GCE

Further Mathematics B (MEI)

Unit **Y416/01**: Statistics b

Advanced Subsidiary GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for AS Level Further Mathematics B (MEI)

a   Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

c   The following types of marks are available.

      M
      A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

      A
      Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

      B
      Mark for a correct result or statement independent of Method marks.

      E
      A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
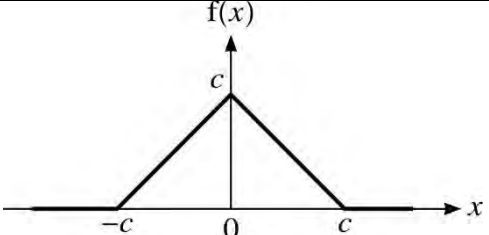
      Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   As noted in the specification, answers should be given exactly when possible. Otherwise answers should be given to a degree of accuracy appropriate to the context. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. Over-specification is penalised no more than once on each question and no more than three times in total. If answers are grossly over-specified, deduct the final answer mark. Probabilities should also be rounded to a sensible degree of accuracy. In general final non probability answers should not be given to more than 4 significant figures. Allow probabilities given to 5 sig fig. **You must highlight any over-specified answers.**

g   Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i   If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j   If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks		Guidance
1	(i)		The underlying parent distribution must be Normal.	E1 [1]	Condone just 'Normal'	
	(ii)		$82.4838 < \mu < 152.4042$	B1 [1]	Allow to 1 dp $82.5 < \mu < 152.4$	
	(iii)		It means 'Degrees of freedom' Its value is equal to 1 less than the number of data values	E1 E1 [2]	ISW anything after this Only allow $n-1$ if n defined Allow 'subtracting the number of dependent variables from n' (since they may know about a 2-sample t-test	Must be general, not just $9-1=8$
	(iv)		Use a larger sample size Use a lower confidence level	E1 E1 [2]	Do not allow 'be less confident' Allow 'Use a smaller confidence level'	
2	(i)		$Y \sim N(0.7 \times 345, (0.7 \times 15)^2)$ or $N(241.5, 110.25)$ 0.791 (0.79089...)	B1 M1 A1 [3]	For Normal and mean For variance $(0.7 \times 15)^2$ Can be implied by correct answer BC	If Normal not specified then can still score M1A1 Allow 0.79
	(ii)		Total for 5 oranges ~ N(1725, 1125) 0.0127 (0.01267...)	B1 B1 [2]	For both BC	Allow N(5×345, 5×225) oe
	(iii)		Mean = $2 \times 345 - 3 \times 241.5$ (= -34.5) Variance = $2 \times 15^2 + 3 \times 10.5^2$ (= 780.75) 2 unpeeled - 3 peeled ~ N(-34.5, 780.75) 0.108 (0.10847...)	M1 M1 A1 A1 [4]	For method for mean For method for variance For correct values BC	Or 27.942^2 Allow N(34.5, 780.75) as may be using peeled - unpeeled
3	(i)	A	f(x) graph	B1 B1 [2]	At least one triangle Fully correct including $-c$, c on x-axis (do not penalise omission of, or error in, c on y-axis)	Must have labels x, f(x) (or pdf on y-axis)

Question			Answer	Marks		Guidance
(i)		**B**	$\frac{1}{2} \times 2c \times c = 1$	M1	For integration must get to at least $\left[cx + \frac{x^2}{2}\right]_{-c}^{0} + \left[cx - \frac{x^2}{2}\right]_{0}^{c} = 1$	Allow $\frac{c \times c}{2} \times 2 = 1$
			So $c = 1$	A1 [2]	AG	
(ii)			$P(X < \frac{1}{4}) = \frac{1}{2} + \left(\frac{1+0.75}{2}\right) \times 0.25$ $= \frac{23}{32}$ or 0.71875	M1 A1 [2]		Allow 0.719
			Or $P(X < \frac{1}{4}) = \frac{1}{2} + \int_0^{0.25}(1-x)dx$ $= \frac{23}{32}$ or 0.71875	M1 A1	www	
			Or $1 - \frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{23}{32}$	M1 A1		
(iii)			Mean $= 0$ $E(X^2) = \int_{-1}^{0}(x^2 + x^3)dx + \int_0^1(x^2 - x^3)dx$ $= \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$ $Var(X) = \frac{1}{6}$ Standard deviation $= 0.408$ or $\frac{1}{\sqrt{6}}$ or $\frac{\sqrt{6}}{6}$	B1 M1 A1 A1FT [4]		
4	(i)		$P(3 < X < 6) = 0.3$	B1 [1]		
	(ii)		$E(X) = 5$ $Var(X) = \frac{25}{3}$	B1 B1 [2]	Allow 8.3 or better	
	(iii)		$P(3 < Y < 6)$ $= \frac{22}{25}$ or 0.88	M1 A1 [2]	For division by 25	

Question		Answer	Marks		Guidance
(iv)		Because the mean value of Y is equal to the mean value of X,	**E1**		
		but the variance of Y is only one eighth of the variance of X.	**E1** **[2]**	Or allow 'as sample size increases, variance decreases'	
(v)		Using CLT, Y approx $\sim$ N(5, $^{25}/_{24}$)	**B1** **M1**	For mean and Normal For variance Can be implied by correct answer	FT their Var(X) for M1 only
		P(3 < Y < 6) approx = 0.811	**A1** **[3]**	BC awrt 0.811	
(vi)		Normal probability plot is very close to a straight line	**E1***		
		So this suggests that the distribution may be Normal and that the Central Limit Theorem may provide a good approximation.	***E1dep** **[2]**		
(vii)		Yes because the estimated probability from the mean of 808 is 0.808 which is close to the clt value of 0.811	**B1** **[1]**	Must compare with 0.811, not 0.88	Do **NOT** FT their answer to part (v)
(viii)		You would expect there to be less variation with 1000 rows	**E1***		Need more than 'more values give a better estimate' or 'or sample size is bigger' or similar for second E1.
		since as the number of rows increases, the estimated probability tends to get closer to the true probability	***E1dep** **[2]**		

Question			Answer	Marks		Guidance
5	(i)		$H_0: \mu = 3.75$ $H_1: \mu < 3.75$	**B1**	**DR** Hypotheses in words only must include "population".	For critical value of X method, critical value = $3.75 - 1.645 \times 0.21/\sqrt{12}$ gets M1 = 3.650 gets A1 with B1 for 1.645 than sixth mark for comparison with 3.638 and significant
			Where μ is the population mean flight time with the new airline in hours	**B1**	For definition in context.	
			Test statistic is $\dfrac{3.638-3.75}{0.21/\sqrt{12}}$ = -1.848	**M1** **A1**	Allow use of exact value for mean = 3.6382 leading to -1.844	For p-value method fifth mark is for p-value of 0.0323 than sixth for comparison with 0.05 and significant
			Critical value (1-tailed) at 5% level is (-)1.645 -1.848 < -1.645 so significant (reject H_0) Sufficient evidence to suggest that the average flight time with the new airline is less than 3.75 hours	**B1** **M1** **A1** **[7]**	No FT if not ±1.645	
	(ii)		Wilcoxon signed rank test	**B1**	Allow Wilcoxon single sample test	Do not allow just 'Wilcoxon test' nor 'Wilcoxon rank test'
			(Underlying distribution) needs to be symmetrical	**B1** **[2]**	Ignore 'around the median' or 'around the mean' following this	Do not allow 'Median = mean'
	(iii)		Single sample t-test	**B1** **[1]**	Condone 't-test' without 'single sample'	
6	(i)		C.I. is given by $0.29 \pm 1.96 \times \dfrac{1.64}{\sqrt{40}}$ $-0.218 \leq \mu \leq 0.798$ or 0.29 ± 0.5082	**M1** **B1** **M1** **A1** **[4]**	For general form For 1.96 For $\dfrac{1.64}{\sqrt{40}}$ can get this indep of everything else	If full working not shown allow SC2 for correct answer At least 2sf required but not just 0.8

Question			Answer	Marks		Guidance
(ii)			Confidence interval does not suggest that the additive reduces fuel consumption … …since the interval contains zero.	**B1** **B1** **[2]**	Condone 'it does not' or similar	Do not allow 'suggests that the additive does not affect fuel consumption' (but second mark still available)
(iii)			The sample is large and mention of Central Limit Theorem or Normal distribution The Central Limit Theorem states that sample means are therefore approximately Normally distributed	**E1** **E1** **[2]**	For full answer	
(iv)			A random sample enables proper inference about the population to be undertaken.	**B2** **[2]**	**B2** for correct explanation as shown in answer column OR **B1** for partially correct explanation e.g. 'a random sample is less likely to be biased' or 'so that the sample is more likely to be representative of the population'	

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