = 2) = \underline{0.318}$ (3 sig fig)

Each week Janice notes the number of days on which she sees a kingfisher.

(iii) Find the probability that Janice sees a kingfisher on exactly 2 days in a week during at least 4 of 6 randomly chosen weeks. **[3]**

This will be found by using our probability from part ii.

$\Rightarrow$ $X \sim B(6, 0.318)$ and $P(X \geq 4)$ with $n = 6$, $x = 4$ and $p = 0.318$

$\Rightarrow$ Probability $P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$$\Rightarrow P(X \geq 4) = \binom{6}{4}(0.318)^4(1-0.318)^2 + \binom{6}{5}(0.318)^5(1-0.318)^1 + \binom{6}{6}(0.318)^6(1-0.318)^0$$

$$= 0.07134581253 + 0.01330672632 + 1.034100433 \times 10^{-3}$$

$$= \underline{0.0857} \quad (3 \text{ sig fig})$$

12 It is known that 20% of plants of a certain type suffer from a fungal disease, when grown under normal conditions. Some plants of this type are grown using a new method. A random sample of 250 of these plants is chosen, and it is found that 36 suffer from the disease. Test, at the 2% significance level, whether there is evidence that the new method reduces the proportion of plants which suffer from the disease. **[7]**

let X be the random variable which denotes the number of diseased plants, then $X \sim B(250, 0.2)$, with $x = 36$.

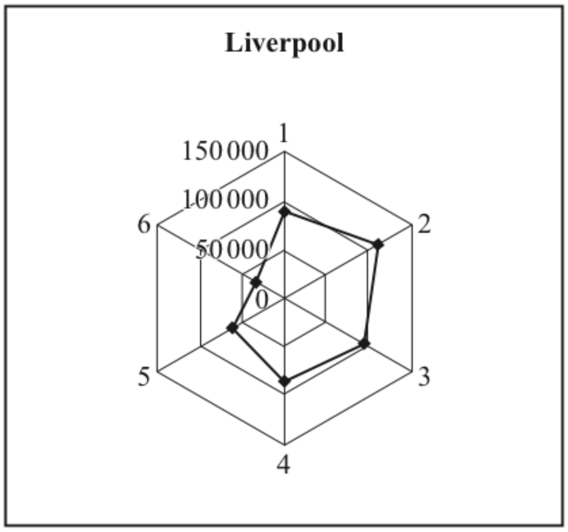
We want to test: $H_0 : p = 0.2$ v.s. $H_1 : 0.2 < 0.2$

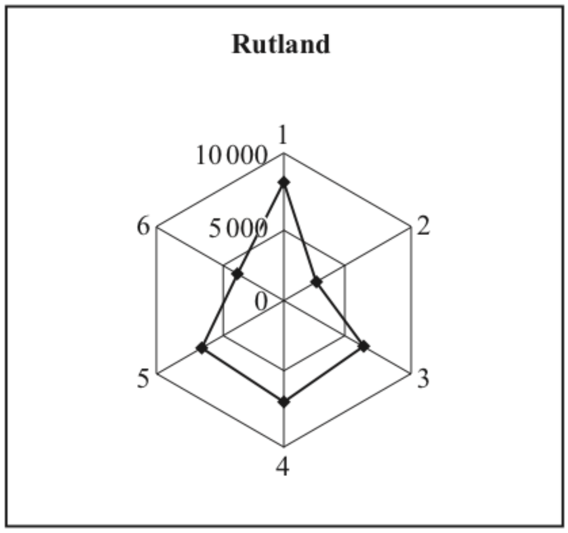
Then $P(X \leq 36) = 0.0139$ from binomial distribution on calculator.

We are testing at $\alpha = 0.02$ (2%) and $0.02 > 0.0139$

Which means that we should reject H_0 and we conclude that there is evidence that the new method reduces the proportion of diseased plants.

13 The radar diagrams illustrate some population figures from the 2011 census results.





Each radius represents an age group, as follows:

Radius	1	2	3	4	5	6
Age group	0–17	18–29	30–44	45–59	60–74	75+

The distance of each dot from the centre represents the number of people in the relevant age group.

(i) The scales on the two diagrams are different. State an advantage and a disadvantage of using different scales in order to make comparisons between the ages of people in these two Local Authorities. **[2]**

• Advantage : having different Scales will make it easier to see/ identify different age groups / distribution of ages in each population in each local authority area / diagram.

• Disadvantage : You cannot directly compare the two diagrams and their Values Since they do not have the Same Scale.

(ii) Approximately how many people aged 45 to 59 were there in Liverpool? **[1]**

This age group has radius 4, and we see the dot lies close to 4 which is 100,000 thus we can say approximately 90,000 people.

(iii) State the main two differences between the age profiles of the two Local Authorities. **[2]**

Take radius 1 (0-17 age group), Rutland has a higher proportion of people in this age group because Rutland's dot is further away from the centre than that of Liverpool.

Then, using the same reasoning, Liverpool has a smaller proportion of people in age group 60-74 compared to Rutland.

(iv) James makes the following claim.

"Assuming that there are no significant movements of population either into or out of the two regions, the 2021 census results are likely to show an increase in the number of children in Liverpool and a decrease in the number of children in Rutland."

Use the radar diagrams to give a justification for this claim. **[2]**

Group 2 (18-29) is proportionally larger in Liverpool and these people are most likely to have babies between 2011-2021, which means theyre will be (proportionally) more children/babies by 2021.

END OF QUESTION PAPER