



Oxford Cambridge and RSA

**Thursday 20 June 2019 – Morning**

**A Level Further Mathematics B (MEI)**

**Y434/01 Numerical Methods**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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Answer **all** the questions.

- 1 Fig. 1 shows some spreadsheet output concerning the values of a function,  $f(x)$ .

	A	B	C
1	$x$	$f(x)$	
2	1	0.367879441	0.367879441
3	2	0.018315639	0.38619508
4	3	0.00012341	0.38631849
5	4	1.12535E-07	0.386318602
6	5	1.38879E-11	0.386318602

**Fig. 1**

The formula in cell B2 is

and equivalent formulae are in cells B3 to B6.

The formula in cell C2 is .

The formula in cell C3 is .

Equivalent formulae are in cells C4 to C6.

(a) Use sigma notation to express the formula in cell C5 in standard mathematical notation. [2]

(b) Explain why the same value is displayed in cells C5 and C6. [2]

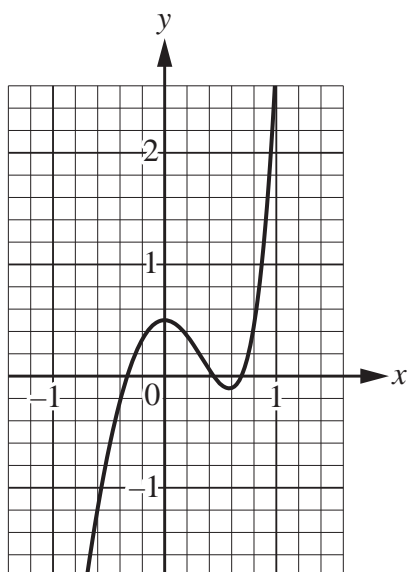
Now suppose that the value in cell C2 is chopped to 3 decimal places and used to approximate the value in cell C2.

(c) Calculate the relative error when this approximation is used. [1]

Suppose that the values in cells B4, B5 and B6 are chopped to 3 decimal places and used as approximations to the original values in cells B4, B5 and B6 respectively.

(d) Explain why the relative errors in these approximations are all the same. [1]

- 2 Fig. 2.1 shows the graph of  $y = x^2e^{2x} - 5x^2 + 0.5$ .



**Fig. 2.1**

There are three roots of the equation  $x^2e^{2x} - 5x^2 + 0.5 = 0$ . The roots are  $\alpha$ ,  $\beta$  and  $\gamma$ , where  $\alpha < \beta < \gamma$ .

- (a) Explain why it is not possible to use the method of false position with  $x_0 = 0$  and  $x_1 = 1$  to find  $\beta$  and  $\gamma$ . [1]

The graph of the function indicates that the root  $\gamma$  lies in the interval  $[0.6, 0.8]$ . Fig. 2.2 shows some spreadsheet output using the method of false position using these values as starting points.

	A	B	C	D	E	F
1	$a$	$f(a)$	$b$	$f(b)$	approx	
2	0.6	-0.10476	0.8	0.469941	0.636457	-0.07876
3	0.636457	-0.07876	0.8	0.469941	0.659931	-0.04748
4	0.659931	-0.04748	0.8	0.469941	0.672783	-0.0249
5	0.672783	-0.0249	0.8	0.469941	0.679184	-0.01211
6	0.679184	-0.01211	0.8	0.469941	0.682218	-0.00567
7	0.682218	-0.00567	0.8	0.469941	0.683623	-0.00261
8	0.683623	-0.00261	0.8	0.469941	0.684266	-0.00119
9	0.684266	-0.00119	0.8	0.469941	0.684559	-0.00054
10	0.684559	-0.00054	0.8	0.469941	0.684692	-0.00025
11	0.684692	-0.00025	0.8	0.469941	0.684753	-0.00011
12	0.684753	-0.00011	0.8	0.469941	0.68478	-5.1E-05

**Fig. 2.2**

- (b) **Without** doing any further calculation, write down the smallest possible interval which is certain to contain  $\gamma$ . [1]

5

- (c) State what is being calculated in column F. [1]

The formula in cell A3 is  $\boxed{=IF(F2<0,E2,A2)}$ .

- (d) Explain the purpose of this formula in the application of the method of false position. [2]

The method of false position uses the same formula for obtaining new approximations as the secant method.

- (e) Explain how the method of false position differs from the secant method. [1]

- (f) Give one **advantage** and one **disadvantage** of using the method of false position instead of the secant method. [2]

- 3 In the first week of an outbreak of influenza, 9 patients were diagnosed with the virus at a medical practice in Pencaster. Records were kept of  $y$ , the total number of patients diagnosed with influenza in week  $n$ . The data are shown in Fig. 3.

$n$	1	2	3	4	5
$y$	9	32	63	96	125

**Fig. 3**

- (a) Complete the difference table in the Printed Answer Booklet. [3]

- (b) Explain why a cubic model is appropriate for the data. [1]

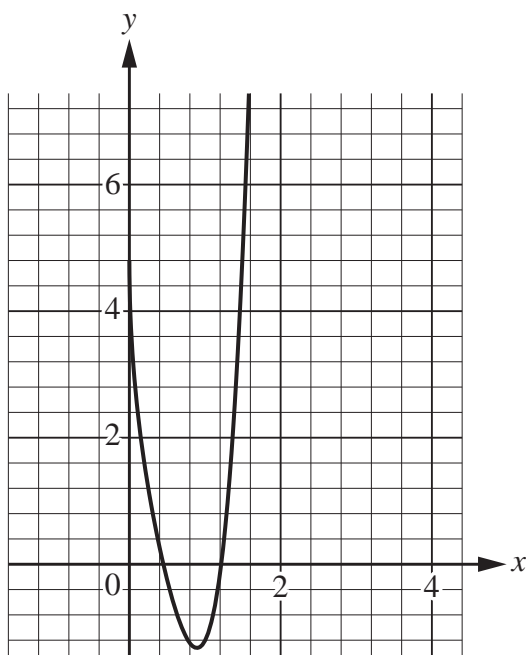
- (c) Use Newton's method to find the interpolating polynomial of degree 3 for these data. [4]

In both week 6 and week 7 there were 145 patients in total diagnosed with influenza at the medical practice.

- (d) Determine whether the model is a good fit for these data. [2]

- (e) Determine the maximum number of weeks for which the model could possibly be valid. [1]

- 4 Fig. 4 shows the graph of  $y = x^5 - 6\sqrt{x} + 4$ .



**Fig. 4**

There are two roots of the equation  $x^5 - 6\sqrt{x} + 4 = 0$ . The roots are  $\alpha$  and  $\beta$ , such that  $\alpha < \beta$ .

- (a) Show that  $0 < \alpha < 1$  and  $1 < \beta < 2$ . [2]
- (b) Obtain the Newton-Raphson iterative formula

$$x_{n+1} = x_n - \frac{x_n^{\frac{11}{2}} - 6x_n + 4\sqrt{x_n}}{5x_n^{\frac{9}{2}} - 3}. \quad [3]$$

- (c) Use the iterative formula found in part (b) with a starting value of  $x_0 = 1$  to obtain  $\beta$  correct to 6 decimal places. [2]
- (d) Use the iterative formula found in part (b) with a starting value of  $x_0 = 0$  to find  $x_1$ . [1]
- (e) Give a geometrical explanation of why the Newton-Raphson iteration fails to find  $\alpha$  in part (d). [1]
- (f) Obtain the iterative formula

$$x_{n+1} = \left( \frac{x_n^5 + 4}{6} \right)^{\frac{1}{2}}. \quad [2]$$

- (g) Use the iterative formula found in part (f) with a starting value of  $x_0 = 0$  to obtain  $\alpha$  correct to 6 decimal places. [2]

- 5 Fig. 5 shows spreadsheet output concerning the estimation of the derivative of a function  $f(x)$  at  $x = 2$  using the forward difference method.

	A	B	C	D
1	$h$	estimate	difference	ratio
2	0.1	6.3050005		
3	0.01	6.0300512	-0.274949	
4	0.001	6.0030018	-0.027049	0.098379
5	0.0001	6.0003014	-0.0027	0.099835
6	0.00001	6.0000314	-0.00027	0.099983
7	0.000001	6.0000044	-2.7E-05	0.099994
8	1E-07	6.0000016	-2.71E-06	0.100352
9	1E-08	6.0000013	-3.02E-07	0.111457
10	1E-09	6.0000018	4.885E-07	-1.61765
11	1E-10	6.0000049	3.109E-06	6.363636
12	1E-11	6.0000005	-4.44E-06	-1.42857
13	1E-12	6.0005334	0.0005329	-120
14	1E-13	5.9952043	-0.005329	-10
15	1E-14	6.1284311	0.1332268	-25
16	1E-15	5.3290705	-0.799361	-6
17	1E-16	0	-5.329071	6.666667

**Fig. 5**

- (a) Write down suitable cell formulae for
- cell C3,
  - cell D4. [2]
- (b) Explain what the entries in cells D4 to D8 tell you about the order of the convergence of the forward difference method. [2]
- (c) Write the entry in cell A10 in standard mathematical notation. [1]
- (d) Explain what the values displayed in cells D10 to D17 suggest about the values in cells B10 to B16. [2]
- (e) Write down the value of the derivative of  $f(x)$  at  $x = 2$  to an accuracy that seems justified, explaining your answer. [2]

The formula in cell B2 is `=LN(SQRT(SINH((2+A2)^3)))-LN(SQRT(SINH(2^3)))/A2` and equivalent formulae are entered in cells B3 to B17.

- (f) Write  $f(x)$  in standard mathematical notation. [1]

The value displayed in cell B17 is zero, even though the calculation results in a non-zero answer.

- (g) Explain how this has arisen. [2]

- 6 The spreadsheet output in Fig. 6 shows approximations to  $\int_0^1 x^{-\sqrt{x}} dx$  found using the midpoint rule, denoted by  $M_n$ , and the trapezium rule, denoted by  $T_n$ .

	A	B	C
1	$n$	$M_n$	$T_n$
2	1	1.632527	1
3	2	1.641461	1.316263
4	4	1.623053	1.478862
5	8	1.610295	1.550957
6	16	1.604132	1.580626
7	32	1.601505	1.592379

**Fig. 6**

- (a) Write down an efficient spreadsheet formula for cell C3. [2]
- (b) By first completing the table in the Printed Answer Booklet using the Simpson's rule, calculate the most accurate estimate of  $\int_0^1 x^{-\sqrt{x}} dx$  that you can, justifying the precision quoted. [8]

**END OF QUESTION PAPER**

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