OCR
Oxford Cambridge and RSA

# Thursday 6 June 2019 – Afternoon

## A Level Further Mathematics B (MEI)

### Y422/01 Statistics Major

**Time allowed: 2 hours 15 minutes**

* 7 7 6 8 8 9 6 7 2 3 *

**You must have:**
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

**You may use:**
- a scientific or graphical calculator

Model Answers

## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

## INFORMATION

- The total mark for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

**Section A** (29 marks)

Answer **all** the questions.

**1** A fair six-sided dice is rolled three times.
The random variable $X$ represents the lowest of the three scores.
The probability distribution of $X$ is given by the formula

$$P(X = r) = k(127 - 39r + 3r^2) \text{ for } r = 1, 2, 3, 4, 5, 6.$$

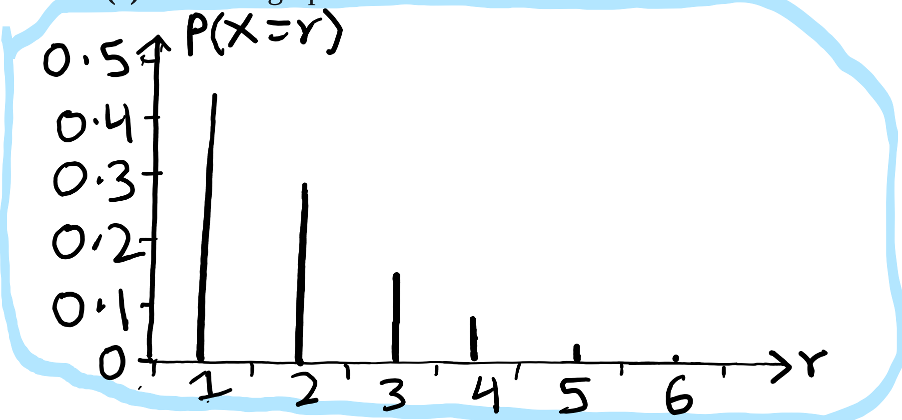
**(a)** Complete the copy of the table in the Printed Answer Booklet. **[1]**

| $r$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(X = r)$ | $91k$ | $61k$ | $37k$ | $19k$ | $7k$ | $k$ |

**(b)** Show that $k = \frac{1}{216}$ **[2]**

$\sum P(X=r)=1$ so $91k+61k+37k+19k+7k+k=1$.
$216k=1 \Rightarrow k=\frac{1}{216}$ as required

**(c)** Draw a graph to illustrate the distribution. **[2]**




**(d)** Comment briefly on the shape of the distribution. **[1]**

Strong positive skew

**(e)** **In this question you must show detailed reasoning.**

Find each of the following.

- $E(X)$
- $Var(X)$ **[5]**

$E(X)=\sum rP(X=r)=\left(1\times\frac{91}{216}\right)+\left(2\times\frac{61}{216}\right)+\left(3\times\frac{37}{216}\right)$
$+\left(4\times\frac{19}{126}\right)+\left(5\times\frac{7}{126}\right)+\left(6\times\frac{1}{126}\right)=\frac{49}{24}$

$E(X^2)=\left(1^2\times\frac{91}{216}\right)+\left(2^2\times\frac{61}{216}\right)+\left(3^2\times\frac{37}{216}\right)+\left(4^2\times\frac{19}{216}\right)$
$+\left(5^2\times\frac{7}{126}\right)+\left(6^2\times\frac{1}{126}\right)=\frac{1183}{216}$

$Var\ X=\frac{1183}{216}-\left(\frac{49}{24}\right)^2=1.308$ to 4 s f

**2** A special railway coach detects faults in the railway track before they become dangerous.

**(a)** Write down the conditions required for the numbers of faults in the track to be modelled by a Poisson distribution. **[2]**

Faults occur randomly, independently and at a uniform average rate.

You should now assume that these conditions do apply, and that the mean number of faults in a 5 km length of track is 1.6.

**(b)** Find the probability that there are at least 2 faults in a randomly chosen 5 km length of track. **[2]**

$X \sim Po(1.6)$. $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.5249$
$= 0.4751$

**(c)** Find the probability that there are at most 10 faults in a randomly chosen 25 km length of track. **[2]**

← ×5 to go from 5 → 25 km

$\lambda = 1.6 \times 5 = 8$ so $Y \sim Po(8)$. $P(Y \leq 10) = 0.8159$

**(d)** On a particular day the coach is used to check 10 randomly chosen 1 km lengths of track. Find the probability that exactly 1 fault, in total, is found. **[3]**

1 fault in 10 km so $Z \sim Po(3.2)$.
$P(Z = 1) = 0.1304$

**3** The weights of bananas sold by a supermarket are modelled by a Normal distribution with mean 205 g and standard deviation 11 g. $X \sim N(205, 11^2)$

**(a)** Find the probability that the total weight of 5 randomly selected bananas is at least 1 kg. **[2]**

$X_1 + X_2 + X_3 + X_4 + X_5 \sim N(5 \times 205, 5 \times 11^2) \Rightarrow \sim N(1025, 605)$
$P(\text{Total} \geq 1000) = 0.8453$

When a banana is peeled the change in its weight is modelled as being a reduction of 35%.

**(b)** Find the probability that the weight of a randomly selected peeled banana is at most 150 g. **[3]**

$\text{Peeled} \sim N(0.65 \times 205, 0.65^2 \times 11^2) \Rightarrow P \sim N(133.25, 7.15^2)$
$P(\text{Peeled} \leq 150) = 0.9904$

Andy makes smoothies. Each smoothie is made using 2 peeled bananas and 20 strawberries from the supermarket, all the items being randomly chosen. The weight of a strawberry is modelled by a Normal distribution with mean 22.5 g and standard deviation 2.7 g.

**(c)** Find the probability that the total weight of a smoothie is less than 700 g. **[4]**

Total $\mu = 2(133.25) + 20(22.5) = 716.5$ and
Total Variance $= 2(51.1225) + 20(2.7^2) = 248.045$
$T \sim N(716.5, 248.045)$. $P(T < 700) = 0.1474$

**Section B** (91 marks)

Answer **all** the questions.

**4** Shellfish in the sea near nuclear power stations are regularly monitored for levels of radioactivity. On a particular occasion, the levels of caesium-137 (a radioactive isotope) in a random sample of 8 cockles, measured in becquerels per kilogram, were as follows.

2.36 2.97 2.69 3.00 2.51 2.45 2.21 2.63

Software is used to produce a 95% confidence interval for the level of caesium-137 in the cockles. The output from the software is shown in Fig. 4. The value for 'SE' has been deliberately omitted.

T Estimate of a Mean ▾

Confidence Level 0.95

Sample

Mean 2.6025

s 0.2793

N 8

Result

T Estimate of a Mean

| | |
|---|---|
| Mean | 2.6025 |
| s | 0.2793 |
| SE | |
| N | 8 |
| df | 7 |
| Interval | 2.6025 ± 0.2335 |

**Fig. 4**

**(a)** State an assumption necessary for the use of the $t$ distribution in the construction of this confidence interval. **[1]**

The underlying distribution of caesium levels need to be Normal

**(b)** State the confidence interval which the software gives in the form $a < \mu < b$. **[1]**

$2.369 < \mu < 2.836$

**(c)** In the software output shown in Fig. 4, SE stands for standard error. Find the standard error in this case. **[2]**

$$SE = \frac{0.2793}{\sqrt{8}} = 0.09875$$

**(d)** Show how the value of 0.2335 in the confidence interval was calculated. **[2]**

T value = 2.365 so 2.365 × 0.09875 = 0.2335

**(e)** State how, using this sample, a wider confidence interval could be produced. **[1]**

Use a higher confidence level to increase the t value.

**5** In an investigation into the possible relationship between smoking and weight in adults in a particular country, a researcher selected a random sample of 500 adults.
The adults in the sample were classified according to smoking status (non-smoker, light smoker or heavy smoker, where light smoker indicates less than 10 cigarettes per day) and body weight (underweight, normal weight or overweight).

Fig. 5 is a screenshot showing part of the spreadsheet used to calculate the contributions for a chi-squared test. Some values in the spreadsheet have been deliberately omitted.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **Observed frequencies** | | | | | |
| 2 | | **Underweight** | **Normal** | **Overweight** | **Totals** | |
| 3 | **Non-smoker** | 8 | 52 | 178 | 238 | |
| 4 | **Light smoker** | 10 | 40 | 68 | 118 | |
| 5 | **Heavy smoker** | 5 | 47 | 92 | 144 | |
| 6 | **Totals** | 23 | 139 | 338 | 500 | |
| 7 | | | | | | |
| 8 | **Expected frequencies** | | | | | |
| 9 | **Non-smoker** | 10.9480 | 66.1640 | 160.8880 | | |
| 10 | **Light smoker** | 5.4280 | | 79.7680 | | |
| 11 | **Heavy smoker** | | 40.0320 | 97.3440 | | |
| 12 | | | | | | |
| 13 | **Contributions to the test statistic** | | | | | |
| 14 | **Non-smoker** | 0.7938 | | 1.8200 | | |
| 15 | **Light smoker** | 3.8510 | 1.5785 | 1.7361 | | |
| 16 | **Heavy smoker** | 0.3982 | 1.2129 | 0.2934 | | |
| 17 | | | | | | |

**Fig. 5**

**(a)** Showing your calculations, find the missing values in each of the following cells.

- B11 $= \dfrac{144 \times 23}{500} = 6.6240$

- C10 $= \dfrac{18 \times 139}{500} = 32.8040$

- C14 $= \dfrac{(52 - 66.164)^2}{66.164} = 3.0321$

**[4]**

**(b)** Complete the hypothesis test at the 1% level of significance. **[6]**

$H_0$: no association between smoking status and weight

$H_1$: some association between smoking status and weight

Test statistic = $\Sigma$ contributions = 14.716.
Critical value at $v = (3-1)(3-1) = 4$ and $p = 1\%$ is 13.28. $14.716 > 13.28$ so this result is significant. There is sufficient evidence to reject $H_0$, which would suggest there is some association between smoking status and weight.

**(c)** For each smoking status, give a brief interpretation of the largest of the three contributions to the test statistic. **[3]**

For non-smokers, the high contribution shows that fewer than expected are normal weight. For light smokers, the high contribution shows that more than expected are underweight. For heavy smokers, the contribution of 1.2129 shows that more than expected are normal weight.

**6** **(a)** A researcher is investigating the date of the 'start of spring' at different locations around the country.
A suitable date (measured in days from the start of the year) can be identified by checking, for example, when buds first appear for certain species of trees and plants, but this istime-consuming and expensive. Satellite data, measuring microwave emissions, canalternatively be used to estimate the date that land-based measurements would give.

The researcher chooses a random sample of 12 locations and obtains land-based measurements for the start of spring date at each location, together with relevant satellite measurements. The scatter diagram in Fig. 6.1 shows the results; the land-based measurements are denoted by $x$ days and the corresponding values derived from satellite measurements by $y$ days.

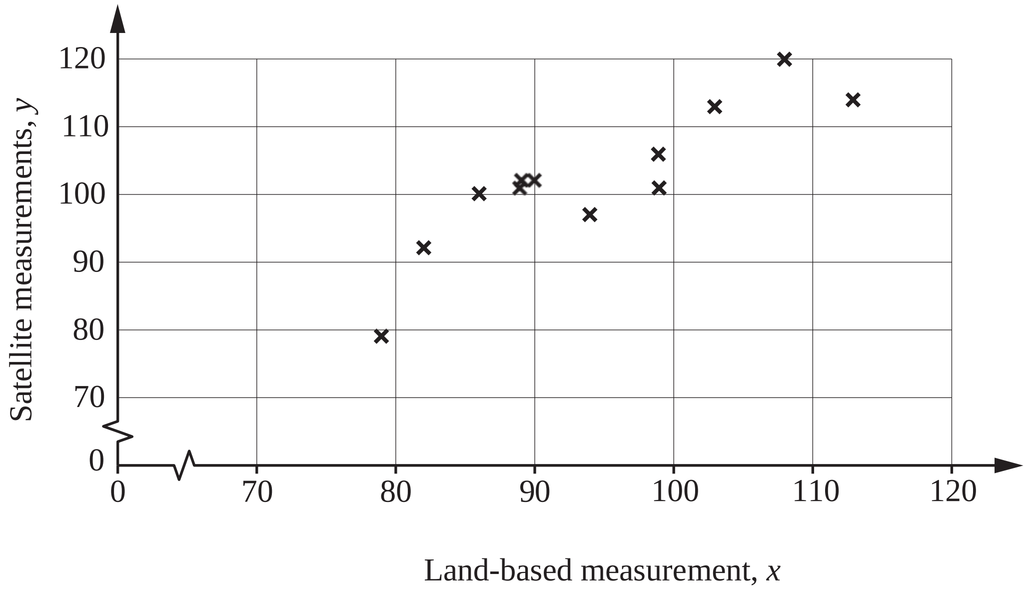



**Fig. 6.1**

Fig. 6.2 shows part of a spreadsheet used to analyse the data. Some rows of the spreadsheet have been deliberately omitted.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | ***x*** | ***y*** | ***x***$^2$ | ***y***$^2$ | ***xy*** |
| 2 | | 90 | 102 | 8100 | 10404 | 9180 |
| 3 | | | | | | |
| 10 | | | | | | |
| 11 | | | | | | |
| 12 | | 94 | 97 | 8836 | 9409 | 9118 |
| 13 | | 99 | 101 | 9801 | 10201 | 9999 |
| 14 | **Sum** | 1131 | 1227 | 107783 | 126725 | 116724 |
| 15 | | | | | | |

**Fig. 6.2**

**(i)** Calculate the equation of a regression line suitable for estimating the land-based date of the start of spring from satellite measurements. [5

$$S_{xy} = 116724 - \frac{1131 \times 1227}{12} = 1079.25$$

$$S_{yy} = 126725 - \frac{1227^2}{12} = 1264.25$$

x on y regression line is needed.

$$b = \frac{S_{xy}}{S_{yy}} = \frac{1079.25}{1264.25} = 0.8537 \text{ (4sf)}$$

$$x - \bar{x} = b(y - \bar{y}) \Rightarrow x - \frac{1131}{12} = 0.8537\left(y - \frac{1227}{12}\right)$$

$$\Rightarrow x = 0.8537y + 6.962$$

**(ii)** Using this equation, estimate the land-based date of the start of spring for the following dates from satellite measurements.

- 95 days $x = 0.8537(95) + 6.962 = 88$
- 60 days $x = 0.8537(60) + 6.962 = 58$ **[2]**

**(iii)** Comment on the reliability of each of your estimates. **[2]**

The first estimate is slightly reliable as points don't lie very close to the line. The second estimate is less reliable as it is extrapolated.

**(b)** The researcher is also investigating whether there is any correlation between the average temperature during a month in spring and the total rainfall during that month at a particular location. The average temperatures in degrees Celsius and total rainfall in mm for a random selection, over several years, of 10 spring months at this location are as follows.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Temperature | 4.2 | 7.1 | 5.6 | 3.5 | 8.6 | 6.5 | 2.7 | 5.9 | 6.7 | 4.1 |
| Rainfall | 18 | 26 | 42 | 76 | 15 | 43 | 84 | 53 | 66 | 36 |

The researcher plots the scatter diagram shown in Fig. 6.3 to check which type of test to carry out.

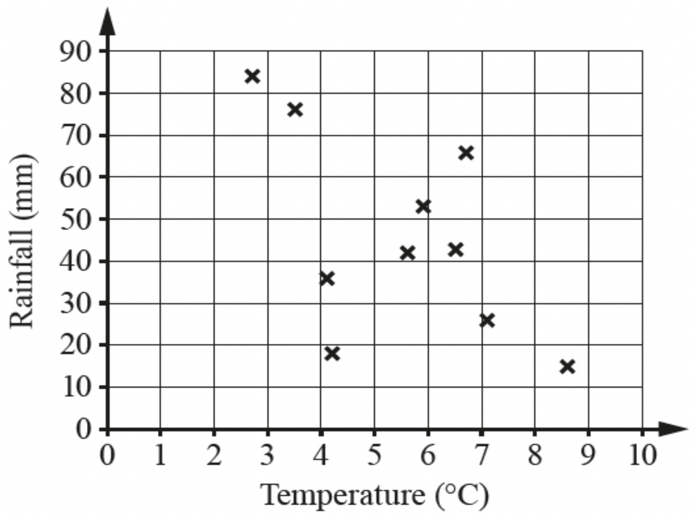



**Fig. 6.3**

**(i)** Explain why the researcher might come to the conclusion that a test based on Pearson's product moment correlation coefficient may be valid. **[2]**

The shape of the scatter diagram is roughly elliptical so the underlying distribution is likely to be Bivariate Normal.

**(ii)** Find the value of Pearson's product moment correlation coefficient. **[2]**

PMCC = −0·5638 using calculator

(can just use calc as only 2 marks)

**(iii)** Carry out a test at the 5% significance level to investigate whether there is any correlation between temperature and rainfall. **[5]**

$H_0: \rho = 0$ and $H_1: \rho \neq 0$ where $\rho$ is the population correlation coefficient between tempurature and rainfall.
Test statistic = −0·5638. Critical value for n=10 and 2 tailed 5% is 0·6319. $|-0{\cdot}5638| < 0{\cdot}6319$ so this result is not significant so insufficient evidence to reject $H_0$, which would suggest there is not a correlation between temperature and rainfall.

**7** A swimming coach believes that times recorded by people using stopwatches are on average 0.2 seconds faster than those recorded by an electronic timing system.
In order to test this, the coach takes a random sample of 40 competitors' times recorded by both methods, and finds the differences between the times recorded by the two methods. The mean difference in the times (electronic time minus stopwatch time) is 0.1442 s and the standard deviation of the differences is 0.2580 s.

**(a)** Find a 95% confidence interval for the mean difference between electronic and stopwatch times. **[4]**

$$0{\cdot}1442 \pm 1{\cdot}96\left(\frac{0{\cdot}2580}{\sqrt{40}}\right)$$ ← Use normal as large n so can use CLT

$\Rightarrow$ 0·0642 to 0·2242

**(b)** Explain whether there is evidence to suggest that the coach's belief is correct. **[2]**

As the confidence interval contains 0·2, the coach's belief may be correct

**(c)** Explain how you can calculate the confidence interval in part **(a)** even though you do not know the distribution of the parent population of differences. **[2]**

As the sample is large ($n=40$), the central limit theorem can be used so the distribution of the sample mean is approximately Normal.

**(d)** If the coach wanted to produce a 95% confidence interval of width no more than 0.12 s, what is the minimum sample size that would be needed, assuming that the standard deviation remains the same? **[3]**

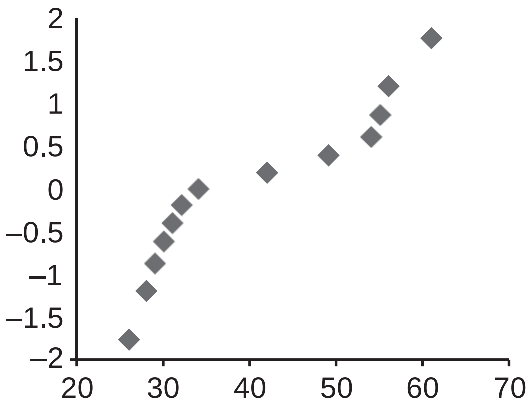
Width of 0.12 so $1.96 \times \frac{0.2580}{\sqrt{n}} \leq 0.06$

so $\sqrt{n} \geq 8.428$ and $n > 71.03$ so minimum sample size $= 72$.

**8** A student doing a school project wants to test a claim which she read in a newspaper that drinking a cup of tea will improve a person's arithmetic skills.
She chooses 13 students from her school and gets each of them to drink a cup of tea. She then gives each of them an arithmetic test. She knows that the average score for this test in students of the same age group as those she has chosen is 33.5.
The scores of the students she tests, arranged in ascending order, are as follows.

26 28 29 30 31 32 34 42 49 54 55 56 61

The student decides to use software to draw a Normal probability plot for these data, and to carry out a Normality test as shown in Fig. 8.




**Kolmogorov-Smirnov Test for Normality**

*p*-value = 0.048

**Fig. 8**

**(a)** The student uses the output from the software to help in deciding on a suitable hypothesis test to use for investigating the claim about drinking tea.
Explain what the student should conclude. **[3]**

A Wilcoxon test should be carried out as a t test or Normal hypothesis test require a Normally distributed population. As the Normal probability plot is not roughly straight and the p value is low, this suggests the data isn't from a Normal distribution.

**(b)** The student's teacher agrees with the student's choice of hypothesis test, but says that even this test may not be valid as there may be some unsatisfactory features in the student's project. Give three features that the teacher might identify as unsatisfactory. **[3]**

- We don't know if the sample is random
- The test scores may not be independent
- The sample size is too small

**(c)** Assuming that the student's procedures can be justified, carry out an appropriate test at the 5% significance level to investigate the claim about drinking tea. **[7]**

$H_0$: population median $= 33.5$

$H_1$: population median $> 33.5$

| Result | Result − 33.5 | $W_+$ | $W_-$ |
|---|---|---|---|
| 26 | −7.5 | | 7 |
| 28 | −5.5 | | 6 |
| 29 | −4.5 | | 5 |
| 30 | −3.5 | | 4 |
| 31 | −2.5 | | 3 |
| 32 | −1.5 | | 2 |
| 34 | 0.5 | 1 | |
| 42 | 8.5 | 8 | |
| 49 | 15.5 | 9 | |
| 54 | 20.5 | 10 | |
| 55 | 21.5 | 11 | |
| 56 | 22.5 | 12 | |
| 61 | 27.5 | 13 | |

$\sum W_+ = 64$ and $\sum W_- = 27$ so test statistic $= 27$. Critical value of $n=13$ and $p=5\%$ is 21. As $27 > 21$, this result is not significant so insufficient evidence to reject $H_0$, which would suggest the claim about drinking tea is false.

**9** Every weekday Jonathan takes an underground train to work. On any weekday the time in minutes that he has to wait at the station for a train is modelled by the continuous uniform distribution over [0, 5]. $\Rightarrow f(x) = \frac{1}{b-a} = \frac{1}{5-0} = \frac{1}{5}$

**(a)** Find the probability that Jonathan has to wait at least 3 minutes for a train. **[1]**

$P(X \geq 3) = \int_3^5 \frac{1}{5}\,dx = \left[\frac{x}{5}\right]_3^5 = \frac{5}{5} - \frac{3}{5} = \frac{2}{5}$ $0.4$

The total time that Jonathan has to wait on two days is modelled by the continuous random variable $X$ with probability density function given by

$$f(x) = \begin{cases} \frac{1}{25}x & 0 \leqslant x \leqslant 5, \\ \frac{1}{25}(10-x) & 5 < x \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

**(b)** Find the probability that Jonathan has to wait a total of at most 6 minutes on two days. **[3]**

$P(X \leq 6) = \int_0^5 \frac{x}{25}\,dx + \int_5^6 \frac{10}{25} - \frac{x}{25}\,dx$

$= 0.5 + \left[\frac{2x}{5} - \frac{x^2}{50}\right]_5^6 = 0.5 + 0.18 = 0.68$

Jonathan's friend suggests that the total waiting time for 5 days, $T$ minutes, will almost certainly be less than 18 minutes. In order to investigate this suggestion, Jonathan constructs the simulation shown in Fig. 9. All of the numbers in the simulation have been rounded to 2 decimal places.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | Mon | Tue | Wed | Thu | Fri | Total T |
| 2 | 1.78 | 4.36 | 2.74 | 3.88 | 4.64 | 17.41 |
| 3 | 0.95 | 1.30 | 4.83 | 4.29 | 1.81 | 13.18 |
| 4 | 4.27 | 4.90 | 4.57 | 1.41 | 3.66 | 18.81 |
| 5 | 0.80 | 0.06 | 3.20 | 1.76 | 0.35 | 6.17 |
| 6 | 0.03 | 4.82 | 1.26 | 3.53 | 0.13 | 9.77 |
| 7 | 3.88 | 4.73 | 1.19 | 3.75 | 1.29 | 14.84 |
| 8 | 4.11 | 3.54 | 4.33 | 0.77 | 4.50 | 17.25 |
| 9 | 3.54 | 0.11 | 3.85 | 2.86 | 1.58 | 11.94 |
| 10 | 1.87 | 1.82 | 3.00 | 3.53 | 1.83 | 12.05 |
| 11 | 4.00 | 2.98 | 4.59 | 1.73 | 1.76 | 15.06 |
| 12 | 1.91 | 3.85 | 2.08 | 1.72 | 2.82 | 12.38 |
| 13 | 0.10 | 4.86 | 2.51 | 0.52 | 2.17 | 10.15 |
| 14 | 1.24 | 4.26 | 0.95 | 1.33 | 1.78 | 9.57 |
| 15 | 2.99 | 0.69 | 3.85 | 3.41 | 2.42 | 13.36 |
| 16 | 4.67 | 1.76 | 2.13 | 3.48 | 3.10 | 15.14 |
| 17 | 1.94 | 1.07 | 0.91 | 0.63 | 3.34 | 7.89 |
| 18 | 0.11 | 2.29 | 0.71 | 4.21 | 0.86 | 8.18 |
| 19 | 0.43 | 4.58 | 4.89 | 1.86 | 2.84 | 14.60 |
| 20 | 4.23 | 0.88 | 2.71 | 4.88 | 4.20 | 16.91 |
| 21 | 3.72 | 4.58 | 3.11 | 4.89 | 3.18 | 19.49 |
| 22 | | | | | | |

**Fig. 9**

**(c)** Use the simulation to estimate $P(T > 18)$ . **[1]**

Estimate $= \frac{2}{20} = 0.1$

**(d)** Explain how Jonathan could obtain a better estimate. **[1]**

Use more rows in the simulation

Jonathan thinks that he can use the Central Limit Theorem to provide a very good approximation to the distribution of $T$.

**(e)** Find each of the following.

- $E(T)$
- $Var(T)$

**[3]**

For one day, $X \sim$ Continuous Uniform $[0, 5]$ so $E(X) = \frac{5}{2}$

and $Var X = \frac{(b-a)^2}{12} = \frac{25}{12}$. $T = X_1 + X_2 + X_3 + X_4 + X_5$

so $E(T) = 5 \times \frac{5}{2} = 12.5$ and $Var T = 5 \times \frac{25}{12} = \frac{125}{12}$

**(f)** Use the Central Limit Theorem to estimate $P(T > 18)$ . **[2]**

$T \sim N(12.5, \frac{125}{12})$. $P(T > 18) = 0.0442$

**(g)** Comment briefly on the use of the Central Limit Theorem in this case. **[1]**

As sample size of 5 is small, CLT may not be an accurate estimate

Jonathan travels to work on 200 days in a year.

**(h)** Find the probability that the total waiting time for Jonathan in a year is more than 510 minutes. **[3]**

Total $\sim N(200 \times 2.5, 200 \times \frac{25}{12})$ so

Total $\sim N(500, \frac{5000}{12})$. $P(\text{Total} > 510) = 0.3121$

**10** The probability density function of the continuous random variable $X$ is given by

$$f(x) = \begin{cases} kx^m & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where $a$, $k$ and $m$ are positive constants.

**(a)** Show that $k = \frac{m+1}{a^{m+1}}$. **[3]**

**(b)** Find the cumulative distribution function of $X$ in terms of $x$, $a$ and $m$. **[4]**

**(c)** Given that $P\left(\frac{1}{4}a < X < \frac{1}{2}a\right) = \frac{1}{10}$,

**(i)** show that $2p^2 - 10p + 5 = 0$, where $p = 2^m$, **[4]**

**(ii)** find the value of $m$. **[3]**

a) $\int_0^a kx^m\,dx = 1$ so $k\int_0^a x^m\,dx = 1 \Rightarrow k\left[\frac{x^{m+1}}{m+1}\right]_0^a = 1$

so $k\left(\frac{a^{m+1}}{m+1}\right) = 1 \Rightarrow k = \frac{m+1}{a^{m+1}}$ as required

b) $\int_0^x \frac{m+1}{a^{m+1}} u^m\,du = \left[\frac{m+1}{a^{m+1}} \frac{u^{m+1}}{m+1}\right]_0^x = \frac{x^{m+1}}{a^{m+1}}$

$$\boxed{F(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{x^{m+1}}{a^{m+1}} & \text{for } 0 \leq x \leq a \\ 1 & \text{for } x > a \end{cases}}$$

c) i) $P\left(\frac{1}{4}a < X < \frac{1}{2}a\right) = \frac{\left(\frac{1}{2}a\right)^{m+1}}{a^{m+1}} - \frac{\left(\frac{1}{4}a\right)^{m+1}}{a^{m+1}}$

$= \frac{1}{2^{m+1}} - \frac{1}{4^{m+1}} = \frac{1}{2(2^m)} - \frac{1}{4(2^m)^2}$

$\Rightarrow \frac{1}{2p} - \frac{1}{4p^2} = \frac{1}{10}$ so $\frac{2p-1}{4p^2} = \frac{1}{10}$

$2p - 1 = \frac{4}{10}p^2$ so $10p - 5 = 2p^2$

$\Rightarrow 2p^2 - 10p + 5 = 0$

ii) $2p^2 - 10p + 5 = 0 \Rightarrow p = 0.5635$ or $4.4365$

so $m = \log_2 0.5635$ or $\log_2 4.4365$

$= 2.149$ or $-0.8275$ (reject)

Therefore $m = 2.149$

**END OF QUESTION PAPER**