**GCE**

**Further Mathematics B (MEI)**

**Y431/01: Mechanics minor**

Advanced GCE

**Mark Scheme for Autumn 2021**

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

**Annotations and abbreviations**

| Annotation in scoris | Meaning |
|---|---|
| ✓ and ✗ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| E | Explanation mark 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| BP | Blank page |
| Highlighting | |
| | |
| **Other abbreviations in mark scheme** | **Meaning** |
| E1 | Mark for explaining a result or establishing a given result |
| dep* | Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark. |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| AG | Answer given |
| awrt | Anything which rounds to |
| BC | By Calculator |
| DR | This indicates that the instruction **In this question you must show detailed reasoning** appears in the question. |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| **1** | **(a)** | | $\mathrm{MLT}^{-2}$ | **B1** | **1.2** | | |
| | | | | **[1]** | | | |
| | **(b)** | | $[\mathrm{RHS}] = \dfrac{\mathrm{M}\left(\mathrm{LT}^{-1}\right)^2}{\mathrm{L}}$ | **M1** | **3.4** | Using given formula with $[m] = \mathrm{M}$ and $[r] = \mathrm{L}$ and $[v] = \mathrm{LT}^{-1}$ | |
| | | | $= ML^2T^{-2}L^{-1} = MLT^{-2} = [LHS]$ | **A1** | **2.2a** | must see $(LT^{-1})^2$ expanded | |
| | | | | **[2]** | | | |
| | **(c)** | | $1.1^2 \div 0.9$ | **M1** | **1.1** | Using correct formula with 1.1 and 0.9 | |
| | | | $= 1.34444\ldots$ so 34.4% | **A1** | **2.2b** | | $34.44444\ldots$ |
| | | | | **[2]** | | | |
| | **(d)** | | $1016 \times 1609 \div 60^2$ | **M1** | **1.1** | Condone denominator which is not squared e.g $60$ *or* $60^3$ for the M mark | |
| | | | $= 454\,\mathrm{N}$ | **A1** | **1.1** | | $454.0955\ldots$ |
| | | | | **[2]** | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| **2** | | | Let the components of the force at A be $F_A$ (parallel to floor) and $R_A$ (parallel to wall). | | | | |
| | | | $F_A = 20$ | **B1** | **1.1** | | |
| | | | $R_A = \sqrt{70^2 - 20^2} = \sqrt{4500} = 30\sqrt{5}$ | **M1** | **3.1b** | Using Pythagoras to find the normal contact force at A | |
| | | | $\Rightarrow W = R_A = 30\sqrt{5}$ or 67 | **A1** | **1.1** | Accept exact or to at least 2 sf | 67.082039… |
| | | | $\mu = \dfrac{F_A}{R_a} = \dfrac{20}{30\sqrt{5}} = \dfrac{2}{3\sqrt{5}} = \dfrac{2\sqrt{5}}{15}$ or 0.30 | **B1** | **3.4** | Using $F = \mu R$ - accept any equivalent exact form or 0.30 (2 sf or better) | 0.29814… $\dfrac{2\sqrt{5}}{15}$ |
| | | | e.g. Taking moments about A: $Wa\cos\theta = F_B\,2a\sin\theta$ | **M1\*** | **3.3** | Taking moments about A (or B etc.) – correct number of terms. Allow cos/sin errors but must reflect ratio of distances. | |
| | | | $\Rightarrow \tan\theta = \dfrac{3}{4}\sqrt{5}$ | **M1dep\*** | **1.1** | Substituting $F_A = 20$ and their value for $W$ and then obtain a value for tan | |
| | | | $\Rightarrow \theta = 59°$ | **A1** | **1.1** | 2 sf or better | 59.19301… |
| | | | | **[7]** | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| **3** | | | | | | | |
| | | | $P = 65gv$ | **B1** | **3.1b** | Use of $P = Fv$ (either one) | |
| | | | $P = 40g(v+3)$ | **B1** | **1.1** | | |
| | | | $65gv = 40g(v+3)$ | **M1** | **3.4** | Equating their two expressions for $P$ – with at least one correct equation | |
| | | | $v = 4.8 \, ms^{-1}$ | **A1** | **1.1** | | |
| | | | $P = 3060 \, W$ | **A1** | **1.1** | or 3.06 kW (but must state kW in this case) | 3057.6 |
| | | | | **[5]** | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 4 | (a) | | $\frac{1}{2}m \times 7.2^2 = mgh$ | M1 | 3.4 | Must use energy method as directed in question. | |
| | | | $h = 2.64489\ldots$ so maximum height is 4.24 m | A1 | 1.1 | | 4.2448979… |
| | | | | [2] | | | |
| | (b) | | Let the amount of work done per metre against air resistance be $W$ | | | | |
| | | | $\frac{1}{2}m \times 7.2^2 - 2.5W = 2.5mg$ | M1 | 3.1b | Work-energy principle. All three terms present. Condone sign errors. | |
| | | | $W = 0.568m$ | A1 | 1.1 | **AG** – sufficient working must be shown | |
| | | | | [2] | | | |
| | (c) | | Let $v$ be the speed of the ball just before impact with ground | | | | |
| | | | $4.1mg - 4.1W = \frac{1}{2}mv^2$ | M1 | 3.3 | Work-energy principle: all three terms present (or all four if starting from when ball leaves the hand). | |
| | | | | B1 | 1.1 | $4.1W$ with $W = 0.568m$ | |
| | | | $\frac{1}{2}mv^2 = 37.8512m \Rightarrow v = 8.70\,\mathrm{ms^{-1}}$ | A1 | 1.1 | | 8.7007126… |
| | | | | [3] | | | |
| | (d) | | Let $V$ be the speed of the ball just after impact with ground. | | | | |
| | | | $\frac{1}{2}mV^2 - 2.8W = 2.8mg$ | M1 | 3.3 | Work-energy principle - all three terms present | |
| | | | $V = 7.6197\ldots$ | A1 | 1.1 | | |
| | | | Coefficient of restitution $= \frac{7.6197\ldots}{8.7007\ldots} = 0.876$ | A1ft | 3.4 | FT their answer to **(c)** | 0.87576318… |
| | | | | [3] | | | |
| | (e) | | $-mv + 12 = mV$ | M1 | 3.3 | Using impulse = change in momentum. Correct number of terms but allow sign errors. FT their values for $v$ and $V$ | |
| | | | $m = \frac{12}{v+V} = 0.735$ | A1 | 1.1 | | |
| | | | | [2] | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| **5** | **(a)** | | Let the coordinates of centre of mass be $\left(\overline{x}, \overline{y}, \overline{z}\right)$ | | | | |
| | | | $\overline{z} = 1.5$ | **B1** | 1.1 | | |
| | | | $210 \begin{pmatrix} \overline{x} \\ \overline{y} \end{pmatrix} = 120 \begin{pmatrix} 2 \\ 5 \end{pmatrix} + 90 \begin{pmatrix} 9 \\ 8.5 \end{pmatrix}$ | **M1** | 1.1 | Any correct equation, using correct ratio of masses of the constituent parts. | e.g. could also have $28 \begin{pmatrix} 2 \\ 3.5 \end{pmatrix} + 42 \begin{pmatrix} 7 \\ 8.5 \end{pmatrix}$ |
| | | | $\overline{x} = 5$ | **A1** | 1.1 | | |
| | | | $\overline{y} = 6.5$ | **A1** | 1.1 | | |
| | | | | **[4]** | | | |
| | **(b)** | | $\theta_{\min} = \arctan \frac{1}{6.5}$ or $\theta_{\max} = \arctan \frac{5}{6.5}$ | **M1** | 3.1b | $\tan\theta = \frac{1}{y}$ or $\tan\theta = \frac{\overline{x}}{y}$ - condone reciprocal fractions for this mark | |
| | | | $\theta_{\min} = 8.75$ | **A1** | 1.1 | | 8.746162… |
| | | | $\theta_{max} = 37.6$ | **A1** | 1.1 | | 37.568592… |
| | | | | **[3]** | | | |
| | **(c)** | | Let the thresholds for breaking equilibrium be sliding and toppling be $P_s$ and $P_t$ | | | | |
| | | | $R + P_s \sin 30° = mg \Rightarrow R = mg - P_s \sin 30°$ | **M1\*** | 3.3 | Resolve vertically – correct number of terms (allow sin/cos errors) | |
| | | | $P_s \cos 30° = F_{max} = \mu \left( mg - P_s \sin 30° \right)$ | **M1\*** | 3.4 | Resolve horizontally and use of $F = \mu R$ | |
| | | | $P_s = \dfrac{\mu mg}{\cos 30° + \mu \sin 30°}$ | **A1** | 1.1 | oe | |
| | | | $14 P_t \sin 30° + 10 P_t \cos 30° = 5mg$ | **M1\*** | 3.1b | Moment – correct number of terms. Condone sin/cos errors (and sign errors) | |
| | | | $P_t = \dfrac{5mg}{14 \sin 30° + 10 \cos 30°}$ | **A1** | 1.1 | oe | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| | | | $P_s > P_t$, so $\dfrac{\mu mg}{\cos 30° + \mu \sin 30°} > \dfrac{5mg}{14\sin 30° + 10\cos 30°}$ <br><br> $\mu mg\left(14\sin 30° + 10\cos 30°\right) > 5mg\left(\cos 30° + \mu \sin 30°\right)$ <br><br> $\mu\left(9\sin 30° + 10\cos 30°\right) > 5\cos 30°$ | M1dep* | 2.1 | Dependent on all previous M marks | |
| | | | $\mu > \dfrac{5}{9\tan 30° + 10}$ <br><br> $\mu > \dfrac{5}{3\sqrt{3} + 10}$ <br><br> So $\mu_{min} = \dfrac{5}{3\sqrt{3} + 10} = \dfrac{50 - 15\sqrt{3}}{73}$ | A1 | 2.2a | Accept any equivalent exact form, or 0.329 (or better) <br><br> Need not be explicitly stated if inequality is present. | 0.3290306… |
| | | | | **[7]** | | | |
| | **(d)** | | Either | | | | |
| | | | If the angle $\theta$, were smaller then $\tan\theta$ would be smaller … | M1 | 2.1 | | |
| | | | … so $\mu_{min}$ would be larger. | A1 | 2.2a | | |
| | | | Or | | | | |
| | | | If the angle were smaller then $P$ would have a larger horizontal component **and** a smaller anticlockwise turning effect … | M1 | 2.1 | | |
| | | | … so $\mu_{min}$ would be larger. | A1 | 2.2a | | |
| | | | | **[2]** | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| **6** | **(a)** | | Let the block move $x$ metres before coming to rest. | | | | |
| | | | $\frac{1}{2}mv^2 - Fx = 0$ | **M1** | **1.1** | Work-energy principle – correct number of terms | |
| | | | Since block is sliding, $F = F_{\max} = \mu mg$ | **M1** | **3.4** | Use of $F = \mu R$ | |
| | | | $\frac{1}{2}mv^2 - \mu mgx = 0 \Rightarrow x = \dfrac{v^2}{2\mu g}$ | **A1** | **1.1** | N.B. answer given. | |
| | | | Alternative method: | | | | |
| | | | Since block is sliding, $F = F_{\max} = \mu mg$ | **M1** | **3.4** | | |
| | | | $a = -\dfrac{\mu mg}{m} = -\mu g$ | **M1** | **1.1** | | |
| | | | So $0^2 = v^2 + 2\,(-\mu g)\,x \Rightarrow x = \dfrac{v^2}{2\mu g}$ | **A1** | **1.1** | | |
| | | | | **[3]** | | | |
| | **(b)** | | $mu = mv_{\mathrm{S}} + 8mv_{\mathrm{B}}$ | **M1\*** | **3.3** | Conservation of linear momentum – correct number of terms (allow sign errors) | |
| | | | $v_{\mathrm{B}} - v_{\mathrm{S}} = 0.8u$ | **M1\*** | **3.3** | Newton's experimental law – must be consistent with CoLM (so signs of $v_S$ in the two equations must be different) | |
| | | | $u = v_s + 8\,(0.8u + v_s) = 9v_s + 6.4u$ | **M1dep\*** | **3.4** | Attempt at eliminating either variable – dependent on both previous M marks | |
| | | | $\Rightarrow v_{\mathrm{S}} = -0.6u$ and $v_{\mathrm{B}} = 0.2u$ | **A1** | **1.1** | Ignore incorrect signs. | |
| | | | S has speed $0.6u$ towards the wall<br>B has speed $0.2u$ away from the wall | **A1** | **2.4** | Both correct. Accept other appropriate descriptions of direction (e.g. 'opposite to original direction of travel', etc.) | |
| | | | | **[5]** | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| **(c)** | | | Each time S returns for impact it has $\frac{3}{5}$ the speed it had previously; therefore after impact, the block will have also have $\frac{3}{5}$ the speed it had just after the previous impact … | **M1** | **3.5a** | Argument using their value of $v_S$ from **(b)** – must relate this value to B | |
| | | | … so by part **(a)**, the block will move only $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$ of the distance moved after the previous impact. | **A1** | **2.2a** | Must reference result in part **(a)** (or convincingly explain where the squaring comes from) | May consider that the ratio of successive distances travelled by B is $v^2$ |
| | | | | **[2]** | | | |
| **(d)** | | | After first impact, speed of block is $11.2 \times 0.2 = 2.24$ | **B1ft** | **3.4** | Follow through their value of $v_B$ from **(b)** | |
| | | | So $x_1 = \dfrac{2.24^2}{2 \cdot \frac{1}{7} \cdot (9.8)} = 1.792$ | **M1** | **1.1** | Using given result in **(a)** to find distance travelled after first collision | |
| | | | $\displaystyle\sum_{n=1}^{\infty} x_n = \dfrac{1.792}{1 - \frac{9}{25}} = 2.8\,(\text{m})$ | **A1** | **2.2a** | **AG** Use of infinite sum of a GP to derive required result | |
| | | | | **[3]** | | | |

**OCR (Oxford Cambridge and RSA Examinations)**
**The Triangle Building**
**Shaftesbury Road**
**Cambridge**
**CB2 8EA**

**OCR Customer Contact Centre**

**Education and Learning**
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

**www.ocr.org.uk**

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored