OCR
Oxford Cambridge and RSA

Wednesday 14 October 2020 – Afternoon

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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Answer **all** the questions.

1 Solve the inequality $2x + 5 < 6x - 3$. **[2]**

$2x + 5 < 6x - 3$

$+3 \qquad +3$

$\Rightarrow 2x + 8 < 6x$

$-2x \qquad -2x$

$8 < 4x$

$\frac{4x}{4} > \frac{8}{4}$

$x > 2$

2 A student measures the upper arm lengths of a sample of 97 women. The results are summarised in the frequency table in Fig. 2.1.

Arm length in cm	30 –	31 –	32 –	33 –	34 –	35 –	36 –	37 –	38 –	39 –	40 – 41
Frequency	1	4	5	9	13	19	17	17	4	3	5

Fig. 2.1

The student constructs two cumulative frequency diagrams to represent the data using different class intervals. These are shown in Fig. 2.2 opposite.

One of these diagrams is correct and the other is incorrect.

(a) State which diagram is incorrect, justifying your answer. **[2]**

(b) Use the correct diagram in Fig. 2.2 to find an estimate of the median. **[1]**

a) Diagram A is incorrect

→ This is because the cumulative frequencies have been plotted at the left hand end of each class interval, not the right hand end.

→ OR Diagram B is right as you have to use upper bound whilst Diagram A used lower bound

b) Median ≈ 35.9 (From diagram)

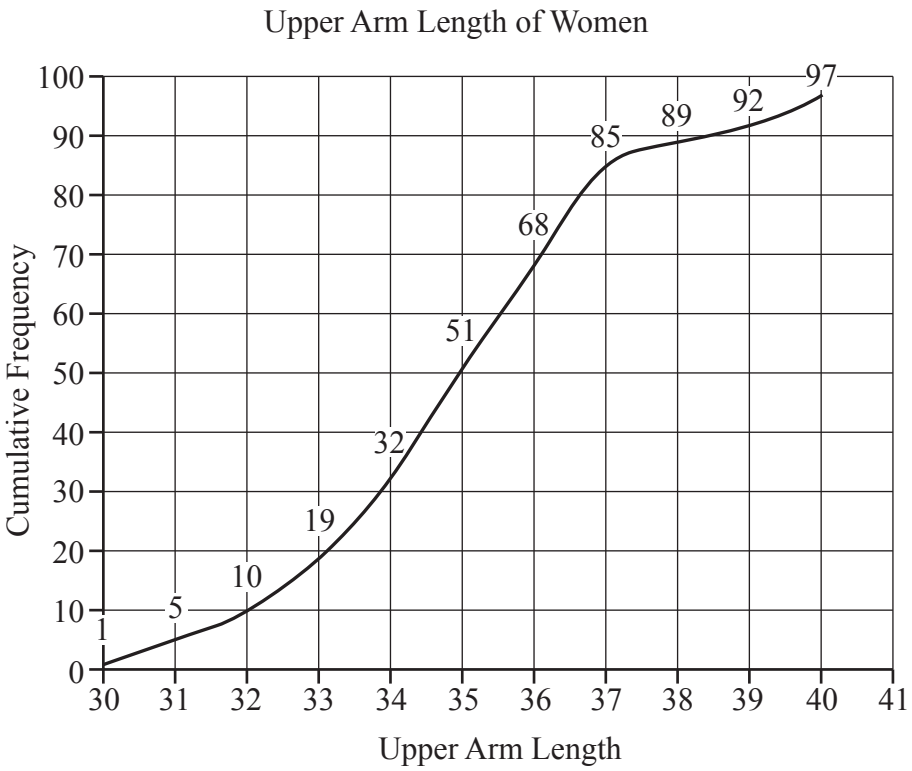


Diagram A

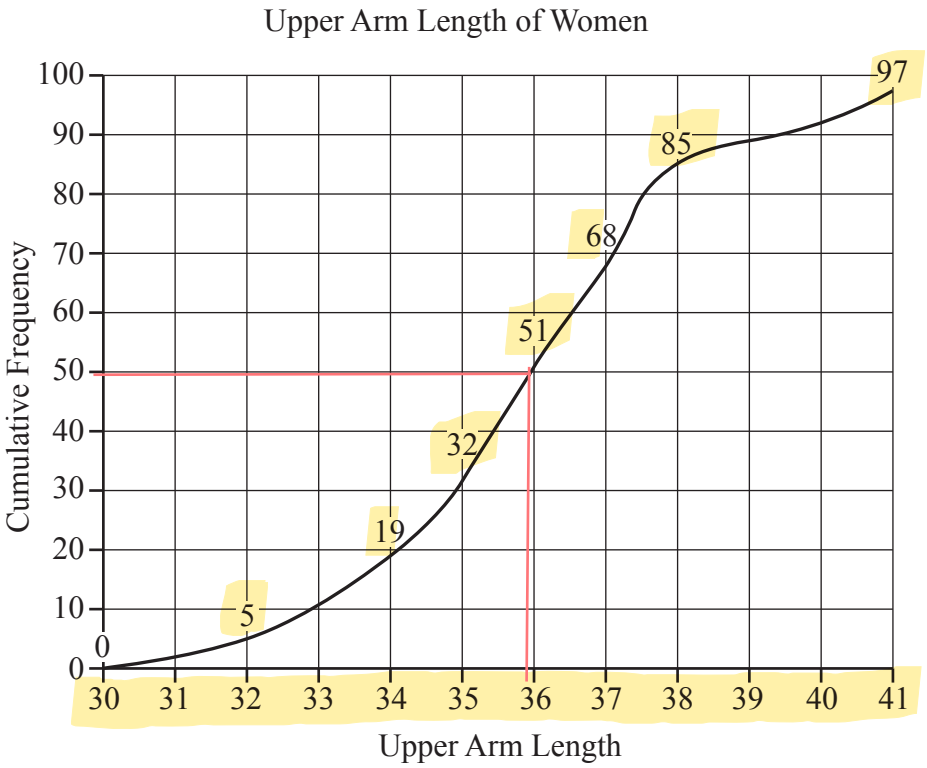


Diagram B

Fig. 2.2

3 A researcher is conducting an investigation into the number of portions of fruit adults consume each day. The researcher decides to ask 50 men and 50 women to complete a simple questionnaire.

(a) State the type of sampling procedure the researcher is using. **[1]**

(b) Write down one disadvantage of this sampling procedure. **[1]**

The researcher represents the data in Fig. 3.1.

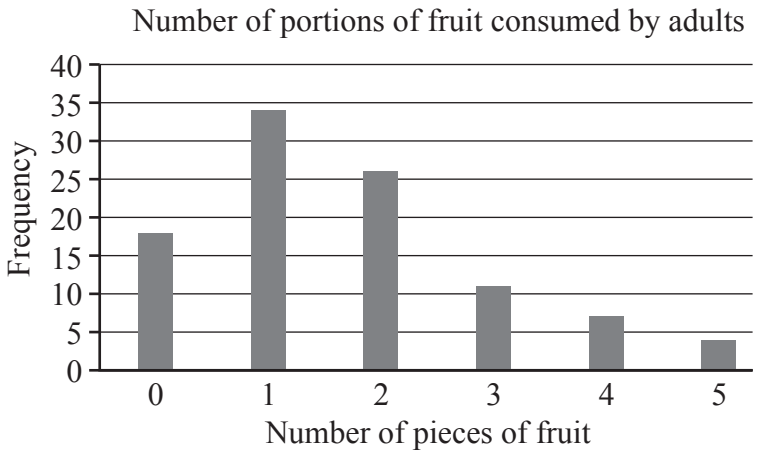


Fig. 3.1

(c) Describe the shape of the distribution. **[1]**

The data are summarised in the frequency table in Fig. 3.2.

Number of portions of fruit	0	1	2	3	4	5
Number of adults	18	34	26	11	7	4

Fig. 3.2

(d) For the data in Fig. 3.2, use your calculator to find

- the mean,
- the standard deviation.

Give your answers correct to 2 decimal places. **[2]**

A second researcher chooses a proportional stratified sample of 100 children from years 5 and 6 in a certain primary school. There are 220 children to choose from. In year 5 there are 125 children, of whom 81 are boys.

(e) How many year 5 girls should be included in the sample? **[1]**

The second researcher found that the mean number of portions of fruit consumed per day by the children in this sample was 1.61 and the standard deviation was 0.53.

(f) Comment on the amount of fruit consumed per day by the children compared to the amount of fruit consumed per day by the adults. **[2]**

a) Quota Sampling

b) → Non-random

c) → Positive skew because the tail extends to the right

d) Mean

x	Number of portions of fruit	0	1	2	3	4	5
f	Number of adults	18	34	26	11	7	4
	fx	0	34	52	33	28	20

$$\frac{\Sigma fx}{\Sigma f} = \frac{0+34+52+33+28+20}{18+34+26+11+7+4} = \frac{167}{100}$$

Mean = 1.67

S.D = $\sqrt{\text{variance}}$

$$\text{Variance} = \frac{\Sigma f(x^2)}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2$$

	x^2	0	1^2	2^2	3^2	4^2	5^2
x	Number of portions of fruit	0	1	2	3	4	5
f	Number of adults	18	34	26	11	7	4
	fx^2	0	34	104	99	112	100

$$\Sigma f(x^2) = 0+34+104+99+112+100$$

$$= \frac{449}{100} = 4.49$$

Variance $= 4.49 - (1.67)^2$

$= 1.7011$

S.D $= \sqrt{1.7011}$

$= 1.31$

e) $44 \times \frac{100}{220} = 20$ ← proportion, ↑ 44, 220 ← total

125 − 81 = 44 girls in year 5

f) Mean is very similar so the average no. of portions of fruit consumed per child is similar to that of an adult

SD is lower ∴ less variability (from mean) in the no. of portions consumed per day by children

4 In a certain country it is known that 11% of people are left-handed.

(a) Calculate the probability that, in a random sample of 98 people from this country, 5 or fewer are found to be left-handed, giving your answer correct to 3 significant figures. **[1]**

An anthropologist believes that the proportion of left-handed people is lower in a particular ethnic group.

The anthropologist collects a random sample of 98 people from this particular ethnic group in order to test the hypothesis that the proportion of left-handed people is less than 11%.

The anthropologist carries out the test at the 1% level.

(b) Determine the critical region for this test. **[3]**

a) $X \sim B(98, 0.11)$

where X is the no. of people that are left handed.

$P(X \leq 5)$

Using calculator = 0.0348 (3sf)

b) $P(X \leq 4) = 0.0133 > 0.01$

$P(X \leq 3) = 0.004 < 0.01$

0.01 = significance level

$\therefore X = 4$ is not in the CR $\therefore$ CR $\Rightarrow X \leq 3$

5 A company needs to appoint 3 new assistants. 8 candidates are invited for interview; each candidate has a different surname. The candidates are to be interviewed one after another. The personnel officer randomly selects the order in which the candidates are to be interviewed by drawing their names out of a hat. One of the candidates is called Mr Browne and another is called Mrs Green.

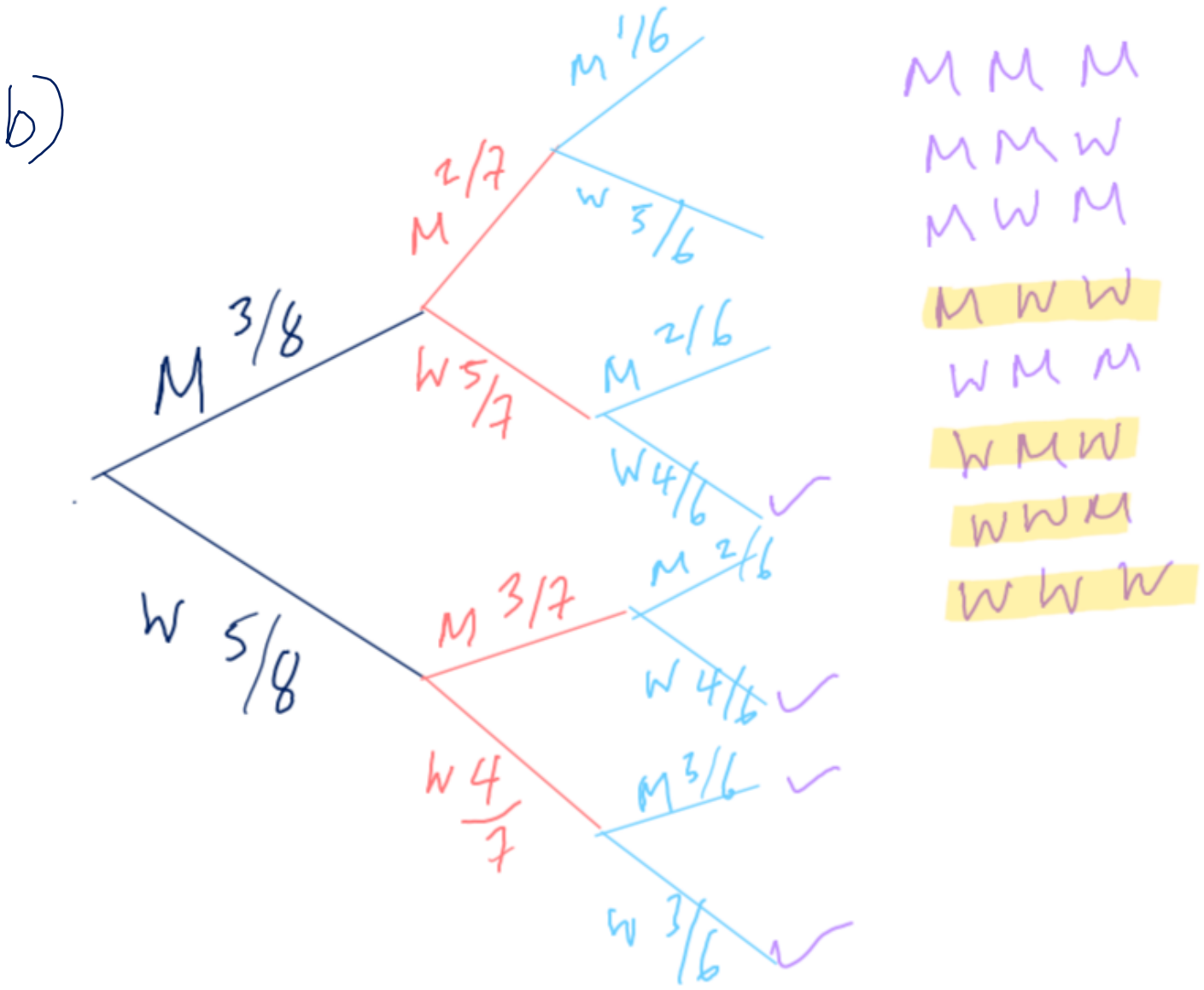
(a) Calculate the probability that Mr Browne is interviewed first and Mrs Green is interviewed last. **[2]**

5 of the 8 candidates invited for interview are women and the other 3 are men. The chief executive can't make up his mind who to appoint so he randomly selects 3 candidates by drawing their names out of a hat.

(b) Determine the probability that more women than men are selected. **[4]**

a) $\frac{(8-2)!}{8!} = \frac{6!}{8!} = \frac{1}{56}$

b)



M M M
M M W
M W M
M W W
W M M
W M W
W W M
W W W

$$\Rightarrow \left(\frac{3}{8} \times \frac{5}{7} \times \frac{4}{6}\right) + \left(\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6}\right) +$$

$$\left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}\right) + \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}\right)$$

$$= \frac{5}{7}$$

6 Use integration to show that the area bounded by the x-axis and the curve with equation $y = (x-1)^2(x-3)$ is $\frac{4}{3}$ square units. **[6]**

① Expand y for simpler integration

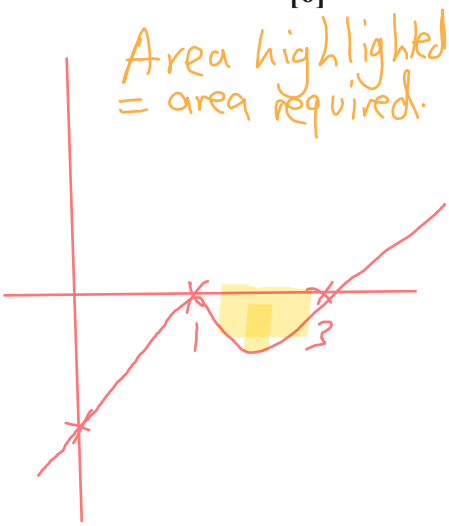
$y = (x-1)^2(x-3)$

$y = (x^2 - 2x + 1)(x-3)$

$\Rightarrow x(x^2-2x+1) - 3(x^2-2x+1)$

$\Rightarrow x^3 - 2x^2 + x - 3x^2 + 6x - 3$

$y = x^3 - 5x^2 + 7x - 3$



② From Factorised Form get x-intercepts and Y intercept

$y = (x-1)^2(x-3)$

∴ X int $y = 0$

$(x-1)^2 = 0$ $\quad (x-3) = 0$

Repeated Root $x = 1$ $\quad x = 3$

Repeated Root $(0,1)$ $\quad (0,3)$

Y int $x = 0$

$y = (0-1)^2(0-3)^2 = -3 \quad (0,-3)$

$\int x^n = \frac{x^{n+1}}{n+1}$

(3) Find Integral From $\int_1^3$

$$\int_1^3 \left(x^3 - 5x^2 + 7x - 3\right) dx$$

$$\left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x\right]_1^3$$

$$\left[\frac{3^4}{4} - \frac{5(3)^3}{3} + \frac{7(3)^2}{2} - 3(3)\right] - \left[\frac{1}{4} - \frac{5}{3} + \frac{7}{2} - 3\right]$$

$-\frac{9}{4} + \frac{11}{12} = -\frac{4}{3}$ as expected but area is scalar $\therefore +\frac{4}{3}$

* Area was expected to be negative from the sketch.

$\therefore$ Area $= \frac{4}{3}$ units^2 as required

7 **In this question you must show detailed reasoning.**

A circle has centre $(2, -1)$ and radius 5.

A straight line passes through the points $(1, 1)$ and $(9, 5)$.

Find the coordinates of the points of intersection of the line and the circle. **[8]**

① Find the equation of the circle

$(x-a)^2 + (y-b)^2 = r^2$

$(a, b) \rightarrow$ centre

$r \rightarrow$ radius

$(x-2)^2 + (y+1)^2 = 25$ - - - ①

② Find equation of straight line

$(x_1, y_1) \quad (x_2, y_2)$

$(1, 1) \quad (9, 5)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-1}{9-1} = \dfrac{4}{8} = \dfrac{1}{2}.$

$y - y_1 = m(x - x_1)$

$y - 1 = \dfrac{1}{2}(x-1)$

$y = \dfrac{1}{2}x - \dfrac{1}{2} + 1 \Rightarrow y = \dfrac{1}{2}x + \dfrac{1}{2}.$ - - - ②

(3) Substitute equation (2) in equation (1)

$$(x-2)^2 + (\tfrac{1}{2}x + \tfrac{1}{2} + 1)^2 = 25$$

$$(x-2)^2 + (\tfrac{1}{2})^2 (x+3)^2 = 25$$

$$(x-2)^2 + \frac{1}{4}(x+3)^2 = 25$$

Expand

$$4\times \quad x^2 - 4x + 4 + \frac{1}{4}(x^2 + 6x + 9) = 25 \qquad \times 4$$

$$4x^2 - 16x + 16 + x^2 + 6x + 9 = 100$$

$$\frac{5x^2 - 10x + 25}{5} = \frac{100}{5}$$

$$x^2 - 2x + 5 - 20 = 0$$

$$x^2 - 2x - 15 = 0$$

$$\frac{2 \pm \sqrt{(2)^2 - 4(-15)}}{2(1)}$$

$x = -3$

$y = \frac{1}{2}(-3) + \frac{1}{2}$

$y = -\frac{3}{2} + \frac{1}{2} = -1$

$x = 5$

$y = \frac{1}{2}(5) + \frac{1}{2}$

$y = \frac{5}{2} + \frac{1}{2} = 3$

$(-3, -1)$

$(5, 3)$

8 **In this question you must show detailed reasoning.**

Solve the equation $3\cos\theta + 8\tan\theta = 0$ for $0° < \theta < 360°$, giving your answers correct to the nearest degree. **[6]**

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\times\cos\theta \quad 3\cos\theta + \frac{8\sin\theta}{\cos\theta} = 0 \quad \times\cos\theta$

$3\cos^2\theta + 8\sin\theta = 0$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$3(1 - \sin^2\theta) + 8\sin\theta = 0$

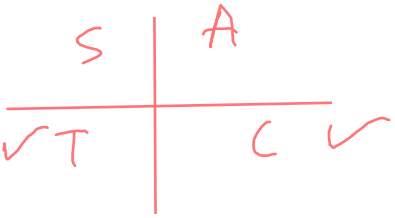
$3 - 3\sin^2\theta + 8\sin\theta = 0$

$3\sin^2\theta - 8\sin\theta - 3 = 0$

$\frac{8 \pm \sqrt{(8)^2 - 4(3x-3)}}{2\times 3}$

$\sin\theta = -\frac{1}{3}$ ✓ $\quad \sin\theta = 3$ not in range

$\sin\theta = -\frac{1}{3}$



$\sin\theta = \frac{1}{3}$

$\sin^{-1}(1/3) = 19.47°$

$\therefore$ From CAST Diagram

$\theta = 180 + 19.47 = 199°$

$\theta = 360 - 19.47 = 341°$

9 The equation of a curve is $y = 24\sqrt{x} - 8x^{\frac{3}{2}} + 16$.

(a) Find $\frac{dy}{dx}$. **[3]**

(b) Find the coordinates of the turning point. **[3]**

(c) Determine the nature of the turning point. **[2]**

9) $y = 24x^{1/2} - 8x^{3/2} + 16$

$y = x^n$

$\frac{dy}{dx} = n x^{n-1}$

$\frac{dy}{dx} = 24(1/2)x^{-1/2} - 8(3/2)x^{1/2}$

$\frac{dy}{dx} = 12x^{-1/2} - 12x^{1/2}$

b) @ TP $\frac{dy}{dx} = 0$

$\frac{12x^{-1/2} - 12x^{1/2}}{12} = \frac{0}{12}$

$x^{-1/2} - x^{1/2} = 0$

$$\left(x^{-1/2}\right)^2 = \left(x^{1/2}\right)^2$$

$$x^{-1} = x$$

$$\frac{1}{x} = x \times x$$

$$x^2 = 1$$

$$x = \pm 1$$

When $x = 1$

$$y = 24 - 8 + 16$$

$$= 32$$

$x \neq -1$ as $24(-1)^{1/2}$ has no real values

$\Rightarrow (1, 32)$

c) $\frac{d^2y}{dx} = 12(-1/2)x^{-3/2} - 12(1/2)x^{-1/2}$

when $x = 1$

$-6 - 6 = -12$

$-12 < 0$

$\therefore$ $(1, 32)$ is a local maximum

10 Fig. 10.1 shows a sample collected from the large data set.

BMI is defined as $\dfrac{\text{mass of person in kilograms}}{\text{square of person's height in metres}}$.

Sex	Age in years	Mass in kg	Height in cm	BMI
Male	38	77.6	164.8	28.57
Male	17	63.5	170.3	21.89
Male	18	68.0	172.3	22.91
Male	18	57.2	172.2	19.29
Male	19	77.6	191.2	21.23
Male	24	72.7	177.0	23.21
Male	25	92.5	177.9	29.23
Male	26	70.4	159.4	27.71
Male	31	77.5	174.0	25.60
Male	34	132.4	182.2	39.88
Male	38	115.0	186.4	33.10
Male	40	112.1	171.7	38.02

Fig. 10.1

(a) Calculate the mass in kg of a person with a BMI of 23.56 and a height of 181.6 cm, giving your answer correct to 1 decimal place. **[2]**

Fig. 10.2 shows a scatter diagram of BMI against age for the data in the table. A line of best fit has also been drawn.

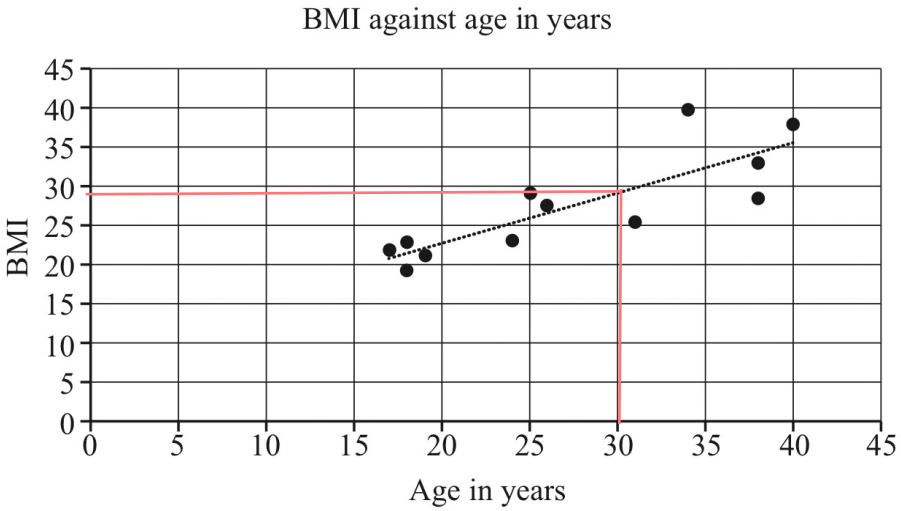


Fig. 10.2

(b) Describe the correlation between age and BMI. **[1]**

(c) Use the line of best fit to estimate the BMI of a 30-year-old man. **[1]**

(d) Explain why it would not be sensible to use the line of best fit to estimate the BMI of a 60-year-old man. **[1]**

(e) Use your knowledge of the large data set to suggest **two** reasons why the sample data in the table may not be representative of the population. **[2]**

(f) Once the data in the large data set had been cleaned there were 196 values available for selection. Describe how a sample of size 12 could be generated using systematic sampling so that each of the 196 values could be selected in the sample. **[2]**

a) $23.56 = \frac{\text{mass}}{1.816^2}$

$\text{mass} = 23.56 \times 1.816^2$

$= 77.7\text{ kg}$

b) Positive correlation

c) $28 \leqslant BMI < 30$

d) Because you'd have to extrapolate which isn't always correct

e) ✓ There are females also in the population
✓ wider age range in the population

f) Generate a random number, n, between 1 and 20, inclusive and select every m^{th} item in the data set for a valid solution

11 A car is travelling along a stretch of road at a steady speed of $11\,\text{ms}^{-1}$.

The driver accelerates, and t seconds after starting to accelerate the speed of the car, V, is modelled by the formula

$V = A + B(1 - \text{e}^{-0.17t})$.

When $t = 3$, $V = 13.8$.

(a) Find the values of A and B, giving your answers correct to 2 significant figures. **[3]**

When $t = 4$, $V = 14.5$ and when $t = 5$, $V = 14.9$.

(b) Determine whether the model is a good fit for these data. **[2]**

(c) Determine the acceleration of the car according to the model when $t = 5$, giving your answer correct to 3 decimal places. **[2]**

The car continues to accelerate until it reaches its maximum speed.

The speed limit on this road is $60\,\text{kmh}^{-1}$. All drivers who exceed this speed limit are recorded by a speed camera and automatically fined £100.

(d) Determine whether, according to the model, the driver of this car is fined £100. **[3]**

a) When $t = 0$ $V = 11$

$t = 3$ $V = 13.8$

$A + B(1 - e^{0})$ $\quad A = 11$ -- (1)

$A + B(1 - e^{-0.17(3)}) = 13.8$

$A + 0.40B = 13.8$ -- (2)

$11 + 0.40B = 13.8$

$0.40B = 2.8$

$B = 7$

b) when $t = 4$

$v = 11 + 7(1 - e^{-0.17(4)})$

$= 14.5$ (1dp) $\therefore$ good fit

when $t = 5$

$v = 11 + 7(1 - e^{-0.17(5)})$

$= 15.0$ (1dp) $\therefore$ good fit

c) $v = 11 + 7 - 7e^{-0.17t}$

$v = 18 - 7e^{-0.17t}$

$\frac{dv}{dt} = -7(-0.17)e^{-0.17t}$

$= 1.19e^{-0.17t}$

when $t = 5$

$= 1.19e^{-0.17(5)}$

$= 0.509 ms^{-2}$

e) $t \to \infty$

$v = 11 + 7(1 - e^{-\infty})$

$e^{-\infty} \to 0$

$v = 11 + 7$

$v = 18\ ms^{-1}$

$18\ m \to 1\ s$

$x \to 60^2\ s \quad (1\ hr)$

$x = 18 \times 60^2 = 64,800\ m$

$= 64.8\ km/h$

$64.8\ km/h > 60\ km/h$

$\therefore$ Motorist is fined