OCR
Oxford Cambridge and RSA

Tuesday 18 June 2019 – Morning

A Level Further Mathematics B (MEI)

Y433/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes

7769828998

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **12** pages.

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Answer **all** the questions.

1 Fig. 1 shows a network. The weights on the arcs are distances.

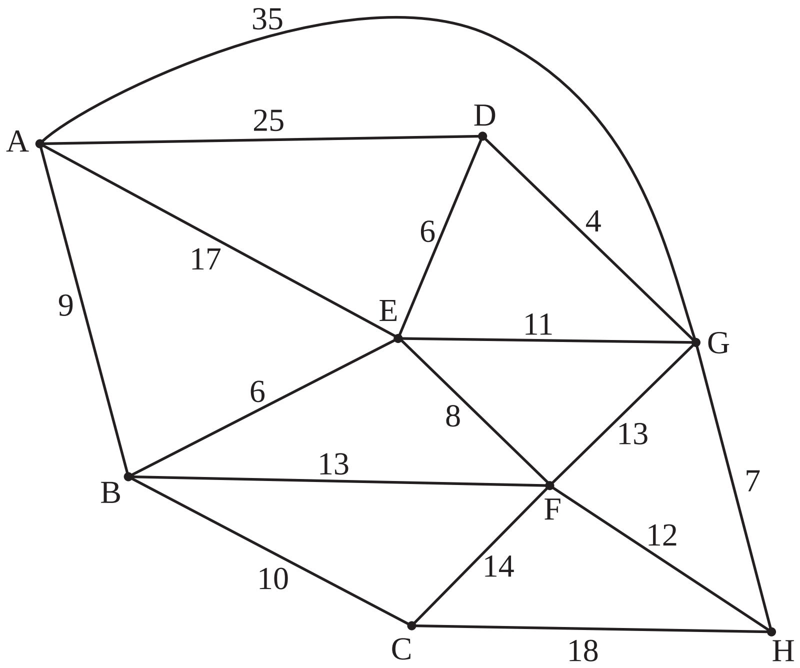

Fig. 1

(a) Apply Dijkstra's algorithm to the copy in the Printed Answer Booklet to find the shortest path from A to H.

State each of the following

- the shortest path and
- its length. **[6]**

The weights on the arcs in Fig. 1 are listed in descending order in the Printed Answer Booklet.

(b) Use the first fit decreasing algorithm to pack these weights into bins that have a capacity of 50. **[2]**

A list of *n* weights, where the weights are arranged in descending order, are to be packed into bins using the first fit decreasing algorithm. The number of comparisons is used as a measure of the complexity of the first fit decreasing algorithm.

(c) Determine, in the worst case, the complexity of the first fit decreasing algorithm in terms of *n*. **[3]**

2 The table in Fig. 2.1 lists the duration (in minutes) and immediate predecessors for each activity in a project.

Activity	Duration (minutes)	Immediate predecessors
A	25	–
B	20	–
C	35	–
D	15	A
E	25	A, B, C
F	25	A, B, C
G	30	C
H	20	E
I	25	E, F, G

Fig. 2.1

(a) Draw an activity network, using activity on arc, to represent the project. **[3]**

(b)
- Carry out a forward pass and a backward pass through the activity network, showing the early event time and the late event time at each vertex of your network.
- List the critical activities. **[5]**

The resource histogram in Fig. 2.2 shows the number of workers required when each activity begins at its earliest possible start time. It is given that each activity requires at least one worker and that when an activity is started it must be completed without interruption.

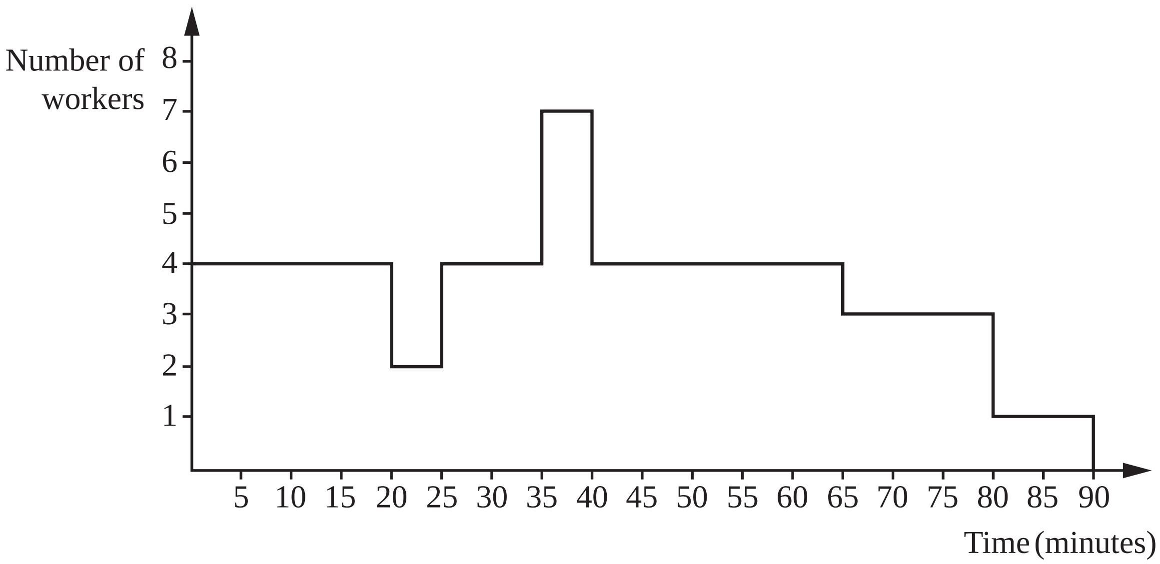

Fig. 2.2

(c) Complete the table in the Printed Answer Booklet showing the number of workers required for each activity in the project. **[3]**

(d) Determine the shortest time in which the project can be completed when only four workers are available. **[2]**

3 Fig. 3 represents a system of pipes through which fluid flows continuously from a source node, S, to a sink node, T.
The weight on each arc shows the capacity of the corresponding pipe, in litres per minute.

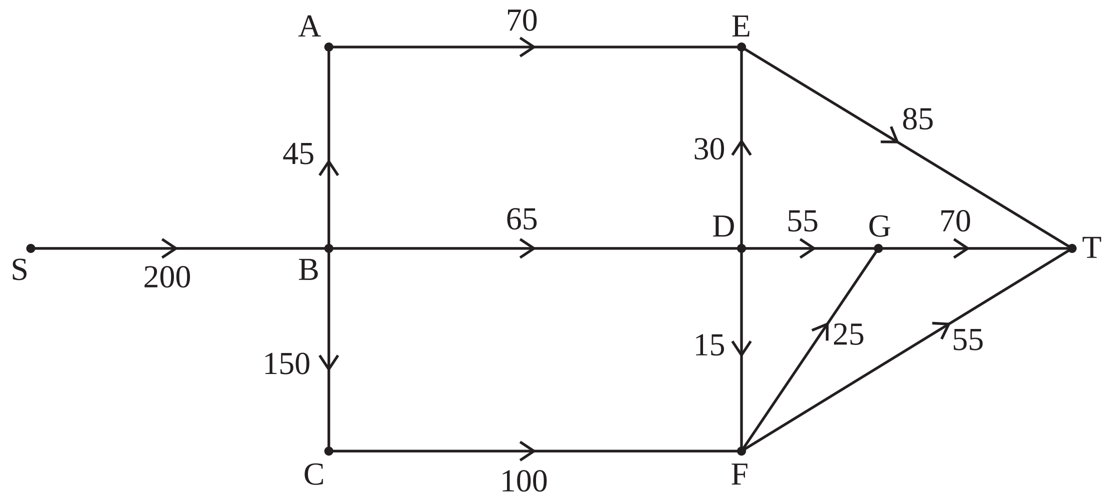

Fig. 3

(a) **(i)** The cut α partitions the vertices into sets {S, B, C}, {A, D, E, F, G, T}.
Calculate the capacity of cut α. **[1]**

(ii) The cut β partitions the vertices into sets {S, A, B, C, E, F}, {D, G, T}.
Calculate the capacity of cut β. **[1]**

(b) Using only the capacities of cuts α and β state what can be deduced about the maximum possible flow through the system. **[1]**

(c) Use the diagram in the Printed Answer Booklet to show how a flow of 190 litres per minute can be achieved. **[2]**

(d) Use a suitable cut to prove that this is the maximum possible flow through the system. **[2]**

An extra pipe is opened linking S to A. Let the capacity of this pipe be x litres per minute.

(e) Find, in terms of x where necessary, the maximum possible flow through the system. **[3]**

4 The table in Fig. 4.1 shows the unit cost, in hundreds of pounds, of transporting goods from each of four suppliers, A, B, C and D to each of three depots, X, Y and Z.
The margins of the table show the stock at each supplier and the demand at each depot.

	Demand	24	12	13
Stock		X	Y	Z
15	A	13	12	11
12	B	15	8	10
8	C	11	12	14
14	D	11	13	12

Fig. 4.1

The following LP formulation can be used to find the minimum total cost of delivering all the required stock.

Minimise $13AX + 12AY + 11AZ + 15BX + 8BY + 10BZ + 11CX + 12CY + 14CZ$
$+ 11DX + 13DY + 12DZ$

Subject to
$AX + AY + AZ = 15$
$BX + BY + BZ = 12$
$CX + CY + CZ = 8$
$DX + DY + DZ = 14$
$AX + BX + CX + DX = 24$
$AY + BY + CY + DY = 12$
$AZ + BZ + CZ + DZ = 13$

(a) Explain the purpose of each of the following lines from the LP formulation.

(i) Minimise $13AX + 12AY + \ldots + 12DZ$. **[2]**

(ii) $AX + BX + CX + DX = 24$. **[1]**

The LP was run in an LP solver and the output is shown in Fig. 4.2.

VARIABLE	VALUE
AX	2.000000
AY	0.000000
AZ	13.000000
BX	0.000000
BY	12.000000
BZ	0.000000
CX	8.000000
CY	0.000000
CZ	0.000000
DX	14.000000
DY	0.000000
DZ	0.000000

Fig. 4.2

(b) **(i)** Interpret the output to give a solution to the transportation problem. **[1]**

(ii) Find the minimum total cost of delivering all the required stock. **[1]**

It is later found that the amount of stock held by supplier B is 16. All other suppliers have the same level of stock as before and there are no changes to the demand at any of the three depots.

Supplier B says that the only required change in the LP formulation is that the line BX + BY + BZ = 12 needs to be replaced with BX + BY + BZ = 16.

(c) Explain why supplier B is incorrect. **[1]**

(d) State the effect this change for supplier B has on

(i) the objective function in the original LP formulation, **[1]**

(ii) the constraints in the original LP formulation. **[1]**

The modified LP problem (where the amount of stock held by supplier B is increased from 12 to 16) is run in the same LP solver and the minimum total cost of delivering all the required stock is found to be £50 300. The only differences from the solution to the original LP problem shown in Fig. 4.2 is in the amount of stock transported from suppliers A and B to depot Z.

(e) Determine the amount of stock for the modified LP problem that is now transported from suppliers A and B to depot Z. **[3]**

5 The initial tableau for a two-stage simplex solution for a maximisation LP problem is shown in Fig. 5.1.

Q	P	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	RHS
1	0	3	1	2	0	–1	–1	0	0	0	50
0	1	–2	–3	$-k$	0	0	0	0	0	0	0
0	0	1	3	1	1	0	0	0	0	0	30
0	0	0	2	1	0	–1	0	0	1	0	20
0	0	3	–1	1	0	0	–1	0	0	1	30
0	0	1	1	2	0	0	0	1	0	0	40

Fig. 5.1

(a) Formulate the information given in Fig. 5.1 as an LP problem by

- Stating the objective function for the original LP problem and
- listing the constraints as simplified inequalities with integer coefficients. **[3]**

(b) Show how the first numerical row of Fig. 5.1 was formed. **[2]**

After three iterations of the first stage of the two-stage simplex method the tableau shown in Fig. 5.2 was produced.

Q	P	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	RHS
1	0	0	0	0	0	0	0	0	–1	–1	0
0	1	0	0	0	$\frac{5}{2}-k$	$\frac{7}{3}-\frac{5}{3}k$	$\frac{1}{6}-\frac{1}{3}k$	0	$-\frac{7}{3}+\frac{5}{3}k$	$-\frac{1}{6}+\frac{1}{3}k$	$\frac{70}{3}+\frac{40}{3}k$
0	0	1	0	0	$\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{6}$	0	$-\frac{2}{3}$	$\frac{1}{6}$	$\frac{20}{3}$
0	0	0	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	0	$-\frac{1}{3}$	$-\frac{1}{6}$	$\frac{10}{3}$
0	0	0	0	1	–1	$-\frac{5}{3}$	$-\frac{1}{3}$	0	$\frac{5}{3}$	$\frac{1}{3}$	$\frac{40}{3}$
0	0	0	0	0	1	$\frac{7}{3}$	$\frac{2}{3}$	1	$-\frac{7}{3}$	$-\frac{2}{3}$	$\frac{10}{3}$

Fig. 5.2

(c) Explain how the tableau in Fig. 5.2 shows that the first stage has been completed. **[2]**

It is given that the tableau in Fig. 5.2 does not give an optimal solution to the LP problem.

(d) Using the tableau in Fig. 5.2 as the starting point for the second stage

- reduce the tableau so that the second stage can be started and
- carry out one iteration of the second stage of the two-stage simplex method, using an entry in the s_2 column as the pivot element. **[4]**

Given that an optimal solution to the LP problem is found after one iteration of the second stage of the two-stage simplex method,

(e) **(i)** determine the range of values of k, **[2]**

(ii) state the optimal values of x, y and z and give, in terms of k, the corresponding maximum value of P. **[2]**

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