Oxford Cambridge and RSA

Tuesday 20 October 2020 – Afternoon

A Level Further Mathematics B (MEI)

Y433/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1

12 15 11 9 20 16 8 23 10 7

(a) Apply the quick sort algorithm to sort the list of numbers into **descending** order. You should use the first value as the pivot for each sublist. **[3]**

The values represent masses, measured in kg.

(b) Show the result of applying the first fit decreasing algorithm to pack these masses into bags that can each hold a maximum of 40 kg. **[2]**

(c) Show the result of applying a full bin strategy to pack these masses into bags that can each hold a maximum of 40 kg. **[2]**

2 The table in Fig. 2 shows the distances, in miles, along the direct roads between ten villages, A to J. A dash (–) indicates that there is no direct road linking the villages.

	A	B	C	D	E	F	G	H	I	J
A	–	9	–	10	13	2	17	6	–	–
B	9	–	14	–	–	–	10	7	2	10
C	–	14	–	8	18	7	–	6	–	5
D	10	–	8	–	–	16	–	11	–	15
E	13	–	18	–	–	–	7	–	–	8
F	2	–	7	16	–	–	13	10	9	–
G	17	10	–	–	7	13	–	13	11	–
H	6	7	6	11	–	10	13	–	–	9
I	–	2	–	–	–	9	11	–	–	–
J	–	10	5	15	8	–	–	9	–	–

Fig. 2

(a) Apply the tabular form of Prim's algorithm to the network, **starting at vertex D**, to find a minimum spanning tree for the network. **[2]**

(b) State the total length of the arcs in the minimum spanning tree. **[1]**

(c) Use Dijkstra's algorithm to determine the length of the shortest path from G to J via D. **[5]**

(d) A computer takes 0.25 secs to solve a shortest path problem on a network with 15 vertices using Dijkstra's algorithm. Approximately how long will it take the computer to solve a shortest path problem on a network with 300 vertices using Dijkstra's algorithm? **[2]**

3

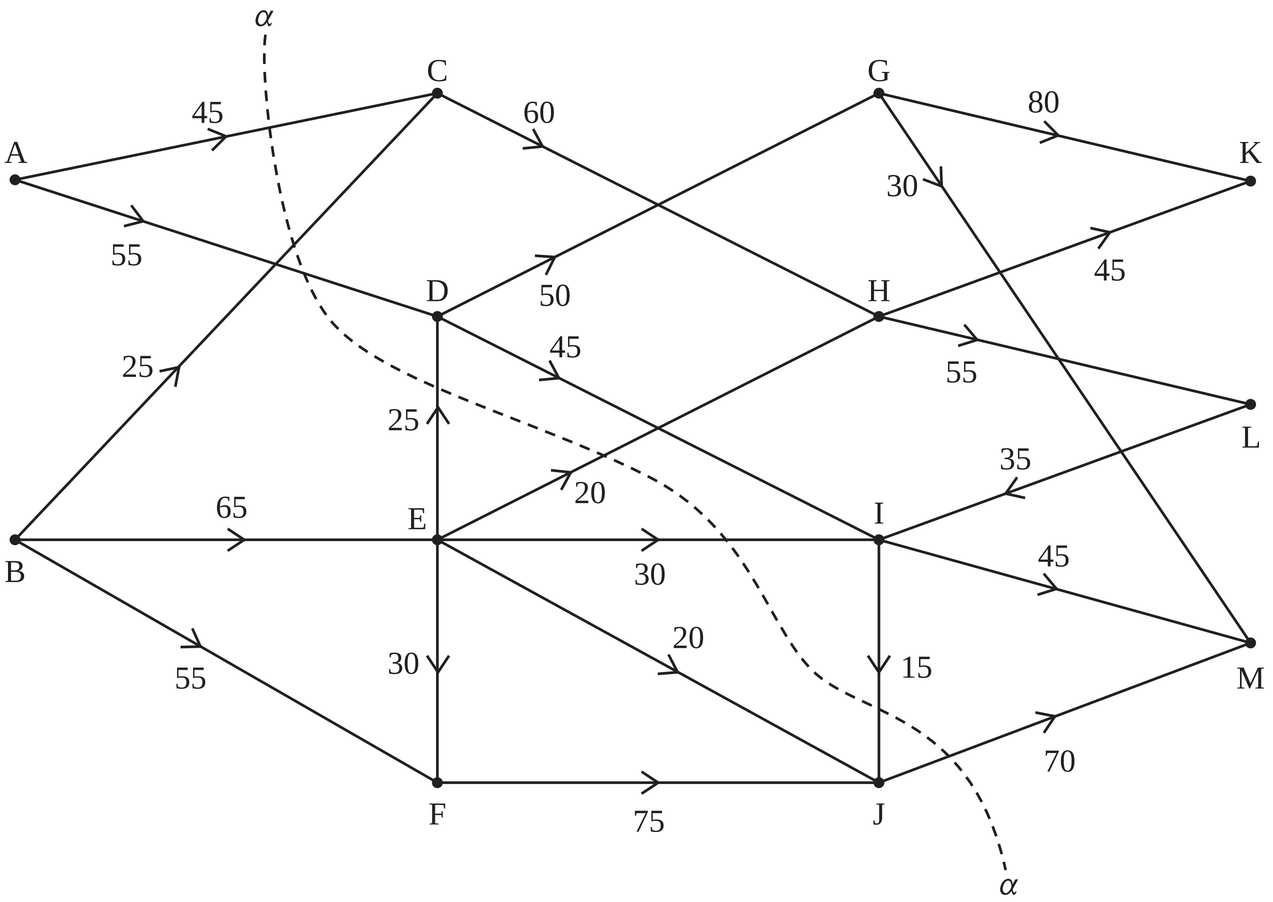

Fig. 3

The diagram in Fig. 3 represents a system of pipes through which a fluid can flow from two sources to two sinks. It also shows a cut α. The weights on the arcs show the capacities of the pipes in gallons per hour.

(a) Add a supersource and a supersink to the network in the Printed Answer Booklet. Give appropriate weightings and directions to the connecting arcs. **[2]**

(b) Calculate the capacity of the cut α. **[1]**

(c) Explain why the arcs DG and DI cannot simultaneously be full to capacity. **[1]**

An LP formulation is set up to find the maximum flow through the network.

(d) **(i)** Write down a suitable objective function for the LP formulation. **[1]**

(ii) Write down the constraint required for the LP formulation that ensures that the flow into vertex I is equal to the flow out of vertex I. **[1]**

The complete LP was run in an LP solver and it was found that arcs AC, BE, BF, ED, CH, DG, HK, LI and JM (and possibly others) were all saturated and that the flow through DI was 5 gallons per hour.

(e) By completing the diagram in the Printed Answer Booklet, determine the maximum value of the flow through the network. **[3]**

(f) Use a suitable cut to prove that this is the maximum flow. **[2]**

4 The following LP formulation can be used to find the minimum completion time, T, for the 11 activities A, B, …, K in a project. The durations of the activities A, B, …, K are in hours.

Minimise T

subject to

$T-I \geqslant 6, \quad T-J \geqslant 11, \quad T-K \geqslant 10, \quad I-D \geqslant 10, \quad J-D \geqslant 10$

$I-H \geqslant 4, \quad J-H \geqslant 4, \quad K-D \geqslant 10, \quad K-H \geqslant 4, \quad K-F \geqslant 9$

$K-G \geqslant 12, \quad H-E \geqslant 11, \quad F-E \geqslant 11, \quad H-A \geqslant 5, \quad F-A \geqslant 5$

$G-A \geqslant 5, \quad H-C \geqslant 7, \quad F-C \geqslant 7, \quad D-B \geqslant 8, \quad E-B \geqslant 8$

(a) Complete the table in the Printed Answer Booklet giving the duration and immediate predecessors for each activity in the project. **[3]**

(b) Draw an activity network, using activity on arc, to represent the project. **[3]**

(c) Carry out a forward pass and a backward pass through the activity network, showing the early event time and late event time at each vertex of your network. **[4]**

It is given that activity C has a non-zero independent float.

(d) **(i)** State the other activity that has a non-zero independent float. **[1]**

(ii) Calculate the corresponding value of this independent float. **[1]**

(iii) Explain, with regards to any delays for this project, the significance of this activity having a non-zero independent float. **[1]**

5

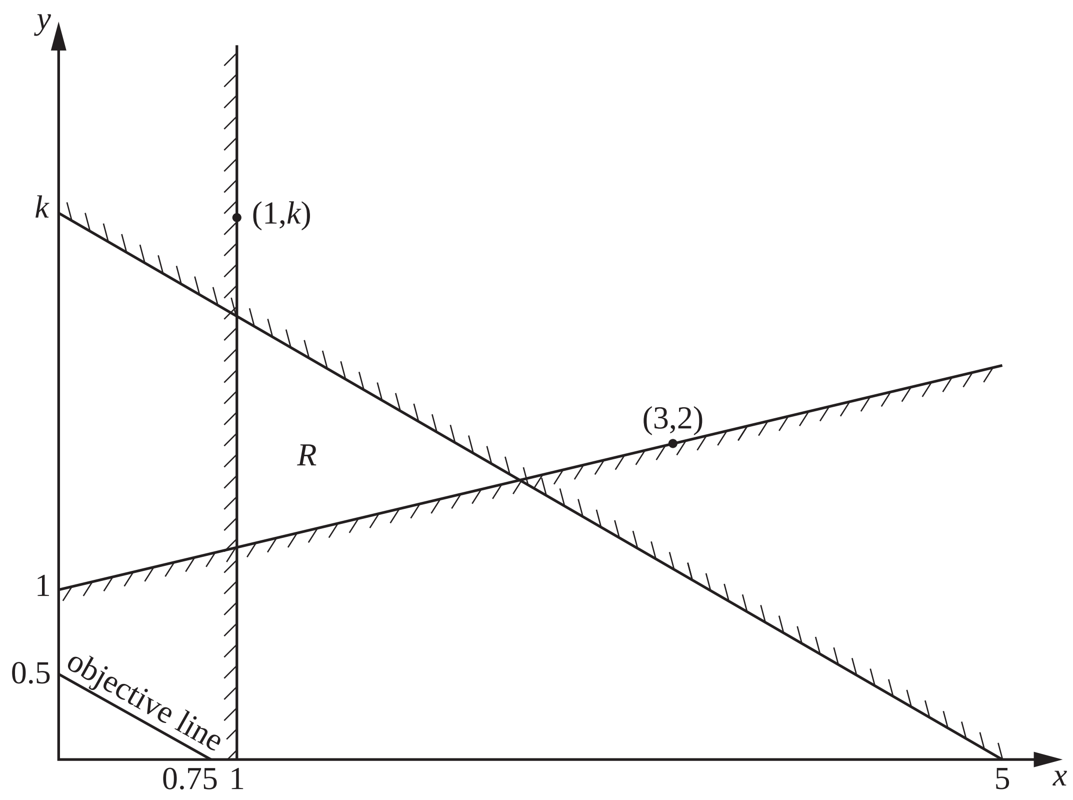

Fig. 5.1

Fig. 5.1 shows the constraints of a linear programming problem, in which the objective is to maximise a function of the variables *x* and *y*.

The feasible region, *R*, is the unshaded region together with its boundaries. An objective line is also shown and labelled on Fig. 5.1.

(a)
- Complete the initial tableau in the Printed Answer Booklet so that the two-stage simplex method may be used to solve this problem.
- Show how the constraints for the problem have been made into equations using slack variables, surplus variables and artificial variables.
- Show how the rows for the two objective functions are formed. **[10]**

After two iterations of the two-stage simplex method a computer produces the tableau in Fig. 5.2.

Q	P	x	y	s_1	s_2	s_3	a_1	a_2	RHS
1	0	0	0	0	0	0	-1	-1	0
0	1	0	0	-3	-1	0	3	1	6
0	0	1	0	-1	0	0	1	0	1
0	0	0	1	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$
0	0	0	0	$k+\frac{5}{3}$	$\frac{5}{3}$	1	$-\left(k+\frac{5}{3}\right)$	$-\frac{5}{3}$	$4k-\frac{20}{3}$

Fig. 5.2

It is given that the tableau in Fig. 5.2 does not give an optimal solution to the LP problem.

(b) Using the tableau in Fig. 5.2 as the starting point for the second stage,

- reduce the tableau, in the Printed Answer Booklet, so that the second stage can be started,
- carry out one iteration of the second stage of the two-stage simplex method, using an entry in the s_2 column as the pivot element. **[4]**

It is given that an optimal solution to the LP problem is not found after one iteration of the second stage of the two-stage simplex method.

(c) Determine the range of values of k. **[3]**

(d) Given that the y-coordinate of the optimal vertex of R is $\frac{12}{7}$, determine the value of k. **[2]**

END OF QUESTION PAPER