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A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r|<1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \left(|x|<1,\ n \in \mathbb{R}\right)$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

Integration

$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\int \mathrm{f}'(x)\left(\mathrm{f}(x)\right)^n \mathrm{d}x = \frac{1}{n+1}\left(\mathrm{f}(x)\right)^{n+1} + c$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B \mid A) = \mathrm{P}(B)\,\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1} S_{xx}$ where $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n,\ p)$ then $\mathrm{P}(X = r) = {}^{n}\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$

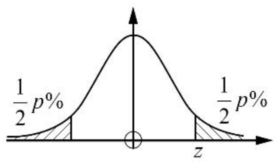
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (23 marks)

1 **Fig. 1** shows a sector of a circle of radius 7 cm. The area of the sector is 5 cm^2.

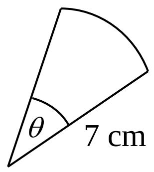


Fig. 1

Find the angle θ in radians. **[2]**

2 A geometric series has first term 3. The sum to infinity of the series is 8.
Find the common ratio. **[3]**

3 Solve the inequality $|2x-1| \geq 4$. **[4]**

4 Differentiate the following.

(a) $\sqrt{1-3x^2}$ **[3]**

(b) $\dfrac{x^2}{3x+2}$ **[3]**

5 A woman is pulling a loaded sledge along horizontal ground. The only resistance to motion of the sledge is due to friction between it and the ground.

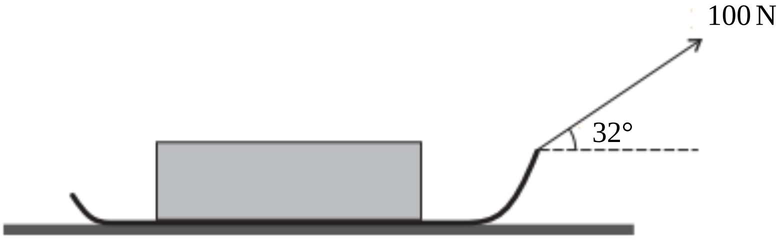


Fig. 5

At first, she pulls with a force of 100 N inclined at 32° to the horizontal, as shown in **Fig.5**, but the sledge does not move.

(a) Determine the frictional force between the ground and the sledge.
Give your answer correct to 3 significant figures. **[2]**

(b) Next she pulls with a force of 100 N inclined at a smaller angle to the horizontal. The sledge still does not move.

Compare the frictional force in this new situation with that in part **(a)**, justifying your answer. **[2]**

6 **Fig. 6** shows a partially completed spreadsheet.

This spreadsheet uses the trapezium rule with four strips to estimate $\int_0^{\frac{1}{2}\pi} \sqrt{1+\sin x}\,\mathrm{d}x$.

	A	B	C	D	E
1		x	$\sin x$	y	
2	0	0.0000	0.0000	1.0000	0.5000
3	0.125	0.3927	0.3827	1.1759	1.1759
4	0.25	0.7854	0.7071	1.3066	1.3066
5	0.375	1.1781	0.9239	1.3870	1.3870
6	0.5	1.5708	1.0000	1.4142	0.7071
7					5.0766
8					

Fig. 6

(a) Show how the value in cell B3 is calculated. **[1]**

(b) Show how the values in cells D2 to D6 are used to calculate the value in cell E7. **[1]**

(c) Complete the calculation to estimate $\int_0^{\frac{1}{2}\pi} \sqrt{1+\sin x}\,\mathrm{d}x$.

Give your answer to 3 significant figures. **[2]**

Answer **all** the questions

Section B (77 marks)

7 **In this question take $g = 10$.**

A small stone is projected from a point O with a speed of 26 m s^{-1} at an angle θ above the horizontal. The initial velocity and part of the path of the stone are shown in **Fig. 7**.

You are given that $\sin\theta = \frac{12}{13}$.

After t seconds the horizontal displacement of the stone from O is x metres and the vertical displacement is y metres.

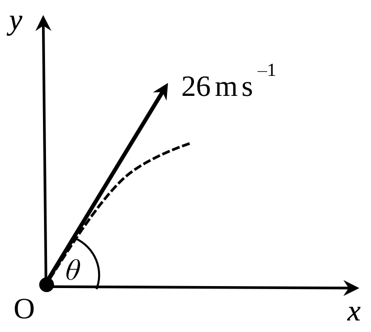


Fig. 7

(a) Using the standard model for projectile motion,

- show that $y = 24t - 5t^2$,
- find an expression for x in terms of t. **[4]**

The stone passes through a point A. Point A is 16 m above the level of O.

(b) Find the two possible horizontal distances of A from O. **[4]**

A toy balloon is projected from O with the same initial velocity as the small stone.

(c) Suggest two ways in which the standard model could be adapted. **[2]**

8 Find $\int x^2 \mathrm{e}^{2x} \, \mathrm{d}x$. **[7]**

9 In an experiment, a small box is hit across a floor. After it has been hit, the box slides without rotation.

The box passes a point A. The distance the box travels after passing A before coming to rest is S metres and the time this takes is T seconds.

The only resistance to the box's motion is friction due to the floor. The mass of the box is m kg and the frictional force is a constant F N.

(a) **(i)** Find the equation of motion for the box while it is sliding.

(ii) Show that $S = kT^2$ where $k = \dfrac{F}{2m}$. **[4]**

(b) Given that $k = 1.4$, find the value of the coefficient of friction between the box and the floor. **[4]**

10 In a certain region, the populations of grey squirrels, P_G and red squirrels P_R, at time t years are modelled by the equations:

$$P_G = 10\,000\left(1 - e^{-kt}\right)$$

$$P_R = 20\,000e^{-kt}$$

where $t \geq 0$ and k is a positive constant.

(a) **(i)** On the axes in your Printed Answer Book, sketch the graphs of P_G and P_R on the same axes.

(ii) Give the equations of any asymptotes. **[4]**

(b) What does the model predict about the long term population of

- grey squirrels
- red squirrels? **[2]**

Grey squirrels and red squirrels compete for food and space. Grey squirrels are larger and more successful than red squirrels.

(c) Comment on the validity of the model given by the equations, giving a reason for your answer. **[1]**

(d) Show that, according to the model, the rate of decrease of the population of red squirrels is always double the rate of increase of the population of grey squirrels. **[4]**

(e) When $t = 3$, the numbers of grey and red squirrels are equal. Find the value of k. **[4]**

11 **Fig. 11** shows the curve with parametric equations

$$x = 2\cos\theta,\ y = \sin\theta,\ 0 \le \theta \le 2\pi.$$

The point P has parameter $\frac{1}{4}\pi$. The tangent at P to the curve meets the axes at A and B.

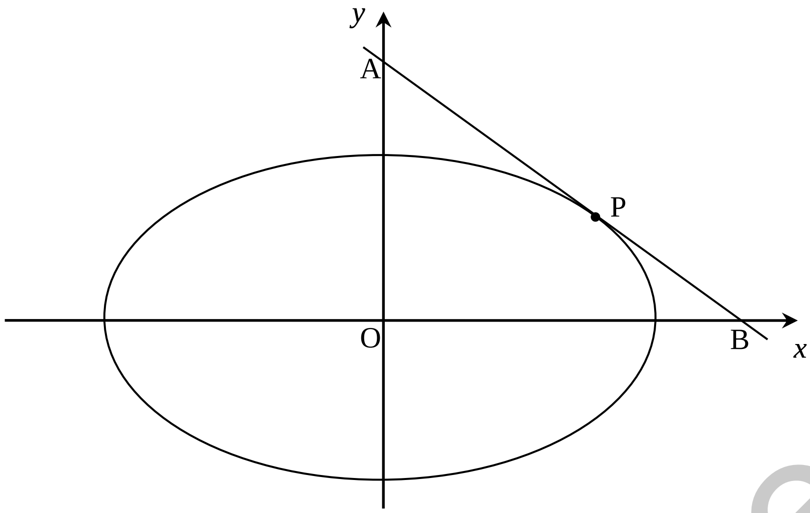


Fig. 11

(a) Show that the equation of the line AB is $x + 2y = 2\sqrt{2}$. **[6]**

(b) Determine the area of the triangle AOB. **[3]**

12 A model boat has velocity $\mathbf{v} = \left((2t-2)\mathbf{i} + (2t+2)\mathbf{j}\right)$ m s^{-1} for $t \ge 0$, where t is the time in seconds. $\mathbf{i}$ is the unit vector east and $\mathbf{j}$ is the unit vector north.
When $t = 3$, the position vector of the boat is (3**i** + 14**j**) m.

(a) Show that the boat is never instantaneously at rest. **[2]**

(b) Determine any times at which the boat is moving directly northwards. **[2]**

(c) Determine any times at which the boat is north-east of the origin. **[5]**

13 **In this question you must show detailed reasoning.**
Determine the values of k for which part of the graph of $y = x^2 - kx + 2k$ appears below the x-axis. **[4]**

14 Blocks A and B are connected by a light rigid horizontal bar and are sliding on a rough horizontal surface.

A light horizontal string exerts a force of 40 N on B.
This situation is shown in **Fig. 14**, which also shows the direction of motion, the mass of each of the blocks and the resistances to their motion.

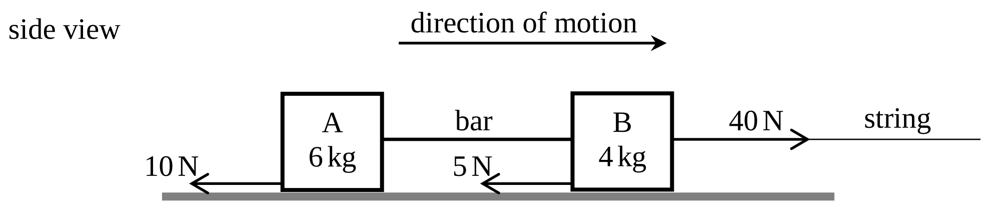


Fig. 14

(a) Calculate the tension in the bar. **[4]**

The string breaks while the blocks are sliding. The resistances to motion are unchanged.

(b) Determine
- the magnitude of the new force in the bar,
- whether the bar is in tension or in compression. **[5]**

15 **Fig. 15** shows a uniform shelf AB of weight W N.
The shelf is 180 cm long and rests on supports at points C and D. Point C is 30 cm from A and point D is 60 cm from B.

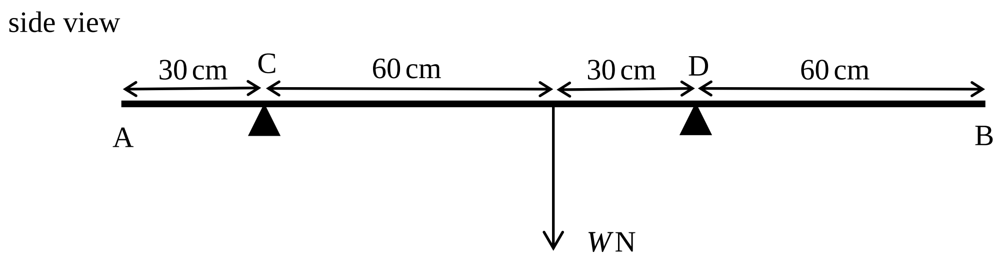


Fig. 15

Determine the range of positions a point load of 3 W could be placed on the shelf without the shelf tipping. **[6]**

END OF QUESTION PAPER

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