OCR
Oxford Cambridge and RSA

Wednesday 07 October 2020 – Afternoon

A Level Mathematics B (MEI)

Model Solutions

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours

8168829224

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

Section A (22 marks)

1 Simplify $\left(\frac{27}{x^9}\right)^{\frac{2}{3}} \times \left(\frac{x^4}{9}\right)$. **[2]**

$$\left(\frac{27}{x^9}\right)^{2/3} \times \left(\frac{x^4}{9}\right)$$

$$27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 9$$
$$(x^9)^{2/3} = x^6$$

$$= \frac{27^{2/3}}{(x^9)^{2/3}} \times \frac{x^4}{9} = \frac{9}{x^6} \times \frac{x^4}{9} = \frac{9x^4}{9x^6} = \underline{\frac{1}{x^2}}$$

2 Express $\frac{a+\sqrt{2}}{3-\sqrt{2}}$ in the form $p+q\sqrt{2}$, giving p and q in terms of a. **[3]**

$$\frac{a+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{(a+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{3a + a\sqrt{2} + 3\sqrt{2} + 2}{7}$$

We want to multiply by the conjugate which allows us to simplify the denominator.

$$= \frac{3a+2}{7} + \frac{a+3}{7}\sqrt{2}$$

$$= p + q\sqrt{2} \text{ where } p = \frac{3a+2}{7} \text{ and } q = \frac{a+3}{7}$$

3 The points A and B have position vectors $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \\ 8 \end{pmatrix}$ respectively.

Show that the exact value of the distance AB is $\sqrt{101}$. **[3]**

We will first calculate the value of the vector $\overrightarrow{AB}$:

$$\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix} -1 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 9 \end{pmatrix}$$

We now will show the distance by finding the magnitude of the vector $\overrightarrow{AB}$:

$$|\overrightarrow{AB}| = \sqrt{(-4)^2 + (2)^2 + (9)^2} = \underline{\sqrt{101}} \text{ as required.}$$

4 Find the second derivative of $(x^2+5)^4$, giving your answer in factorised form. [5]

To find the second derivative we must first find the first derivative :

let $f(x) = (x^2+5)^4$ then $f'(x) = 4 \times 2x\,(x^2+5)^3$

$\Rightarrow f'(x) = 8x\,(x^2+5)^3$

We now want to differentiate again and we will have to use the product rule :

If $h(x) = g(x) \cdot k(x)$ then $h'(x) = g(x) \cdot k'(x) + g'(x) \cdot k(x)$

$\Rightarrow$ Our $h(x) = f'(x) = 8x\,(x^2+5)^3$ $\Rightarrow$ $g(x) = 8x$, $k(x) = (x^2+5)^3$ hence $g'(x) = 8$, $k'(x) = 6x\,(x^2+5)^2$

$$\Rightarrow h'(x) = f''(x) = 8x\left(6x\,(x^2+5)^2\right) + 8(x^2+5)^3$$
$$= 48x^2(x^2+5)^2 + 8\,(x^2+5)^3$$

let $y = x^2+5$ then $f''(x) = 48x^2y^2 + 8y^3$

$f''(x) = 8y^2(6x^2+y)$

manipulate into factored form.

$f''(x) = 8(x^2+5)^2(6x^2+x^2+5)$

$\Rightarrow f''(x) = \underline{8(x^2+5)^2(7x^2+5)}$

5 A child is running up and down a path. A simplified model of the child's motion is as follows:

- he first runs north for 5 s at 4 m s^{-1};
- he then suddenly stops and waits for 8 s;
- finally he runs in the opposite direction for 7 s at 3.5 m s^{-1}.

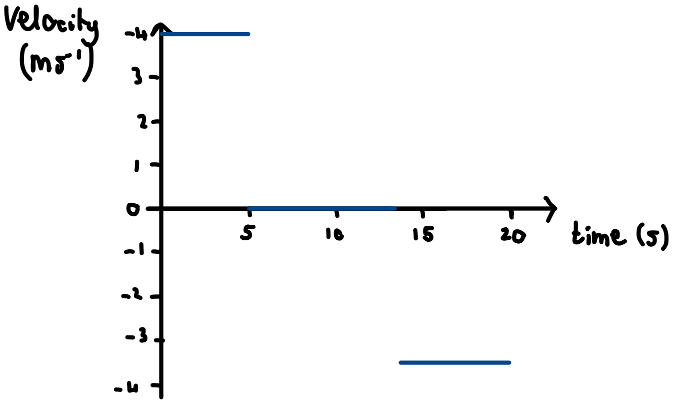
(a) Taking north to be the positive direction, sketch a velocity-time graph for this model of the child's motion. **[2]**

Using this model,

(b) calculate the total distance travelled by the child, **[2]**

(c) find his final displacement from his original position. **[1]**

a) Let North be the positive direction.



b) The total distance travelled by the child is a scaler quantity where direction does not matter. We sum the distances from each part of the childs movement.

=> Total Distance = Speed x time

=> $d = V_1 t_1 + V_2 t_2$ where $V_1 = 4ms^{-1}$ and $t_1 = 5s$
$V_2 = 3.5ms^{-1}$ and $t_2 = 7s$

=> $d = 4 \times 5 + 3.5 \times 7 = 44.5$

=> Total Distance travelled by child is <u>44.5m</u>

c) Displacement is found by finding the distance and direction of our child in relation to where they started.

Displacement $= s = 4 \times 5 + -(3.5 \times 7) = -4.5m$.

=> Since we have negative displacement, we conclude that the childs displacement is <u>4.5m South</u>.

6 A uniform ruler AB has mass 28 g and length 30 cm. As shown in Fig. 6, the ruler is placed on a horizontal table so that it overhangs a point C at the edge of the table by 25 cm.

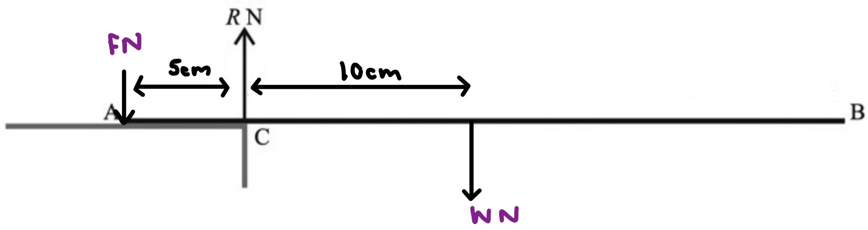
A downward force of F N is applied at A. This force just holds the ruler in equilibrium so that the contact force between the table and the ruler acts through C.

Fig. 6

(a) Complete the force diagram in the Printed Answer Booklet, labelling the forces and all relevant distances. **[2]**

(b) Calculate the value of F. **[2]**

a)



• Weight W acts downwards
• Force F acts downwards.

b) Here, we want to take moments about point C on the diagram.
Noting that 28g = 0.028kg, and g is the gravitational constant, we have that:

$$10 \times 0.028g = 5F$$

$$\Rightarrow \quad F = \frac{10 \times 0.028g}{5} = \frac{10 \times 0.028 \times 9.8}{5} = 0.5488$$

$$\Rightarrow \quad F = \underline{0.549\,N}$$

Section B (78 marks)

7 **In this question you must show detailed reasoning.**

The function $f(x)$ is defined by $f(x) = x^3 + x^2 - 8x - 12$ for all values of x.

(a) Use the factor theorem to show that $(x+2)$ is a factor of $f(x)$. **[2]**

(b) Solve the equation $f(x) = 0$. **[4]**

a) $f(x) = x^3 + x^2 - 8x - 12$

We have that $(x+2)$ is a factor $\Rightarrow$ $x = -2$ will give us that the value of $f(x)$ evaluated at $x = -2$ will be 0, i.e. $f(-2) = 0$.

$\Rightarrow f(-2) = (-2)^3 + (2)^2 - 8(-2) - 12 = 0$

$\Rightarrow f(-2) = 0$ hence $(x+2)$ is a factor of $f(x)$.

b) $f(x) = x^3 + x^2 - 8x - 12 = 0$

$\Rightarrow f(x) = (x+2)(x^2 - x - 6)$ factorise trinomial

$= (x+2)(x+2)(x-3)$

$\Rightarrow$ $\underline{\underline{x = -2}}$ (repeated) and $\underline{\underline{x = 3}}$

We use this to find out what happens when we take out $(x+2)$ as a factor.

$$
\begin{array}{r|l}
 & x^2 - x - 6 \\
\hline
x+2 & x^3 + x^2 - 8x - 12 \\
 & -\ x^3 - 2x^2 \\
\hline
 & -x^2 - 8x - 12 \\
 & +x^2 + 2x \\
\hline
 & -6x - 12 \\
 & +6x + 12 \\
\hline
 & 0
\end{array}
$$

8 Fig. 8.1 shows the cross-section of a straight driveway 4 m wide made from tarmac.

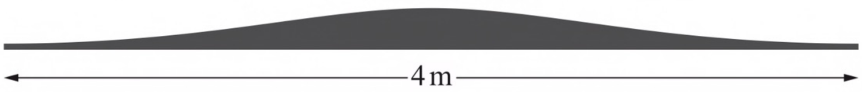


Fig. 8.1

The height h m of the cross-section at a displacement x m from the middle is modelled by $h = \frac{0.2}{1+x^2}$ for $-2 \leqslant x \leqslant 2$.

A lower bound of $0.3615\,\text{m}^2$ is found for the area of the cross-section using rectangles as shown in Fig. 8.2.

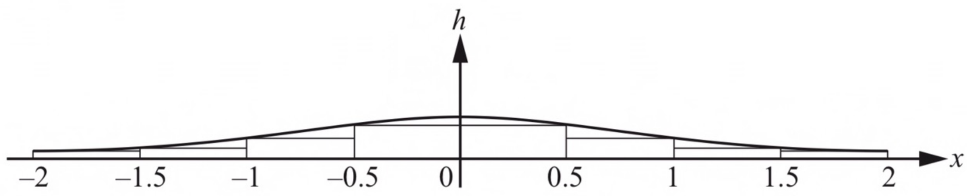


Fig. 8.2

(a) Use a similar method to find an upper bound for the area of the cross-section. **[3]**

(b) Use the trapezium rule with 4 strips to estimate $\int_0^2 \frac{0.2}{1+x^2}\,dx$. **[2]**

(c) The driveway is 10 m long. Use your answer in part **(b)** to find an estimate of the volume of tarmac needed to make the driveway. **[2]**

a) We want to find an upper bound for the area of the cross-section.

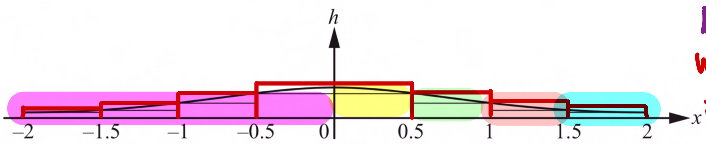


Diagram for lower bound
We will annotate upper bound in Red.

We have that $h = \frac{0.2}{1+x^2}$, hence we can fill out the following table.

x	0	± 0.5	± 1	± 1.5	± 2
h	$1/5$	$4/25$	$1/10$	$4/65$	$1/25$

$\Rightarrow$ Area $= \left(0.5 \times 1/5 + 0.5 \times 4/25 + 0.5 \times 1/10 + 0.5 \times 4/65\right) \times 2$

$= 2 \times 0.5\left(1/5 + 4/25 + 1/10 + 1/25\right)$

$= 0.5215\ldots$

$\Rightarrow$ An upper bound for the area of the cross-section is $\underline{0.522}$ (3 s.f.)

b) We now want to use the trapezium rule with four strips to estimate $\int_0^2 \frac{0.2}{1+x^2}\,dx$

Trapezium rule with $n=4$: $\int_a^b f(x)\,dx = \frac{1}{2}\cdot h\left[y_0 + \ldots + y_4 \right]$ where $h = 2/4 = 0.5$

$y_0 = 1/5,\ y_1 = 4/25,\ y_2 = 1/10,\ y_3 = 4/65,\ y_4 = 1/25$

$$\Rightarrow \int_0^2 \frac{0.2}{1+x^2}\,dx = \frac{0.5}{2}\left(1/5 + 1/25 + 2\times(4/25 + 1/10 + 4/65)\right) = 0.220769\ldots$$

$$\Rightarrow \int_0^2 \frac{0.2}{1+x^2}\,dx = \underline{0.221} \quad (3\text{ s.f}).$$

8c) We now want to estimate the volume of tarmac needed to make the driveway.

Area : 0.221 $\Rightarrow$ Volume $= (0.221 \times 10) \times 2 = 4.42\,m^3$.

Area length

9 A particle is moving in a straight line. The acceleration a m s^{-2} of the particle at time t s is given by $a = 0.8t + 0.5$. The initial velocity of the particle is 3 m s^{-1} in the positive x-direction.

Determine whether the particle is ever stationary. **[6]**

We know that

integrate ↓ acceleration / Velocity / Displacement ↑ differentiate

$\Rightarrow V = \int a\,dt = \int 0.8t + 0.5\,dt = 0.4t^2 + 0.5t + c = V.$

$\Rightarrow$ For $t=0,\ V=3$ hence $0.4t^2 + 0.5t + c = 3$

$\Rightarrow c = 3$ by substituting in $t=0$, hence $V = 0.4t^2 + 0.5t + 3$.

We will now look to see if the particle is ever stationary, which would occur when $V=0$.

Hence, $V=0 \Rightarrow 0.4t^2 + 0.5t + 3 = 0$

We can now work out the discriminant of this quadratic with $a = 0.4$, $b = 0.5$ and $c = 3$

$\Rightarrow b^2 - 4ac = (0.5)^2 - 4(0.4)(3) = \underline{-4.55 < 0.}$

$\Rightarrow$ This equation has no real roots since $b^2 - 4ac < 0$, hence the velocity is never 0 and the particle will never be stationary.

10 In this question you must show detailed reasoning.

Fig. 10 shows the curve given parametrically by the equations $x = \frac{1}{t^2}$, $y = \frac{1}{t^3} - \frac{1}{t}$, for $t > 0$.

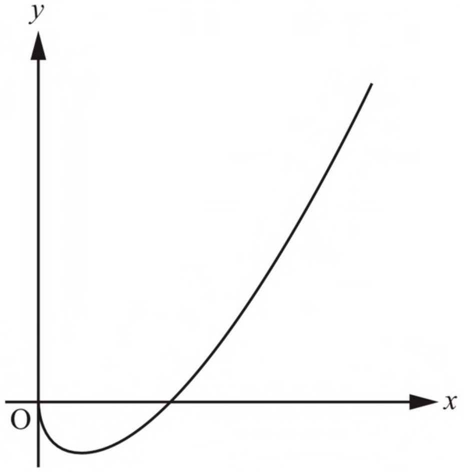


Fig. 10

(a) Show that $\frac{dy}{dx} = \frac{3-t^2}{2t}$. **[3]**

(b) Find the coordinates of the point on the curve at which the tangent to the curve is parallel to the line $4y + x = 1$. **[3]**

(c) Find the cartesian equation of the curve. Give your answer in factorised form. **[3]**

a) We want to find $\frac{dy}{dx}$ where $x = 1/t^2$, $y = 1/t^3 - 1/t$ for $t > 0$.

We know that $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$, so we will first find $\frac{dy}{dt}$ and $\frac{dt}{dx}$.

$\Rightarrow$ $y = \frac{1}{t^3} - \frac{1}{t} = t^{-3} - t^{-1}$ then $\frac{dy}{dt} = -3t^{-4} + t^{-2}$ and $x = \frac{1}{t^2} = t^{-2}$ then $\frac{dx}{dt} = -2t^{-3}$

$\Rightarrow \frac{dt}{dx} = \frac{-t^3}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = (-3t^{-4} + t^{-2}) \times -\frac{t^3}{2} = \left(-3t^{-4} \times -\frac{t^3}{2}\right) - \frac{t^{-2} \times t^3}{2} = \frac{3}{2t} - \frac{t}{2} = \frac{6 - 2t^2}{4t} = \frac{3 - t^2}{2t}$.

$\Rightarrow \frac{dy}{dx} = \frac{3-t^2}{2t}$ as required.

b) line: $4y + x = 1 \Rightarrow y = \frac{1-x}{4} = \frac{1}{4} - \frac{1}{4}x \Rightarrow \frac{dy}{dx} = -1/4$

$\Rightarrow$ The tangent will be parallel when $\frac{dy}{dx} = -1/4$, so we can set the derivative of the equation of the curve equal to this: $\frac{dy}{dx} = \frac{3-t^2}{2t} = -\frac{1}{4}$ and we now want to solve for t, and hence x and y.

$\Rightarrow 2(3 - t^2) = -t \Rightarrow 2t^2 - t - 6 = 0 \Rightarrow$ Quadratic formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(2)(-6)}}{4} = \frac{1 \pm 7}{4} \Rightarrow t = 2$ and $t = -3$

Since $t > 0$, we conclude that the only valid solution is $t = 2$, which we can now use to find x and y.
Hence, $t = 2 \Rightarrow x = 1/t^2 = \frac{1}{4}$ and $y = 1/t^3 - 1/t = -3/8$

$\Rightarrow$ The coordinates are $(1/4, -3/8)$

c) Recall, $x = \frac{1}{t^2}$ and $y = \frac{1}{t^3} - \frac{1}{t}$

We first will find t in terms of y which will allow us to substitute this into our equation for y.

$x = 1/t^2 \Rightarrow t^2 = \frac{1}{x} \Rightarrow t = x^{-1/2} \Rightarrow y = \frac{1}{(x^{-1/2})^3} - \frac{1}{x^{-1/2}} = x^{3/2} - x^{1/2}$ factorise

$\Rightarrow y = \sqrt{x}(x-1)$

11 A block of mass 2 kg is placed on a rough horizontal table. A light inextensible string attached to the block passes over a smooth pulley attached to the edge of the table. The other end of the string is attached to a sphere of mass 0.8 kg which hangs freely.

The part of the string between the block and the pulley is horizontal. The coefficient of friction between the table and the block is 0.35. The system is released from rest.

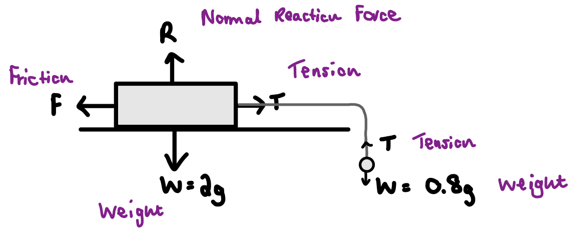
(a) Draw a force diagram showing all the forces on the block and the sphere. **[3]**

(b) Write down the equations of motion for the block and the sphere. **[2]**

(c) Show that the acceleration of the system is $0.35\,\text{m}\,\text{s}^{-2}$. **[4]**

(d) Calculate the time for the block to slide the first 0.5 m. Assume the block does not reach the pulley. **[2]**

a) Key info:
- 2kg block on table
- 0.8kg freehanging sphere
- Friction is 0.35
- At rest



b) We now want to find an equation of motion for block and the sphere.

- The block will be moving horizontally, hence $T - F = 2a$
- The sphere will be moving vertically, hence $0.8g - T = 0.8a$

c) We now will find the acceleration of the block

2g since W = 2g and its not moving horizontally

- We first work out friction; $F = \mu R = 0.35 \times 2g = 0.7g$

$\Rightarrow T - 0.7g = 2a$ and $0.8g - T = 0.8a$

$\Rightarrow T = 2a + 0.7g$ and $T = 0.8g - 0.8a$

$\Rightarrow 2a + 0.7g = 0.8g - 0.8a$

$\Rightarrow 2.8a = 0.1g$

$\Rightarrow a = \frac{0.1}{2.8} \times 9.8 = 0.35\,\text{ms}^{-2}$ as required.

d) $s = 0.5\text{m}$ We choose our equation of motion:

$u = 0\ \text{ms}^{-1}$ $s = ut + \frac{1}{2}at^2$

$v = x$ $0.5 = \frac{1}{2}(0.35)t^2$

$a = 0.35\text{ms}^{-2}$ $\Rightarrow t^2 = \frac{20}{7}$

$t = ?$

Hence $t = \underline{1.69\text{s}}$

12 A function is defined by $f(x) = x^3 - x$.

(a) By considering $\frac{f(x+h) - f(x)}{h}$, show from first principles that $f'(x) = 3x^2 - 1$. **[4]**

(b) Sketch the gradient function $f'(x)$. **[2]**

(c) Show that the curve $y = f(x)$ has a single point of inflection which is not a stationary point. **[3]**

a) $f(x) = x^3 - x$ then $f(x+h) = (x+h)^3 - (x+h)$
$= x^3 + 3x^2h + 3xh^2 + h^3 - x - h$

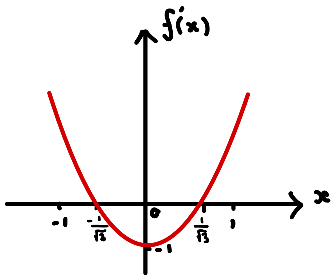
$\Rightarrow f'(x) = \lim_{h \to 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3 - x - h) - (x^3 - x)}{h} = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - h}{h} = \lim_{h \to 0} (3x^2 + 3xh + h^2 - 1)$

$\Rightarrow f'(x) = \underline{3x^2 - 1}$ as required.

b) Here, we want to sketch the gradient function $f'(x) = 3x^2 - 1$

- $\cup$ parabola with turning point $(0, -1)$
- Roots: $x^2 = 1/3 \Rightarrow x = \pm 1/\sqrt{3} \Rightarrow \left(\frac{1}{\sqrt{3}}, 0\right)$ and $\left(-\frac{1}{\sqrt{3}}, 0\right)$

$\Rightarrow$



c) We know that a point of inflection occurs when $f''(x) = 0$.

$f'(x) = 3x^2 - 1 \Rightarrow f''(x) = 6x$

Then $f''(x) = 6x = 0$ has one root, namely $x = 0$, which is our point of inflection.

This is not a stationary point since $f'(0) = 3(0)^2 - 1 = -1 \neq 0$.

13 A projectile is fired from ground level at $35\,\mathrm{m\,s^{-1}}$ at an angle of $\theta°$ above the horizontal.

(a) State a modelling assumption that is used in the standard projectile model. **[1]**

(b) Find the cartesian equation of the trajectory of the projectile. **[4]**

The projectile travels above horizontal ground towards a wall that is 110 m away from the point of projection and 5 m high. The projectile reaches a maximum height of 22.5 m.

(c) Determine whether the projectile hits the wall. **[6]**

a) Generally, we should assume that :

- we neglect air resistance
- Gravity is constant
- we treat our projectile as a particle.

b) Our Cartesian equation can be formed by first creating equations in both the vertical and horizontal plane.

Horizontal : $u_x = 35\cos\theta \Rightarrow x = (35\cos\theta)t \Rightarrow t = \dfrac{x}{35\cos\theta}$

Vertical : $u_y = 35\sin\theta \qquad y = (35\sin\theta)t - \frac{1}{2}gt^2$

Substitute in

$$\Rightarrow y = (35\sin\theta)\cdot\frac{x}{35\cos\theta} - \frac{1}{2}g\left(\frac{x}{35\cos\theta}\right)^2$$

$\dfrac{\sin}{\cos} = \tan$

$g = 9.8$

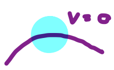
$\dfrac{\frac{1}{2}\times 9.8}{35^2} = \dfrac{1}{250}$

$$\Rightarrow y = x\tan(\theta) - \frac{x^2}{250\cos^2\theta}$$

c) $S = 22.5\mathrm{m}$ (max height)

$u = 35\sin\theta\ \mathrm{ms^{-1}}$ (Vertical direction as dealing with height)

$V = 0$ (Since the projectile is at peak of flight)



$a = -9.8\,\mathrm{ms^{-2}}$

$\Rightarrow V^2 = u^2 + 2as$

$0 = (35\sin\theta)^2 + 2(-9.8)(22.5)$

$\Rightarrow 1125\sin^2\theta = 441$

$\Rightarrow \sin(\theta) = 0.6 \Rightarrow \theta = 36.9°$

We now use this angle with our Cartesian equation $y = x\tan(\theta) - \dfrac{x^2}{250\cos^2\theta}$ and $x = 110\mathrm{m}$ as this is the distance the wall is from the start point.

$\Rightarrow y = 110\tan(36.9) - \dfrac{110^2}{250\cos^2(36.9)} = 6.905 = 6.9\mathrm{m} > 5\mathrm{m} \Rightarrow$ the projectile goes over the wall.

Check

14 Douglas wants to construct a model for the height of the tide in Liverpool during the day, using a cosine graph to represent the way the height changes.

He knows that the first high tide of the day measures 8.55 m and the first low tide of the day measures 1.75 m.

Douglas uses t for time and h for the height of the tide in metres. With his graph-drawing software set to degrees, he begins by drawing the graph of $h = 5.15 + 3.4\cos t$.

(a) Verify that this equation gives the correct values of h for the high and low tide. **[1]**

Douglas also knows that the first high tide of the day occurs at 1 am and the first low tide occurs at 7.20 am. He wants t to represent the time in hours after midnight, so he modifies his equation to $h = 5.15 + 3.4\cos(at + b)$.

(b) **(i)** Show that Douglas's modified equation gives the first high tide of the day occurring at the correct time if $a + b = 0$. **[1]**

(ii) Use the time of the first low tide of the day to form a second equation relating a and b. **[1]**

(iii) Hence show that $a = 28.42$ correct to 2 decimal places. **[2]**

(c) Douglas can only sail his boat when the height of the tide is at least 3 m.

Use the model to predict the range of times that morning when he cannot sail. **[3]**

(d) The next high tide occurs at 12.59 pm when the height of the tide is 8.91 m.

Comment on the suitability of Douglas's model. **[2]**

a) We have that $h = 5.15 + 3.4\cos t$

For high tide we have $\cos t = 1 \Rightarrow h_{max} = 5.15 + 3.4 = 8.55\text{m}$
For low tide we have $\cos t = -1 \Rightarrow h_{min} = 5.15 - 3.4 = 1.75\text{m}$

$\Rightarrow$ These match the high/low tide values given in the question.

b)

i) We have that high-tide occurs when $t = 1$.

When $t = 1$: $8.55 = 5.15 + 3.4\cos(a+b)$ 8.55 - 5.15 = 3.4 which cancels out

$\Rightarrow \cos(a+b) = 1$

$\Rightarrow a+b = \cos^{-1}(1) = 0$

$\Rightarrow a+b = 0$, hence Douglas's modified equation is correct.

ii) We have that a minimum will occur when $at + b = 180°$, since $\cos(x)$ is minimised at $180°$.
and we have t such that $t = 7\frac{1}{3} = \frac{22}{3}$.

Therefore, an equation for low tide will be $1.75 = 5.15 + 3.4\cos\left(\frac{22}{3}a + b\right)$.

$\left(\frac{22}{3}a + b = 180°\right)$

iii) We have $a+b=0$ and $\frac{22}{3}a+b=180°$

$b=-a$ and $b=180°-\frac{22}{3}a$

$-a+\frac{22}{3}a=180$

$\frac{19}{3}a=180°$, hence $\underline{\underline{a=28.42}}$ (2 d.p) as required.

$a+b=0$, hence $b=-28.42$

c) We will now predict the range of times that Douglas can sail.
We set $h=3$, which gives us:

$3=5.15+3.4\cos(at+b)$

$\Rightarrow 3=5.15+3.4\cos(28.42t-28.42)$ (we now solve for t.)

$\Rightarrow \cos(28.42t-28.42)=-\frac{43}{68}$

S	A ✓	0 + 129.224
T	C ✓	360 - 129.224

$\Rightarrow 28.42t-28.42=129.224$ and $28.42t-28.42=230.76$

$\Rightarrow t=5.55$ and $t=9.12$

5.55 hours $=$ 5hrs $+\frac{55}{100}\times 60$ mins

$=$ 5hrs 33mins

9.12 hours $=$ 9hrs $+\frac{12}{100}\times 60$

$=$ 9hrs 7 mins

$\Rightarrow$ Douglas does not sail between $\underline{5:33\text{am}}$ and $\underline{9:07\text{am}}$.

d)

- Our model predicted that every high tide is 8.55m.
- The next high tide (8.91m) is higher, which means that the model is not perfectly modelling reality.

15 Fig. 15 shows a particle of mass m kg on a smooth plane inclined at 30° to the horizontal. Unit vectors **i** and **j** are parallel and perpendicular to the plane, in the directions shown.

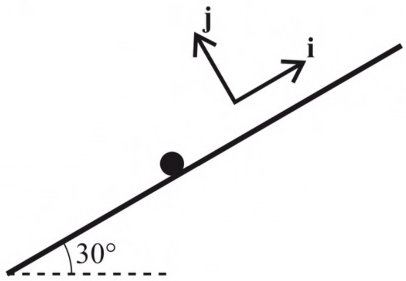


Fig. 15

(a) Express the weight **W** of the particle in terms of m, g, **i** and **j**. **[2]**

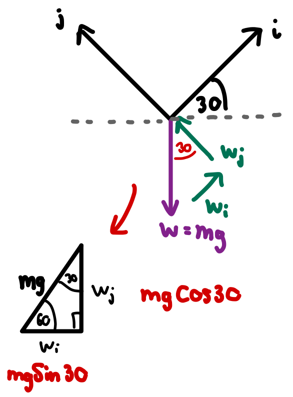
The particle is held in equilibrium by a force **F**, and the normal reaction of the plane on the particle is denoted by **R**. The units for both **F** and **R** are newtons.

(b) Write down an equation relating **W**, **R** and **F**. **[1]**

(c) Given that $\mathbf{F} = 6\mathbf{i} + 8\mathbf{j}$,

- show that $m = 1.22$ correct to 3 significant figures,
- find the magnitude of **R**. **[6]**

a) $\underline{i}$: Parallel to plane and $\underline{j}$: perpendicular to plane



$\underline{W} = -W_i\underline{i} - W_j\underline{j}$ and $W_i =$

$\underline{W} = (-mg\sin 30)\underline{i} + (-mg\cos 30)\underline{j}$

$\Rightarrow \underline{W} = \left(-\frac{1}{2}mg\right)\underline{i} + \left(-\frac{\sqrt{3}}{2}mg\right)\underline{j}$

b) The particle is in equilibrium, hence the net force will add to zero.

$\Rightarrow \underline{W} + \underline{R} + \underline{F} = \underline{0}$.

c) $\underline{F} = 6\underline{i} + 8\underline{j}$ and our resultant force $\underline{R}$ is perpendicular to plane hence $\underline{R} = R\underline{j}$.

Then we substitute into the equation $\underline{W} + \underline{R} + \underline{F} = \underline{0}$.

$\Rightarrow \left(-\frac{1}{2}mg\right)\underline{i} + \left(-\frac{\sqrt{3}}{2}mg\right)\underline{j} + 6\underline{i} + 8\underline{j} + R\underline{j} = \underline{0}$

$\underline{i}$: $-\frac{1}{2}mg + 6 = 0$

$\Rightarrow m = \frac{6 \times 2}{9.8} = 1.22$ as required.

$\underline{j}$: $-\frac{\sqrt{3}}{2}mg + 8 + R = 0$

$\Rightarrow -\frac{\sqrt{3}}{2} \times 1.22 \times 9.8 + 8 = -R \Rightarrow R = \underline{2.35N}$