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A Level Further Mathematics B (MEI)
Y433 Modelling with Algorithms
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 15 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae A Level Further Mathematics B (MEI)

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics B (MEI)
- Scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

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Answer **all** the questions.

1 The following instructions generate a sequence. It uses Floor(x), which is the largest integer less than or equal to x. For example Floor(8.95) = 8.

Step 10 Let $n = 1$
Step 20 Let $r = \sqrt{2}$
Step 30 Let $\mathrm{u}(n) = \mathrm{Floor}(n \times r)$
Step 40 Print u(n)
Step 50 Let $n = n + 1$
Step 60 Go to Step 30

(i) Write down the first five terms of the sequence. **[1]**

(ii) Amend the instructions to produce an algorithm to give the first five terms of the sequence only. **[2]**

The original instructions are amended by replacing Step 20, Step 30 and Step 40. The new instructions are as follows.

Step 10 Let $n = 1$
Step 20 Let $s = 2 + \sqrt{2}$
Step 30 Let $\mathrm{v}(n) = \mathrm{Floor}(n \times s)$
Step 40 Print v(n)
Step 50 Let $n = n + 1$
Step 60 Go to Step 30

These new instructions generate a new sequence.

(iii) Find the first five terms of this new sequence. **[1]**

It is conjectured that every positive integer m appears in exactly one of the two sequences u(n) and v(n).

(iv) Show that the conjecture is true for $m = 100$. **[3]**

SPECIMEN

2 Activity X is part of a large project. The project has been modelled by an activity network. Fig. 2 shows that part of the activity network relating to X.

The events at the start and end of activity X are shown, together with all activities starting and ending at these events. Activity durations are shown. Some early and late event times are shown; these have been calculated from the full activity network.

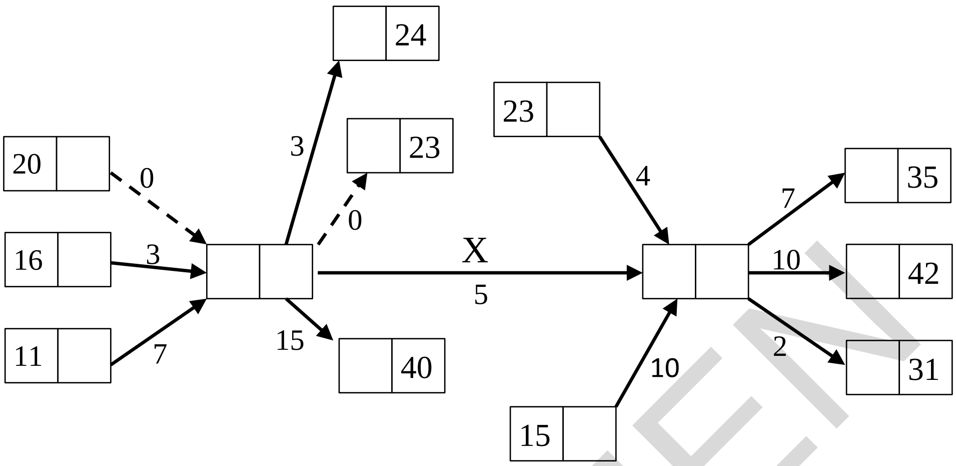

Fig. 2

Calculate the early and late times for the events at the start and end of activity X. **[4]**

SPECIMEN

3 Direct transport links between six cities A, B, C, D, E, F are shown in Fig. 3. The weights on the arcs are the times, in hours, for moving along those links.

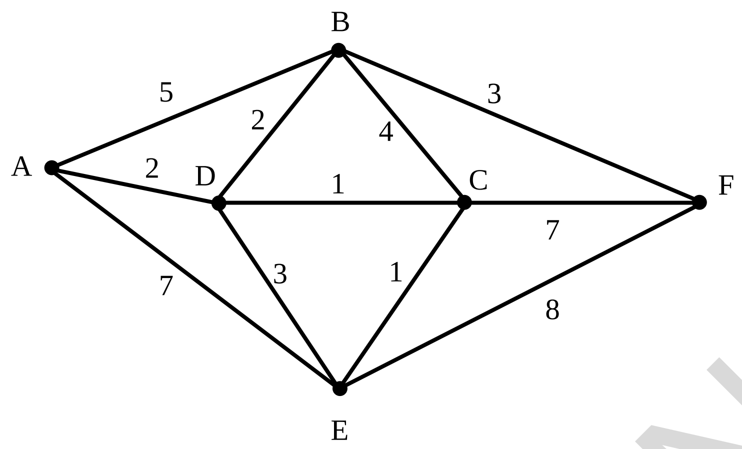

Fig. 3

The table below shows the complete set of shortest times between the cities. Some entries have been omitted.

	A	B	C	D	E	F
A	-	4	3	2	4	
B	4	-	3	2	4	
C	3	3	-	1	1	
D	2	2	1	-	2	
E	4	4	1	2	-	
F						-

(i) Use Dijkstra's algorithm to find the missing entries, and complete the table in the Printed Answer Booklet. **[3]**

A single application of Dijkstra's algorithm has complexity $O(n^2)$ where n is the number of vertices in the network. A computer program obtains a table of the complete set of shortest times by repeatedly applying Dijkstra's algorithm at each vertex. It takes the program t seconds to obtain a table with the complete set of shortest times for a network of p vertices.

(ii) Approximately how long will it take the program to carry out the task for a network of q vertices? Explain your reasoning. **[2]**

(iii) Explain why your answer to part **(ii)** is only an approximation. **[1]**

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4 A list of n numbers is sorted by making passes through an algorithm as follows.

A pass consists of the following.

Compare the first and second number. If necessary, swap them so that the first number is less than or equal to the second number.
Compare the new second and third number. If necessary, swap them so that the second number is less than or equal to the third number.
Compare the third and fourth number. If necessary, swap them so that the third number is less than or equal to the fourth number.
…
Compare the $(n-1)$th and nth number. If necessary, swap them so that the $(n-1)$th number is less than or equal to the nth number.

Repeat this until a pass occurs with no swaps.

The algorithm is applied to the list of 10 numbers below.

30 29 28 27 26 25 24 23 22 21

(i) After one pass, in what position is
- the largest number,
- the smallest number? **[2]**

The algorithm is applied to a list of n numbers, with the largest number at the beginning of the list (in position 1) and the smallest at the end (in position n).

(ii) How many comparisons are made in sorting this list? **[4]**

The algorithm is applied to a list of n numbers. Nothing is known about the position of the numbers before the algorithm is applied.

(iii) What are the minimum and maximum number of comparisons which might be required to apply the algorithm? **[2]**

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5 A network has 10 vertices, A to J. The table below shows the distance between each pair of vertices for which there is a connecting arc.

	A	B	C	D	E	F	G	H	I	J
A			8	4						
B					10				7	
C	8			6			12			
D	4		6					6		7
E		10				8			9	
F					8				8	
G			12					6		9
H				6			6			7
I		7			9	8				
J				7			9	7		

(i) Apply the tabular form of Prim's algorithm to the network, starting at vertex A. **[3]**

(ii) Explain how the table shows that the algorithm terminates before connecting all the vertices. **[1]**

(iii) (*A*) Draw a minimum connector for the vertices which are connected to A. **[1]**

(*B*) Give the total length of the minimum connector in part **(iii)** (*A*). **[1]**

(iv) (*A*) Draw the network for the vertices not connected to A. **[1]**

(*B*) Draw a minimum connector for these vertices. **[1]**

(*C*) Give the total length of the minimum connector in part **(iv)** (*B*). **[1]**

A single arc connecting A and B is added to the original network.

(v) Explain why a minimum connector for the new network is given by the two minimum connectors from parts **(iii)** and **(iv)** together with the new arc. **[2]**

As well as the arc connecting A and B, another arc connecting C and E is added to the network. A tree is formed by the minimum connectors from parts **(iii)** and **(iv)**, together with the shorter of the two arcs AB and CE.

(vi) Determine whether it is always true, sometimes true or never true that this tree is a minimum connector for the new network. Give reasons for your answer. **[3]**

SPECIMEN

6 Three liquid medicines, X, Y and Z, are to be manufactured. All the medicines require ingredients A, B, C and D which are in limited supply. The table below shows how many grams of each ingredient are required for one litre of each medicine. It also shows how much of each ingredient is available.

	A	B	C	D
Each litre of X requires	2	0	2	4
Each litre of Y requires	5	2	4	3
Each litre of Z requires	3	1	2	2
Amount, in grams, of each ingredient available	20	10	70	30

When the medicines are sold, the profits are £5 per litre of X manufactured, £2 per litre of Y and £3 per litre of Z.

(i) Formulate an LP to maximise the total profit subject to the constraints imposed by the availability of the ingredients. Use x as the number of litres of X, y as the number of litres of Y and z as the number of litres of Z. **[3]**

The simplex algorithm is used to solve this LP. After the first iteration the tableau below is produced.

P	x	y	z	s_1	s_2	s_3	s_4	RHS
1	0	1.75	–0.5	0	0	0	1.25	37.5
0	0	3.5	2	1	0	0	–0.5	5
0	0	2	1	0	1	0	0	10
0	0	2.5	1	0	0	1	–0.5	55
0	1	0.75	0.5	0	0	0	0.25	7.5

(ii) (A) Perform a second iteration. **[2]**

(B) Give the maximum profit, and the number of litres of X, Y and Z which should be manufactured to achieve this profit. **[1]**

(iii) An extra constraint is imposed by a contract to supply at least 5 litres of Y. Produce an initial tableau which could be used to solve this new problem by using the two-stage simplex method. **[3]**

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7 A series of drainage pipes allows rainwater to flow downhill under gravity from a field on a hillside to a river in a valley below. The directed network in Fig. 7 models this system, with S representing the field and T the river. The weights represent the capacities of the pipes, in litres per minute.

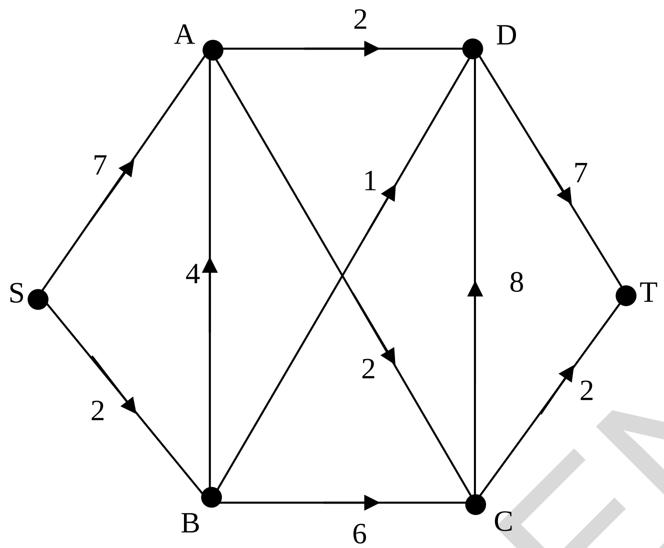

Fig. 7

(i) What aspect of the network models the fact that the point represented by C is higher up the hillside than the point represented by D? **[1]**

The following LP formulation finds the maximum flow through the network. The variable SA represents the flow along the arc from vertex S to vertex A, and similarly for other arcs.

Maximise $SA + SB$

subject to
$SA + BA - AD - AC = 0$
$SB - BA - BC - BD = 0$
$BC + AC - CD - CT = 0$
$AD + BD - DT + CD = 0$
$SA \leq 7$
$AD \leq 2$
$BC \leq 6$
$CT \leq 2$
$SB \leq 2$
$AC \leq 2$
$CD \leq 8$
$BA \leq 4$
$BD \leq 1$
$DT \leq 7$

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(ii) Explain the purpose of each of the following lines from the LP formulation.

(*A*) Maximise $\mathrm{SA} + \mathrm{SB}$ **[2]**

(*B*) $\mathrm{SA} + \mathrm{BA} - \mathrm{AD} - \mathrm{AC} = 0$ **[2]**

(*C*) $\mathrm{CT} \le 2$ **[1]**

The LP is run in an Online LP Solver and in a Spreadsheet LP Solver, and the following outputs are obtained.

Online LP Solver

Objective	6
Variable	Value
SA	4
SB	2
BA	0
AD	2
AC	2
BD	1
BC	1
CD	3
DT	6
CT	0

This question is continued on the next page

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Spreadsheet LP Solver

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$M$5	capacity	0	6

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$4	SA	0	4	Contin
$D$4	SB	0	2	Contin
$E$4	BA	0	0	Contin
$F$4	AD	0	2	Contin
$G$4	AC	0	2	Contin
$H$4	BD	0	1	Contin
$I$4	BC	0	1	Contin
$J$4	CD	0	1	Contin
$K$4	DT	0	4	Contin
$L$4	CT	0	2	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$M$10	SA<=7	4	$M$10<=$N$10	Not Binding	3
$M$11	AD <=2	2	$M$11<=$N$11	Binding	0
$M$12	BC<=6	1	$M$12<=$N$12	Not Binding	5
$M$13	CT<=2	2	$M$13<=$N$13	Binding	0
$M$14	SB <=2	2	$M$14<=$N$14	Binding	0
$M$15	AC<=2	2	$M$15<=$N$15	Binding	0
$M$16	CD<=8	1	$M$16<=$N$16	Not Binding	7
$M$17	BA<=4	0	$M$17<=$N$17	Not Binding	4
$M$18	BD<=1	1	$M$18<=$N$18	Binding	0
$M$19	DT<=7	4	$M$19<=$N$19	Not Binding	3
$M$6	SA+BA-AD-AC=0	0	$M$6=$N$6	Binding	0
$M$7	SB-BA-BC-BD=0	0	$M$7=$N$7	Binding	0
$M$8	BC+AC-CD-CT=0	0	$M$8=$N$8	Binding	0
$M$9	AD+BD-DT+CD=0	0	$M$9=$N$9	Binding	0

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(iii) (*A*) For each solver, interpret the output to give a maximum flow on the diagram in the Printed Answer Booklet. **[2]**

(*B*) State the maximum capacity of the network. **[1]**

(iv) Give a cut with capacity equal to the maximum flow. **[1]**

(v) In order to save money it is proposed to shut down as many drainage pipes as possible, while still allowing the maximum flow through the system. Show how this can be done. **[2]**

END OF QUESTION PAPER

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