



Oxford Cambridge and RSA

Monday 4 October 2021 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 Using standard summation formulae, find $\sum_{r=1}^n (r^2 - 3r)$, giving your answer in fully factorised form. [3]

- 2 The equation $3x^2 - 4x + 2 = 0$ has roots α and β .

Find an equation with integer coefficients whose roots are $3 - 2\alpha$ and $3 - 2\beta$. [3]

- 3 Three planes have the following equations.

$$\begin{aligned} 2x - 3y + z &= -3, \\ x - 4y + 2z &= 1, \\ -3x - 2y + 3z &= 14. \end{aligned}$$

- (a) (i) Write the system of equations in matrix form. [1]

- (ii) Hence find the point of intersection of the planes. [2]

- (b) **In this question you must show detailed reasoning.**

Find the acute angle between the planes $2x - 3y + z = -3$ and $x - 4y + 2z = 1$. [4]

- 4 Anika thinks that, for two square matrices \mathbf{A} and \mathbf{B} , the inverse of \mathbf{AB} is $\mathbf{A}^{-1}\mathbf{B}^{-1}$. Her attempted proof of this is as follows.

$$\begin{aligned} (\mathbf{AB})(\mathbf{A}^{-1}\mathbf{B}^{-1}) &= \mathbf{A}(\mathbf{BA}^{-1})\mathbf{B}^{-1} \\ &= \mathbf{A}(\mathbf{A}^{-1}\mathbf{B})\mathbf{B}^{-1} \\ &= (\mathbf{AA}^{-1})(\mathbf{BB}^{-1}) \\ &= \mathbf{I} \times \mathbf{I} \\ &= \mathbf{I} \end{aligned}$$

$$\text{Hence } (\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$$

- (a) Explain the error in Anika's working. [2]

- (b) State the correct inverse of the matrix \mathbf{AB} and amend Anika's working to prove this. [3]

- 5 Prove by induction that $\sum_{r=1}^n r \times 2^{r-1} = 1 + (n-1)2^n$ for all positive integers n . [5]
- 6 A transformation T of the plane has associated matrix $\mathbf{M} = \begin{pmatrix} 1 & \lambda+1 \\ \lambda-1 & -1 \end{pmatrix}$, where λ is a non-zero constant.
- (a) (i) Show that T reverses orientation. [3]
(ii) State, in terms of λ , the area scale factor of T . [1]
- (b) (i) Show that $\mathbf{M}^2 - \lambda^2 \mathbf{I} = \mathbf{0}$. [2]
(ii) Hence specify the transformation equivalent to two applications of T . [1]
- (c) In the case where $\lambda = 1$, T is equivalent to a transformation S followed by a reflection in the x -axis.
- (i) Determine the matrix associated with S . [3]
(ii) Hence describe the transformation S . [2]
- 7 (a) (i) Find the modulus and argument of z_1 , where $z_1 = 1 + i$. [2]
(ii) Given that $|z_2| = 2$ and $\arg(z_2) = \frac{1}{6}\pi$, express z_2 in $a + bi$ form, where a and b are exact real numbers. [2]
- (b) Using these results, find the exact value of $\sin \frac{5}{12}\pi$, giving the answer in the form $\frac{\sqrt{m} + \sqrt{n}}{p}$, where m , n and p are integers. [5]

8 In this question you must show detailed reasoning.

The equation $x^3 + kx^2 + 15x - 25 = 0$ has roots α , β and $\frac{\alpha}{\beta}$. Given that $\alpha > 0$, find, in any order,

- the roots of the equation,
- the value of k . [7]

9 (a) On a single Argand diagram, sketch the loci defined by

- $\arg(z-2) = \frac{3}{4}\pi$,
- $|z| = |z+2-i|$. [4]

(b) In this question you must show detailed reasoning.

The point of intersection of the two loci in part (a) represents the complex number w .

Find w , giving your answer in exact form. [5]

END OF QUESTION PAPER

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