GCE

Mathematics B (MEI)

H630/01: Pure Mathematics and Mechanics

Advanced Subsidiary GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			**DR** Rearrange as $2x^2 - x + 7 = 0$ Discriminant is $(-1)^2 - 4\times 2\times 7$ $= -55 < 0$ so no real roots	**M1** **M1 (dep)** **A1** **[3]**	**1.1a** **1.1a** **2.2a**	Must be clearly argued from a correct discriminant which need not be evaluated if clearly negative	Expression for discriminant must be precise if not evaluated
			Alternative method Rearrange as $2x^2 - x + 7 = 0$ Attempt to complete the square $2(x-0.25)^2 + 6.875 = 0$ $x - 0.25 = \pm\sqrt{-3.4375}$ so no real roots	**M1** **M1 (dep)** **A1** **[3]**		Allow for $2(x-0.25)^2 + \ldots$ soi Must be clearly argued from correct working	
			Second alternative method Rearrange as $[y =]2x^2 - x + 7 = 0$ Differentiate $\frac{dy}{dx} = 4x - 1 = 0$ Stationary point at $(0.25, 6.875)$ Stationary point is minimum so $y \geq 6.875$ so is never zero	**M1** **M1 (dep)** **A1** **[3]**		Must equate to zero Must be clearly argued from correct working	

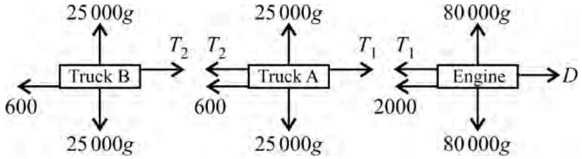
2			**DR** $4\sin x° = 3\cos x°$ $\tan x° = \frac{3}{4}$ $x° = 36.9°,\ 216.9°$	M1 M1 A1 [3]	1.1a 1.1a 1.1	Attempt to solve simultaneously Using trig identity to form equation in $\tan x$ Both answers required with no extras in the interval. ISW answers outside $0° \le x° \le 360°$	Answers must be given to 1 dp (in question)
			Alternative method $4\sin x° = 3\cos x°$ $\Rightarrow 16\sin^2 x° = 9\cos^2 x°$ $\sin^2 x° = \frac{9}{25}$ or $\cos^2 x° = \frac{16}{25}$ $x° = 36.9°,\ 143.1°, 216.9°, 323.9°$ Check solutions satisfy the original equation to give $x° = 36.9°,\ 216.9°$	M1 M1 A1 [3]	1.1a 1.1a 1.1	Attempt to solve simultaneously Using trig identity $\sin^2 x° + \cos^2 x° = 1$ to form equation in $\sin^2 x°$ or $\cos^2 x°$ Both answers required with no extras in the interval. ISW answers outside $0° \le x° \le 360°$	

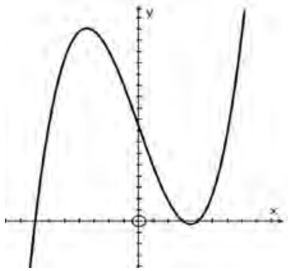
3			$\sqrt{8}=2\sqrt{2}$ $\dfrac{(3\sqrt{2}-k)(2\sqrt{2}-1)}{(2\sqrt{2}+1)(2\sqrt{2}-1)}$ $=\dfrac{12+k}{7}-\dfrac{3+2k}{7}\sqrt{2}$	B1 M1 A1 A1 [4]	3.1a 1.1a 1.1 1.1	soi Multiplying by conjugate. Allow for using $\sqrt{8}-1$ Correct denominator Fully correct in the form $a+b\sqrt{2}$ Condone $\dfrac{12+k-(3+2k)\sqrt{2}}{7}$	Do not allow final A1 for $\dfrac{12-3\sqrt{2}+k-2k\sqrt{2}}{7}$ oe
			$\sqrt{8}=2\sqrt{2}$ $\dfrac{3\sqrt{2}}{(\sqrt{8}+1)}-\dfrac{k}{(\sqrt{8}+1)}=\dfrac{12-3\sqrt{2}}{7}-\left(\dfrac{-1+2\sqrt{2}}{7}\right)k$ $=\dfrac{12+k}{7}-\dfrac{3+2k}{7}\sqrt{2}$	B1 M1 A1 A1 [4]		soi Splitting the fraction into two terms and simplifying each term BC Correct denominator in both terms and k included correctly Rearranging into the form $a+b\sqrt{2}$	Allow M1 if both fractions seen

4	(a)		$\cos \text{BAC} = \dfrac{5^2 + 9^2 - 10^2}{2 \times 5 \times 9}$ $= \dfrac{1}{15}$	M1 A1 [2]	1.1a 1.1	Oe Fraction must be seen in lowest terms isw 86.2º found	Do not allow for a different angle found
4	(b)		$\sin \text{BAC} = \sqrt{1 - \cos^2 \text{BAC}}$ $\left[= \dfrac{4\sqrt{14}}{15} \right]$ Area $= \frac{1}{2} \times 5 \times 9 \times \sin \text{BAC}$ $= 6\sqrt{14}$ cm^2	B1 M1 A1 [3]	3.1a 1.1a 1.1	FT their **(a)** Allow if value used for their angle Cao. Must be from exact working Condone missing units	Use of $\frac{1}{2} \times 5 \times 9 \times \sin 86°$ or similar, using their value for another angle found B0 M1 A0

5	(a)		$(\mathbf{W} =)\ -2.5g\mathbf{j}$ N $\quad (= -24.5\mathbf{j}$ N$)$	B1 [1]	1.2	Condone missing units. Must be negative. Allow any correct vector notation	
5	(b)		Newton's second law $-24.5\mathbf{j} + (3\mathbf{i} - 2\mathbf{j}) + (-\mathbf{i} + 18\mathbf{j}) = 2.5\mathbf{a}$ $\mathbf{a} = \dfrac{1}{2.5}(2\mathbf{i} - 8.5\mathbf{j}) = (0.8\mathbf{i} - 3.4\mathbf{j})$ m s^{-2}	M1 A1 [2]	1.1a 1.1	Attempt to use N2L with $m = 2.5$ Condone missing weight or an incorrect weight vector Cao – do not award if the magnitude of acceleration given as acceleration	Must be sum of forces = $2.5a$

PMT

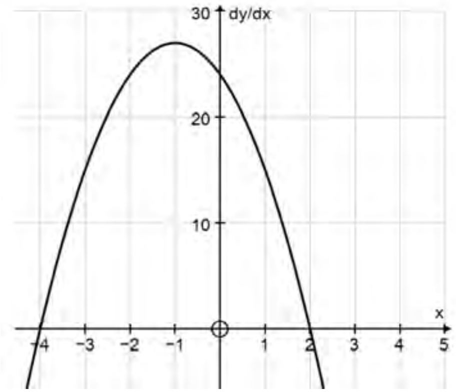
6	**(a)**			**B1** **B1** **B1** **[3]**	**1.1a** **1.1a** **1.1a**	Weights and normal reactions shown Tensions (must be two distinct) shown Driving force (D or 16 200) and three resistances correctly shown	Allow N_1, N_1 and N_2 instead of values to match weights for the normal reaction (condone N_1, N_2, N_3)
6	**(b)**		N2L for the whole train: $D-2000-600-600=130\,000\times 0.1$ $D=16\,200$	**M1** **A1** **[2]**	**1.1a** **1.1**	All forces present and no extras– allow slip in the mass as long as it is their attempt at total mass of the train. Signs must be consistent	
			Alternative solution N2L for each part of the train $D-2000-T_1=80\,000\times 0.1$ $T_1-T_2-600=25\,000\times 0.1$ $T_2-600=25\,000\times 0.1$ Add equations to eliminate tension $D=16\,200$	**M1** **A1** **[2]**		All forces present and no extras in each equation Signs must be consistent Award the M mark only when an attempt is made to solve the equations simultaneously	Other equations may be used instead Both trucks A and B $T_1-2\times 600=5000$ Engine plus A $D-T_2-2600=10500$
6	**(c)**		N2L for truck A: $T_1-T_2-600=25\,000\times 0.1$ Difference T_1-T_2 is 3100 N	**M1** **A1** **[2]**	**3.1b** **1.1**	All forces present and no extras Signs must be consistent from correct working	Values for tensions need not be found
			Alternative solution For the engine $D-2000-T_1=8000$ $T_1=6200$ N For truck B $T_2-600=2500$ $T_2=3100$ N Difference T_1-T_2 is 3100 N	**M1** **A1** **[2]**		All forces present and no extras in one equation leading to a value for either T_1 or T_2. Signs must be consistent FT their D	The equations may have been found in part (b) Also allow T_1 is twice T_2 oe from correct values

7	(a)		**DR** Nigel should substitute $x=-7$ not $x=7$; his calculation shows that $(x-7)$ is not a factor $(-7)^3-37\times(-7)+84=-343+259+84=0$, so $(x+7)$ is a factor	**B1** **B1** **[2]**	**2.3** **2.1**	Any suitable comment that references the factor theorem Clear argument needed	Allow for correct algebraic division if comment made about no remainder.
7	(b)		**DR** $x^3-37x+84=(x+7)(x^2-7x+12)$ $=(x+7)(x-3)(x-4)$ Crosses x-axis at (–7, 0), (3, 0), (4, 0) Crosses y-axis at (0, 84) 	**M1** **M1** **A1** **(dep)** **B1** **B1** **[5]**	**3.1a** **1.1a** **1.1** **1.1** **1.1**	Attempt to divide cubic by $(x+7)$ or find a quadratic factor by inspection Attempt to factorise their quadratic factor oe and find points on x-axis FT Accept values shown on sketch graph www Accept value shown on sketch graph www Axes labelled and correct general shape FT their values as long as right way up.	May be seen above (Allow if at least two correct terms) The A mark is dependent on the second M mark Exact position of the stationary points is not needed

7	(c)		**DR** Equation of the form $y=(x-1)^3-37(x-1)+84$ $=(x^3-3x^2+3x-1)-37(x-1)+84$ $y=x^3-3x^2-34x+120$	**M1** **M1** **A1** **A1** **[4]**	**3.1a** **1.1a** **1.1** **1.1**	Substituting $(x-1)$ for x twice Attempt to expand Correct expansion of their cubic term cao (including $y=$)	
			Alternative solution Equation of the form $y=(x-1+7)(x-1-3)(x-1-4)$ $=(x+6)(x^2-9x+20)$ or $(x-4)(x^2+x-30)$ or $(x-5)(x^2+2x-24)$ $y=x^3-3x^2-34x+120$	**M1** **M1** **A1** **A1** **[4]**		Substituting $(x-1)$ for x everywhere Attempt to multiply out their factors Correct quadratic factor cao (including $y=$)	Allow for $(x+6)(x-4)(x-5)$

8			**DR** $y = x^2 - 4x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = 2x - 4 \times \frac{1}{2} x^{-\frac{1}{2}}$	**M1**	**2.1**	Uses fractional power in an attempt to differentiate	
			$\frac{dy}{dx} = 0$ gives $x^{\frac{3}{2}} = 1 \Rightarrow x = 1$	**M1**	**3.1a**	Attempt to solve their $\frac{dy}{dx} = 0$	Allow SC1 for verifying that $x = 1$ gives $\frac{dy}{dx} = 0$
			There is only one solution to this so there is only one stationary point on the graph	**A1**	**2.2a**	Must obtain $x = 1$ from correct working and indicate that this is the only stationary point	
			When $x = 1$, $y = 1 - 4 = -3$	**B1**	**2.1**	From correct working seen (AG)	
			$\frac{d^2y}{dx^2} = 2 - 2\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$	**M1**	**1.1a**	Attempt to find second derivative	
			When $x = 1$, $\frac{d^2y}{dx^2} = 3 > 0$	**M1**	**1.1a**	Substituting into their expression	
			So the stationary point is a minimum point	**A1** **[7]**	**2.2a**	Conclusion from correct working (AG)	
			Alternative for final three marks Attempt to evaluate $\frac{dy}{dx}$ at a point $x \neq 1$	**M1**			As there is only one stationary point, allow for similarly evaluating *y* and comparing with -3
			Attempt to evaluate $\frac{dy}{dx}$ at a point the other side of $x = 1$	**M1**			
			Correct conclusion from correct values	**A1**			

9	(a)		**DR** Using given values in $v = 1.2t^2 - kt^3$: $25.6 = 1.2\times 8^2 - k\times 8^3$ $k = \dfrac{25.6-76.8}{-8^3} = 0.1$	**M1** **A1** **[2]**	**3.3** **2.1**	method must be clear **AG**	Allow M1 for substitution of $t = 8, k = 0.1$ to obtain $v = 25.6$ A1 for comment made that $k = 0.1$ is correct value.
9	(b)		**DR** Uses $v = u + at$ $u = 0, v = 25.6, t = 8$ $\Rightarrow 25.6 = 0 + 8a$ $a = 3.2 \text{ m s}^{-2}$	**M1** **A1** **[2]**	**3.3** **1.1**	Cao	
9	(c)		**DR** First model $s = \int_0^8 (1.2t^2 - 0.1t^3)\,\mathrm{d}t = \left[0.4t^3 - 0.025t^4\right]_0^8$ $(0.4\times 8^3 - 0.025\times 8^4) - (0.4\times 0 - 0.025\times 0)$ =102.4 m Second model: use *suvat* equation(s) with 3 from $u = 0, v = 25.6, t = 8, a =$ their value , giving e.g. $s = 0\times 8 + \frac{1}{2}\times 3.2\times 8^2$ or $s = \frac{1}{2}(0+25.6)8$ = 102.4 m, [which is the same value as for 1st model]	**M1** **M1** **A1** **M1** **A1** **[5]**	**3.4** **1.1** **1.1** **3.4** **1.1**	Attempt to integrate – expression for indefinite integral must be seen Both limits seen or establishes that c = 0 if indefinite integral used. Allow following first M mark Method must be clear Allow without comment	Allow SC1 for $\int_0^8 (1.2t^2 - 0.1t^3)\,\mathrm{d}t = 102.4$

10	**(a)**		**DR** $\frac{dy}{dx} = 24 - 6x - 3x^2$ When $x = 0$, $\frac{dy}{dx} = 24$ When $\frac{dy}{dx} = 0,\ -3(x^2 + 2x - 8) = 0 \Rightarrow (x+4)(x-2) = 0$ $x = -4,\ 2$ 	**M1** **A1** **M1** **A1** **B1** **[5]**	**1.1a** **1.1** **1.1a** **3.1a** **1.1**	Expression for derivative seen May be shown on graph or in the working Method for solving their quadratic equation (allow any algebraic method) Must be seen on graph Correct shape. Maximum point should be to the left of the *y*-axis but need not be exact.	
10	**(b)**		**DR** Decreasing function when $\frac{dy}{dx} < 0$ $\{x : x < -4\} \cup \{x : x > 2\}$	**M1** **A1** **[2]**	**1.1a** **2.5**	Attempt to give the values of *x* for which $\frac{dy}{dx} < 0$ from their graph FT their graph if quadratic Condone use of $\leq$ for M mark Allow for "$x < -4$ or $x > 2$" Must be correct use of language or set notation here.	Do not allow A1 for $x < -4$**,** $x > 2$ $x < -4$ **and** $x > 2$ $-4 > x > 2$

11	(a)		$m = \frac{k}{t}$, so $t = 5$, $m = 2.1 \Rightarrow k = 10.5$	M1	2.1	Using algebraic expression to represent proportionality and one pair of values in attempt to find k	oe with $t = 50, m = 0.21$
			When $t = 50, m = \frac{10.5}{50} = 0.21$, oe	M1	2.1	Uses the model to predict the value of m for the other value of t, or uses the other pair of values to check the value of k	
			EITHER The model fits the measurements because the prediction agrees with given value OR The model fits the measurements because the same value of k is obtained in each case	A1 [3]	2.2a	Makes suitable statement about consistency of results	e.g. $\therefore \quad m = \frac{10.5}{t}$
			Alternative argument When the value of t is multiplied by 10, the value of m is divided by 10 So [consistent with the model that] mass is inversely proportional to time.	M2 A1 [3]		Argument in words need not reference the constant of proportionality Must make a clear conclusion about inverse proportionality	
11	(b)	(i)	When t is small, $m = \frac{10.5}{t}$ is large, so the mass will not be modelled correctly.	B1 [1]	3.5a	Any suitable comment that identifies a problem at $t = 0$ or as $t \to 0$	
11	(b)	(ii)	Melts completely when $m = 0$, but $t = \frac{10.5}{0}$ is not defined so the model cannot be used	B1 [1]	3.5a	Any suitable comment explaining that the model will not give a time for $m = 0$	Allow for "the model does not give a time for which $m = 0$" oe

11	(c)		Substitute $t = 5, m = 2.1 \Rightarrow 2.1 = 5a + b$ Substitute $t = 50, m = 0.21 \Rightarrow 0.21 = 50a + b$ Solving simultaneously: $a = -0.042,\ b = 2.31$	M1 M1 A1 [3]	3.3 1.1a 1.1	Uses either data point to form equation Uses the other data point to find 2nd equation and solve simultaneously Both values required	Solution may be **BC**
			Alternative method gradient of line joining points (5,2.1) and 50,0.21) $\frac{2.1-0.21}{5-50} = -0.042$ So line is of the form $y - 2.1 = -0.042(x-5)$ $y = -0.042x + 2.31$	M1 M1 A1 [3]		Attempt to find gradient Uses either data point to complete the equation Award for the correct equation without reference to *a* and *b*	
11	(d)		*a* is the rate at which ice melts: 0.042 kg of ice is lost per hour *b* is the initial mass of the block: 2.31 kg	B1 B1 [2]	3.3 3.3	Must refer to the value found; FT their negative *a* FT their positive *b*	If no reference is made to the numerical values, allow **SC1** for '*a* is the rate at which ice melts and *b* is the initial mass of the block' oe
11	(e)		$m = 0 \Rightarrow 0 = -0.042t + 2.31 \Rightarrow t = \frac{2.31}{0.042} = 55$ so time for block to melt is 55 hours	B1 [1]	3.4	FT their *a* and *b* only if *t* is positive	

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