



Oxford Cambridge and RSA

Monday 13 May 2019 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes



You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions

1 In this question you must show detailed reasoning.

Find $\sum_{r=1}^{100} \left(\frac{1}{r} - \frac{1}{r+2} \right)$, giving your answer correct to 4 decimal places. [3]

2 The roots of the equation $3x^2 - x + 2 = 0$ are α and β .
Find a quadratic equation with integer coefficients whose roots are $2\alpha - 3$ and $2\beta - 3$. [3]

3 In this question you must show detailed reasoning.

A and **B** are matrices such that $\mathbf{B}^{-1}\mathbf{A}^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$.

(a) Find **AB**. [3]

(b) Given that $\mathbf{A} = \begin{pmatrix} \frac{1}{3} & 1 \\ 0 & 1 \end{pmatrix}$, find **B**. [3]

4 (a) Find \mathbf{M}^{-1} , where $\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ -2 & 1 & 2 \end{pmatrix}$. [1]

(b) Hence find, in terms of the constant k , the point of intersection of the planes

$$\begin{aligned} x + 2y + 3z &= 19, \\ -x + y + 2z &= 4, \\ -2x + y + 2z &= k. \end{aligned} \quad [3]$$

(c) **In this question you must show detailed reasoning.**

Find the acute angle between the planes $x + 2y + 3z = 19$ and $-x + y + 2z = 4$. [4]

5 Prove by induction that, for all positive integers n , $\sum_{r=1}^n \frac{1}{3^r} = \frac{1}{2} \left(1 - \frac{1}{3^n} \right)$. [6]

- 6 A linear transformation T of the x - y plane has an associated matrix \mathbf{M} , where $\mathbf{M} = \begin{pmatrix} \lambda & k \\ 1 & \lambda - k \end{pmatrix}$, and λ and k are real constants.

(a) You are given that $\det \mathbf{M} > 0$ for all values of λ .

(i) Find the range of possible values of k . [3]

(ii) What is the significance of the condition $\det \mathbf{M} > 0$ for the transformation T ? [1]

For the remainder of this question, take $k = -2$.

(b) Determine whether there are any lines through the origin that are invariant lines for the transformation T . [4]

(c) The transformation T is applied to a triangle with area 3 units². The area of the resulting image triangle is 15 units².
Find the possible values of λ . [3]

7 (a) Sketch on a single Argand diagram

(i) the set of points for which $|z - 1 - 3i| = 3$, [3]

(ii) the set of points for which $\arg(z + 4) = \frac{1}{4}\pi$. [3]

(b) Find, in exact form, the two values of z for which $|z - 1 - 3i| = 3$ and $\arg(z + 4) = \frac{1}{4}\pi$. [6]

8 **In this question you must show detailed reasoning.**

You are given that i is a root of the equation $z^4 - 2z^3 + 3z^2 + az + b = 0$, where a and b are real constants.

(a) Show that $a = -2$ and $b = 2$. [4]

(b) Find the other roots of this equation. [7]

END OF QUESTION PAPER

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