Oxford Cambridge and RSA

Friday 17 June 2022 – Afternoon

AS Level Further Mathematics B (MEI)

Y416/01 Statistics b

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Each working day, Beth takes a bus to her place of work. She believes that the mean time that her journey takes is 30 minutes. In order to check this, Beth selects a random sample of 8 journeys. The times in minutes for these 8 journeys are as follows.

31.9 28.5 35.9 31.0 30.2 34.9 28.9 31.3

(a) What assumption does Beth need to make in order to construct a confidence interval for the mean journey time based on the *t* distribution? **[1]**

(b) **In this question you must show detailed reasoning.**

Given that the assumption in part **(a)** is valid, determine a 95% confidence interval for the mean journey time. **[7]**

(c) Explain whether the confidence interval suggests that Beth may be correct in the belief that her mean journey time is 30 minutes. **[2]**

2 The continuous random variable X has cumulative distribution function given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < a, \\ \dfrac{x-a}{b-a} & a \leqslant x \leqslant b, \\ 1 & x > b, \end{cases}$$

where a and b are constants with $0 < a < b$.

(a) Find $\mathrm{P}\left(X < \frac{1}{2}(a+b)\right)$. **[2]**

(b) Sketch the graph of the probability density function of X. **[3]**

(c) Find the variance of X when $a = 2$ and $b = 8$. **[1]**

3 A local council collects domestic kitchen waste for composting. Householders place their kitchen waste in a 'compost bin' and this is emptied weekly by the council.

The average weight of kitchen waste collected per household each week is known to be 3.4 kg. The council runs a campaign to try to increase the amount of kitchen waste per household which is put in the compost bin. After the campaign, a random sample of 40 households is selected and the weights in kg of kitchen waste in their compost bins are measured.

A hypothesis test is carried out in order to investigate whether the campaign has been successful, using software to analyse the sample. The output from the software is shown below.

Z Test of a Mean ▼

Null Hypothesis μ = 3.4

Alternative Hypothesis ○ < ◉ > ○ ≠

Sample

Mean	3.565
s	1.05
N	40

Result

Z Test of a Mean

Mean	3.565
s	1.05
SE	0.1660
N	40
Z	0.994
p	0.160

(a) Explain why the test is based on the Normal distribution even though the distribution of the population of amounts of kitchen waste per household is not known. **[2]**

(b) Using the output from the software, complete the test at the 5% significance level. **[2]**

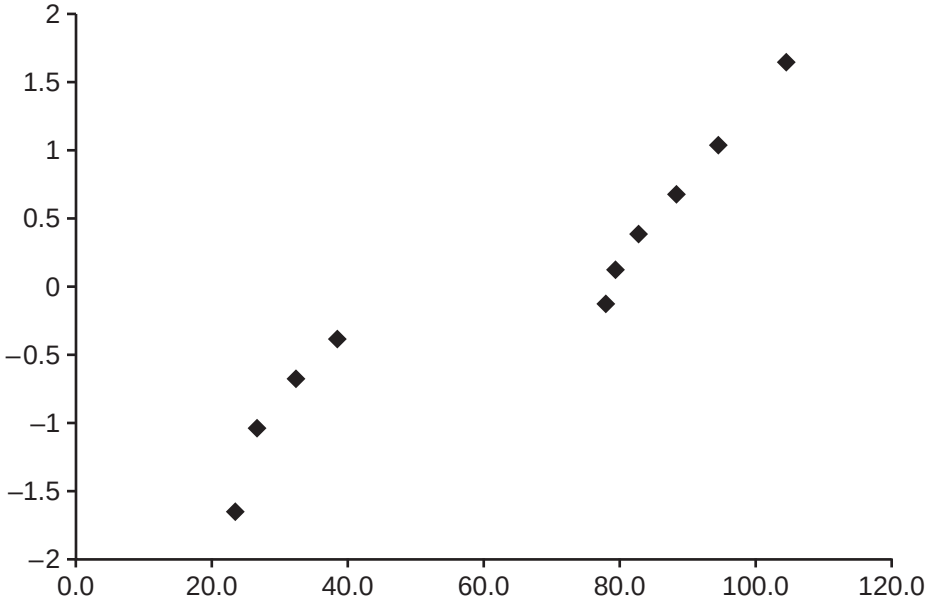
(c) Show how the value of Z in the software output was calculated. **[2]**

(d) Calculate the least value of the sample mean which would have resulted in the conclusion of the test in part **(b)** being different. You should assume that the standard error is unchanged. **[2]**

4 A wood contains a large number of mature beech trees. The diameters in centimetres of a random sample of 10 of these trees are as follows.

82.6 79.2 77.8 38.4 88.1 32.2 26.5 23.4 94.3 104.2

A tree surgeon wants to know if the average diameter of mature beech trees in this wood is 50 cm. The tree surgeon produces a Normal probability plot for these data.



(a) Explain why the tree surgeon should not carry out a test based on the *t* distribution. **[2]**

(b) Carry out a suitable test at the 5% significance level to investigate whether the average diameter of mature beech trees in this wood is 50 cm. **[8]**

5 Layla works at an internet café. Each terminal at the café has its own keyboard, and keyboards need to be replaced whenever faults develop.

Layla knows that the number of weeks for which a keyboard lasts before it needs to be replaced can be modelled by the random variable X, which has an exponential distribution with mean 20 and variance 400. She wants to investigate how likely it is that the keyboard at a terminal will need to be replaced at least 3 times within a year (taken as being a period of 52 weeks).

Layla designs the simulation shown in the spreadsheet below. Each of the 20 rows below the heading row consists of 3 values of X together with their sum T. All of the values in the spreadsheet have been rounded to 1 decimal place.

	A	B	C	D
1	X_1	X_2	X_3	T
2	10.9	21.5	5.3	37.7
3	23.9	52.4	85.3	161.6
4	5.2	10.4	24.0	39.6
5	2.9	14.4	0.8	18.1
6	9.0	43.3	49.7	102.0
7	0.4	16.2	12.4	29.0
8	44.1	39.5	22.1	105.7
9	9.2	43.6	13.9	66.7
10	40.4	10.9	6.1	57.4
11	3.2	54.8	15.7	73.7
12	5.3	6.1	1.6	13.0
13	20.5	28.9	22.9	72.3
14	37.3	2.1	28.6	68.0
15	7.1	13.6	50.1	70.8
16	18.6	2.0	9.3	29.9
17	9.0	1.2	49.9	60.1
18	1.9	9.5	69.8	81.2
19	9.0	2.1	10.4	21.5
20	28.7	1.4	93.8	123.9
21	1.8	2.9	34.8	39.5

(a) Explain why T represents the number of weeks after which the third keyboard at a terminal will need to be replaced. **[1]**

(b) Use the information in the spreadsheet to write down an estimate of $\mathrm{P}(T > 52)$. **[1]**

(c) Explain how you could obtain a more reliable estimate of $\mathrm{P}(T > 52)$. **[1]**

(d) The internet café has 50 terminals. You are given that faults in keyboards occur independently of each other.

Determine an estimate of the probability that the mean number of weeks before which the third keyboard at a terminal needs to be replaced is more than 52. **[5]**

6 The length L of a particular type of fence panel is Normally distributed with mean 179.2 cm and standard deviation 0.8 cm. You should assume that the lengths of individual fence panels are independent of each other.

(a) Find the probability that the length of a randomly chosen fence panel is at least 180 cm. **[1]**

(b) Find the probability that the total length of 5 randomly chosen fence panels is less than 895 cm. **[3]**

The width W of a fence post is Normally distributed with mean 9.8 cm and standard deviation 0.3 cm. A straight fence is constructed using 6 posts and 5 panels with no gaps between them. **Fig. 6** shows a view from above of the first two posts, the first panel and the start of the second panel. You should assume that the lengths of fence panels and widths of fence posts are independent.

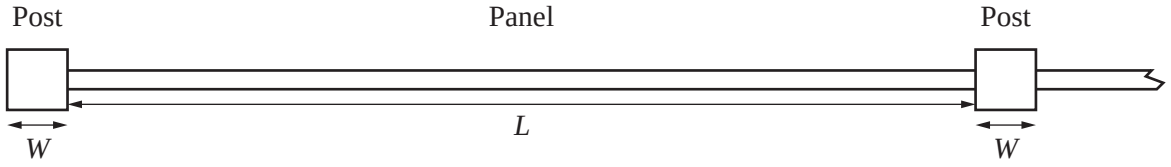


Fig. 6

(c) Determine the probability that the total length of the fence, including the posts, is less than 9.5 m. **[4]**

(d) State another assumption that is necessary for the calculation of the probability in part **(c)** to be valid. **[1]**

7 Many cars have pollen filters to try to remove as much pollen as possible from the passenger compartment. In a test car, the amount of pollen is regularly monitored. The amount of pollen is measured using a scale from 0 to 1, and is modelled by the continuous random variable X with probability density function given by

$$\mathrm{f}(x) = \begin{cases} k(5x^4 - 16x^2 + 11x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $k = \frac{6}{7}$. **[2]**

(b) Determine $\mathrm{P}\big(X < \mathrm{E}(X)\big)$. **[4]**

(c) Verify that the median amount of pollen according to the model lies between 0.417 and 0.418. **[3]**

END OF QUESTION PAPER

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