



Oxford Cambridge and RSA

**Monday 15 May 2023 – Afternoon**

**AS Level Further Mathematics B (MEI)**

**Y410/01 Core Pure**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator



**INSTRUCTIONS**

- Use black ink. You can use an HB pencil but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

**ADVICE**

- Read each question carefully before you start your answer.

## 2

1 The transformation  $R$  of the plane is reflection in the line  $x = 0$ .

(a) Write down the matrix  $\mathbf{M}$  associated with  $R$ . [1]

(b) Find  $\mathbf{M}^2$ . [1]

(c) Interpret the result of part (b) in terms of the transformation  $R$ . [1]

2 In this question you must show detailed reasoning.

The equation  $x^2 - kx + 2k = 0$ , where  $k$  is a non-zero constant, has roots  $\alpha$  and  $\beta$ .

Find  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  in terms of  $k$ , simplifying your answer. [4]

3 In this question you must show detailed reasoning.

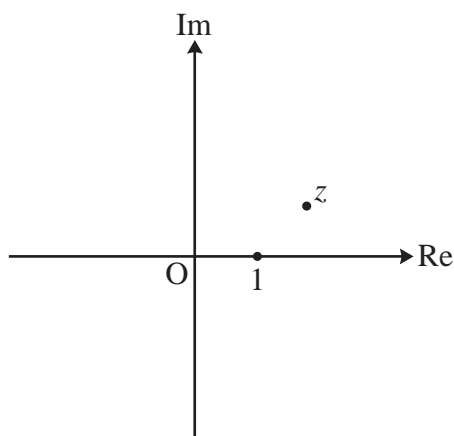
The function  $f(z)$  is given by  $f(z) = 2z^3 - 7z^2 + 16z - 15$ .

By first evaluating  $f\left(\frac{3}{2}\right)$ , find the roots of  $f(z) = 0$ . [6]

4 You are given that  $\sum_{r=1}^n (ar + b) = n^2$  for all  $n$ , where  $a$  and  $b$  are constants.

By finding  $\sum_{r=1}^n (ar + b)$  in terms of  $a$ ,  $b$  and  $n$ , determine the values of  $a$  and  $b$ . [6]

- 5 The Argand diagram below shows the points representing 1 and  $z$ , where  $|z| = 2$ .



Mark the points representing the following complex numbers on the copy of the diagram in the Printed Answer Booklet, labelling them clearly.

- $z^*$
- $\frac{1}{z}$
- $1 + z$
- $iz$  [4]

- 6 The matrix  $\mathbf{M}$  is  $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$ .

- (a) Calculate  $\mathbf{M}^2$ ,  $\mathbf{M}^3$  and  $\mathbf{M}^4$ . [2]
- (b) Hence make a conjecture about the matrix  $\mathbf{M}^n$ . [1]
- (c) Prove your conjecture. [5]

- 7 In this question you must show detailed reasoning.

The complex number  $\sqrt{3} + i$  is denoted by  $z$ .

- (a) By expanding  $(\sqrt{3} + i)^5$ , express  $z^5$  in the form  $a + bi$  where  $a$  and  $b$  are real and exact. [3]
- (b) (i) Express  $z$  in modulus-argument form. [3]
- (ii) Hence find  $z^5$  in modulus-argument form. [2]
- (iii) Use this result to verify your answers to part (a). [2]

8 The equations of three planes are

$$2x + y + 3z = 3,$$

$$3x - y - 2z = 2,$$

$$-4x + 3y + 7z = k,$$

where  $k$  is a constant.

(a) By considering a suitable determinant, show that the planes do **not** meet at a single point. [2]

(b) Given that the planes form a sheaf, determine the value of  $k$ . [4]

9 A transformation  $T$  of the plane is represented by the matrix  $\mathbf{M} = \begin{pmatrix} k+1 & -1 \\ 1 & k \end{pmatrix}$ , where  $k$  is a constant.

Show that, for all values of  $k$ ,  $T$  has no invariant lines through the origin. [6]

10 The plane  $P$  has normal vector  $2\mathbf{i} + a\mathbf{j} - \mathbf{k}$ , where  $a$  is a positive constant, and the point  $(3, -1, 1)$  lies in  $P$ . The plane  $x - z = 3$  makes an angle of  $45^\circ$  with  $P$ .

Find the cartesian equation of  $P$ . [7]

## END OF QUESTION PAPER

---

# OCR

Oxford Cambridge and RSA

### Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.