



Oxford Cambridge and RSA

Monday 16 May 2022 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 (a) (i) Write the following simultaneous equations as a matrix equation.

$$\begin{aligned}x + y + 2z &= 7 \\2x - 4y - 3z &= -5 \\-5x + 3y + 5z &= 13\end{aligned}\quad [1]$$

- (ii) Hence solve the equations. [2]

- (b) Determine the set of values of the constant k for which the matrix equation

$$\begin{pmatrix} k+1 & 1 \\ 2 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 23 \\ -17 \end{pmatrix}$$

has a unique solution. [3]

- 2 (a) Show that the vector $\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ is parallel to the plane $2x + y - 3z = 10$. [3]

- (b) Determine the acute angle between the planes $2x + y - 3z = 10$ and $x - y - 3z = 3$. [4]

- 3 The complex number z satisfies the equation $5(z - i) = (-1 + 2i)z^*$.

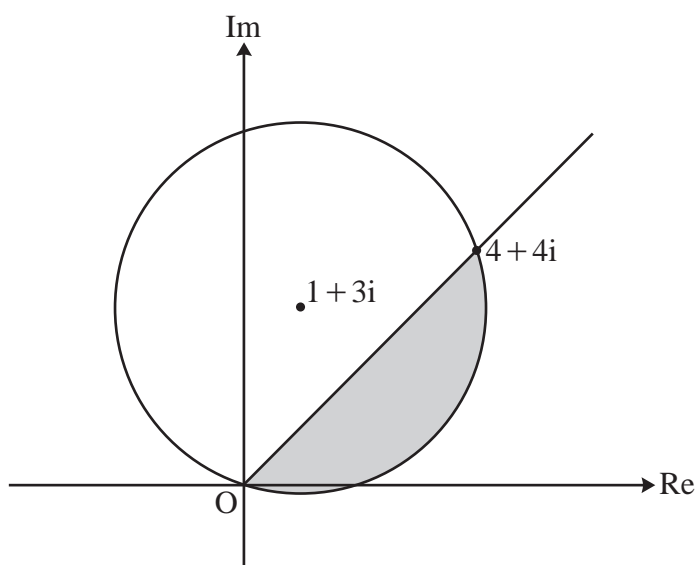
Determine z , giving your answer in the form $a + bi$, where a and b are real. [5]

- 4 **In this question you must show detailed reasoning.**

The equation $z^3 + 2z^2 + kz + 3 = 0$, where k is a constant, has roots α , $\frac{1}{\alpha}$ and β .

Determine the roots in exact form. [6]

- 5 An Argand diagram is shown below. The circle has centre at the point representing $1 + 3i$, and the half line intersects the circle at the origin and at the point representing $4 + 4i$.



State the **two** conditions that define the set of complex numbers represented by points in the shaded segment, including its boundaries.

[5]

- 6 (a) Using standard summation formulae, show that $\sum_{r=1}^n r(r+2) = \frac{1}{6}n(n+1)(2n+7)$. [4]
- (b) Use induction to prove the result in part (a). [6]

7 On an Argand diagram, the point A represents the complex number z with modulus 2 and argument $\frac{1}{3}\pi$. The point B represents $\frac{1}{z}$.

(a) Sketch an Argand diagram showing the origin O and the points A and B. [2]

(b) The point C is such that OACB is a parallelogram. C represents the complex number w .

Determine each of the following.

- The modulus of w , giving your answer in exact form.
- The argument of w , giving your answer correct to 3 significant figures. [7]

8 A transformation T of the plane has matrix \mathbf{M} , where $\mathbf{M} = \begin{pmatrix} \cos \theta & 2 \cos \theta - \sin \theta \\ \sin \theta & 2 \sin \theta + \cos \theta \end{pmatrix}$.

(a) Show that T leaves areas unchanged for all values of θ . [2]

(b) Find the value of θ , where $0 < \theta < \frac{1}{2}\pi$, for which the y-axis is an invariant line of T. [4]

The matrix \mathbf{N} is $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(c) (i) Find \mathbf{MN}^{-1} . [2]

(ii) Hence describe fully a sequence of two transformations of the plane that is equivalent to T. [4]

END OF QUESTION PAPER

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