

GCE

Further Mathematics B (MEI)

Y421/01: Mechanics major

Advanced GCE

Mark Scheme for Autumn 2021

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0,B1	Independent mark awarded 0, 1
Е	Explanation mark 1
SC	Special case
٨	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in	Meaning
mark scheme	
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
a wrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

Q	uestio	n Answer	Marks	AOs	Guidance
1		J = 0.25(4.2 - (-5))	M1	3.3	Use of Impulse = change in momentum
		J = 0.02F	M1	3.3	Use of Impulse = Ft
		$F = \frac{2.3}{10.0} = 115 (\text{N})$	A1	1.1	cao
		$\frac{1}{0.02}$			
			[3]		
2		$10m\bar{x} = 1(3m) + 2(5m) + 5(2m)$	M1	1.1	Use of $\overline{x} \sum m_i = \sum x_i m_i$
		x = 2.3	A1	1.1	cao
		$10my^{-} = 2(3m) + (-2)(5m) + 3(2m)$	M1	1.1	Use of $\overline{y} \sum m_i = \sum y_i m_i$
		$\overline{y} = 0.2$	A1	1.1	cao
			[4]		
3	(a)	T=4g	B1	1.1	Resolve vertically (possibly implied by subsequent working)
		$\frac{\lambda (0.02)}{0.3} = 4g$	M1	3.3	Use of Hooke's law with their 4g
		$\lambda = 588(N)$	A1	1.1	cao oe e.g. 60 <i>g</i>
			[3]		
3	(b)	e.g. spring stretched beyond its elastic limit	B1	2.2b	oe (any correct equivalent statement for
		e.g. Hooke's law no longer applies			why the extension of the spring may not
			[1]		be 0.1 m)
			[1]		

Question	Answer	Marks	AOs	Guidance	
4	DR				
	$A = \int_{1} (4 - x_{2}) - 3 \int_{0} \frac{x}{1} dx = \left[4x - \frac{1}{3} x^{3} - 2x^{\frac{2}{2}} \right]^{1}$	M1*	2.1	Correct integral expression for the area and attempt to integrate (at least two terms correct)	Ignore limits for first two M marks
	$A = 4 - \frac{1}{3} - 2 = \frac{5}{3}$	A1	1.1		SC M1 A0 if correct integral and value seen but with no
	$A\overline{x} = \int_0^1 4x - x^3 - 3x^2 dx = \left[2x^2 - \frac{1}{4}x^4 - \frac{6}{5}x^2 \right]_0^1$	M1*	1.1	Correct integral expression for $A\bar{x}$ and attempt to integrate (at least two terms correct)	intermediate working
	$Ax = 2 - \frac{1}{4} - \frac{6}{5} = \frac{11}{20}$	A1	1.1		SC M1 A0 if correct integral and value seen but with no intermediate working
	$\pi = \frac{A\pi}{A} = \frac{\frac{11}{20}}{\frac{5}{3}}$	M1dep*	1.1	Correct use of $x = \frac{Ax}{A}$	Dependent on both previous M marks
	$=\frac{33}{100}$	A1	2.2a	oe	This mark can be awarded even if the two previous A marks were not awarded
		[6]			

Qı	estion	Answer	Marks	AOs	Guidance	
5		Let w_A and w_B be the horizontal components of the				
		velocity of A and B after collision				
		$w_{\rm B} = 2.5$	B1	1.2		
			M1	3.3	Use of conservation of linear momentum (parallel to the line of centres) – correct number of terms	
		$2(6) + 4(0) = 2w_A + 4(2.5)$	A1	1.1	Allow with $w_{\rm B}$ instead of 2.5	For reference: $w_A = 1$
			M1	3.3	Use of Newton's experimental law (parallel to the line of centres) – correct number of terms	
		$w_{\rm A} - 2.5 = -e(6 - 0)$	A1	1.1	Use of NEL must be consistent with CLM – allow with w_B instead of 2.5 and possibly their w_A	
		e = 0.25	A1 [6]	1.1		

Q	uestio	n	Answer	Marks	AOs	Guidance
6	(a)		$[F] = MLT^{-2}$	B1	1.2	
			-	[1]		
6	(b)		$[G] = M^{-1}L^3T^{-2}$	B1		May use $F = \frac{Gm_1m_2}{d^2}$ to obtain the
						$\frac{1}{d^2}$
						dimensions of G
			1	[1]		
6	(c)		$G = \left(6.67 \times 10^{-11}\right) \times 0.454 \times \frac{1}{\left(0.305\right)^3}$	M1	3.1a	SC B1 for
			$(0.305)^3$			$G = \left(6.67 \times 10^{-11}\right) \times \frac{1}{0.454} \times \left(0.305\right)^3$
						$=4.17\times10^{-12}$
			$G = 1.07 \times 10^{-9} \text{ (lb}^{-1} \text{ ft}^3 \text{ s}^{-2}\text{)}$	A1	1.1	awrt 1.07×10 ⁻⁹
				[2]		
6	(d)		$\left\lceil \frac{kGM}{r} \right\rceil_{1} = \frac{\left(M^{-1}L^{3}T^{-2}\right)M}{L}$	M1	2.1	Attempt to calculate the dimension of
			$\left \begin{array}{cc} r \end{array} \right _{\Gamma} = \frac{\Gamma}{\Gamma}$			either $\frac{kGM}{}$ or its square root with
						r
						$\begin{bmatrix} k \end{bmatrix} = 1$ and two other terms correct
			$\left \sqrt{\frac{kGM}{r}}\right = LT^{-1}$	A1	1.1	$ \operatorname{Or}\left[\frac{kGM}{r}\right] = L^{2}T^{-2} $
			$[v] = LT^{-1}$ so the formula is dimensionally consistent	A1	2.2a	Or allow showing consistency for
			[/] 21 so the formation dimensionally consistent			$v^2 = \frac{kGM}{r}$
						$V = \frac{1}{r}$
				[3]		

Ques 6 (e	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,		I Wiarks	AOs	l (Juidance	
	<i>a</i>)	Answer	Marks M1	3.4	Guidance	
	e)	$11186 = \sqrt{\frac{k(6.67 \times 10^{-11})(5.97 \times 10^{24})}{6.371000}}$	MII	3.4		
		$k \approx 2$	A1	1.1		k = 2.0019677
		2667,10-11 (620,1023)	M1	1.1		
		$v = \sqrt{\frac{2(6.67 \times 10^{-11})(6.39 \times 10^{23})}{3389500}}$				
		$v = 5015 (\text{m s}^{-1})$	A1	2.2a	Allow to 3 sf or better (allow 5015 to	If using
					5017 inclusive)	k = 2.0019677
						expect to see
						5017.346122
			[4]			
7 (a	a)	Driving force of engine is $\frac{kmg}{v}$	B1	1.1		
		kmg dv	M1	3.3	Use of N2L, correct number of terms,	
		$\frac{kmg}{v} - mg = mv \frac{\mathrm{d}v}{\mathrm{d}x}$			allow D (oe) for $\frac{kmg}{}$ and a (oe) for the	
		,			and u (oc) for the	
					acceleration	
		$\int_{V} dv \int_{V} dv \int_{V} dv$	A1	2.2a	AG – sufficient working must be shown	
		$kg - gv = v^2 \frac{dv}{dx} \Rightarrow v^2 \frac{dv}{dx} = (k - v) g$			as answer given	
		th th	[3]		Č	
			[0]			

Que	estior	Answer	Marks	AOs	Guidance	
7 ((b)	$gx = k^{2} \ln \left(\frac{k}{k-v}\right) - kv - \frac{1}{2}v^{2}$ $x = 0, v = 0 \Rightarrow g(0) = k^{2} \ln \left(\frac{k}{k-0}\right) - k(0) - \frac{1}{2}(0)^{2} \text{ so}$	B1	1.1		
		initial conditions are consistent with given equation $g\frac{dx}{dv} = k^{2} \begin{bmatrix} 1 \\ \frac{k}{k-v} \end{bmatrix} k \binom{k-v}{k-v}^{2} - k-v$	M1*		Attempt to differentiate using chain rule cao oe e.g. $g = k^{2} \left(\frac{k - v}{k} \right) \left(\frac{-k \left(-\frac{dv}{dx} \right)}{(k - v)^{2}} \right) - k \frac{dv}{dx} - v \frac{dv}{dx}$	Or equivalent (e.g. solving using separation of variables)
		$g\frac{\mathrm{d}x}{\mathrm{d}v} = \frac{-kv + v^2 - k^2 + kv + k^2}{(k - v)}$	M1dep*	1.1	Correct method to obtain an expression for $\frac{dx}{dv}$ as a single fraction or as a single fraction with $\frac{dv}{dx}$	
		$v^{2} = g (k - v) \frac{dx}{dv} \Rightarrow v^{2} \frac{dv}{dx} = (k - v) g$	A1 [5]	2.2a	e.g. $g = \left(\frac{k^2 - k^2 + kv - kv + v^2}{k - v}\right) \frac{dv}{dx}$ AG – sufficient working required as answer given	

Q	uestio	n	Answer	Marks	AOs	Guidance
7	(c)		Work done by engine is kmgt	B1	1.1	
			$kgmt = \frac{1}{2}mV^2 + mgx$	M1*	3.3	Use work-energy principle – correct number of terms
			$kgt = \frac{1}{2}V^2 + k^2 \ln\left(\frac{k}{k - V}\right) - kV - \frac{1}{2}V^2$	M1dep*	3.4	Use given result from (b) in work-energy equation to eliminate <i>x</i>
			$kgmt = \frac{1}{2}mV^{2} + mgx$ $kgt = \frac{1}{2}V^{2} + k^{2}\ln\left(\frac{k}{k - V}\right) - kV - \frac{1}{2}V^{2}$ $kgt = k^{2}\ln\left(\frac{k}{k - V}\right) - kV \Rightarrow t = \frac{k}{g}\ln\left(\frac{k}{k - V}\right) - \frac{V}{g}$	A1	2.2a	AG – sufficient working required as answer given
				[4]		SC if correctly found by solving $\frac{kmg}{v} - mg = m\frac{dv}{dt} \text{ this can score } 3/4 \text{ max.}$
8	(a)			B1	1.2	All remaining forces adding on correctly
						(with arrows to indicate directions) to the
				F11		figure in the Printed Answer Booklet
8	(b)			[1] M1*	3.3	Resolve horizontally and vertically
						(correct number of terms in both equations)
			$F_{\rm D} + R_{\rm C} = W$	A1	1.1	Where $R_{\rm C}$ is the normal contact force at
			$R_{\rm D} = F_{\rm C}$			C, etc.
			$F_{\rm D} = \frac{1}{3} R_{\rm D}$ and $F_{\rm C} = \frac{1}{3} R_{\rm C}$	B1	3.4	Correct use of $F = \mu R$ at C and D
			$\frac{1}{3}F_{C} + R_{C} = W \Rightarrow \frac{1}{9}R_{C} + R_{C} = W$	M1dep*	3.4	Combine results to get an equation in $R_{\rm C}$ only
			$R_{\rm C} = \frac{9}{10}W$	A1	1.1	
				[5]		

Q	uestior	Answer	Marks	AOs	Guidance	
8	(c)		M1*	3.1b	Taking moments about D (or any other equivalent point) – correct number of terms	
		$(r + h\sin\theta)W + (r + 2h\cos\theta)F_{\rm C} = (r + 2h\sin\theta)R_{\rm C}$	A1	1.1	oe	
		$(r + h\sin\theta)W + (r + 2h\cos\theta)F_{C} = (r + 2h\sin\theta)R_{C}$ $(r + h\sin\theta)W + (r + 2h\cos\theta)\left(\frac{3}{10}W\right)$ $= (r + 2h\sin\theta)\left(\frac{9}{10}W\right)$	M1dep*	3.4	Substitute expressions for $F_{\rm C}$ and $R_{\rm C}$	
		$r = h(2\sin\theta - 1.5\cos\theta)$	A1	1.1		
		$2h\sin\theta - 1.5h\cos\theta > 0$	M1	2.3	Setting their expression for $r > 0$	
		$4\sin\theta - 3\cos\theta > 0 \Rightarrow \tan\theta > \frac{3}{4}$	A1	2.2a	AG	
			[6]			

				1 37 3	1 40		1
_	uestio		nswer	Marks	AOs	Guidance	
9	(a)	$\ddot{x} = -g \sin \alpha , \ddot{y} = -g \cos \alpha$		B1	2.1		
				M1*	3.4	Attempt to integrate (twice) and use of	
						initial conditions	
		$\dot{x} = 5\cos\theta - gt\sin\alpha$, $\dot{y} =$	$=5\sin\theta-gt\cos\alpha$	A1	1.1		
		$x = 5t\cos\theta - 0.5gt^2\sin\alpha$		A1	1.1	Or M1 for use of s = ut + 1 at 2 perallel	Similarly M1 A1 for
						Or M1 for use of $s = ut + \frac{1}{2}at^2$ parallel	correct expression for
		$y = 5t\sin\theta - 0.5gt^2\cos\alpha$				to line of greatest slope and then A1 for	y (following SUVAT
						correct expression for x	perpendicular to
						correct expression for x	slope)
		$y = 0 \Rightarrow t = \dots$		M1dep*	3.3	Sets $y = 0$ and solve for t	
				A1	1.1	, , , , , , , , , , , , , , , , , , ,	
		$t = \frac{10\sin\theta}{a\cos\alpha}$		111	1		
		$g \cos \alpha$					
		$(10\sin\theta)$	$(10\sin\theta)^2$	M1	3.4	Substitute expression for <i>t</i> into equation	Dependent on both
		$x = 5 \left(\frac{10\sin\theta}{g\cos\alpha} \right) \cos\theta - 0.5$	$g \left(\frac{1}{g \cos \alpha} \right) \sin \alpha$			for x	previous M marks
		$x = \frac{50\sin\theta}{\cos\theta} (\cos\theta \cos\alpha - \frac{1}{2}\cos\theta \cos\alpha)$		A1	2.2a	AG	
		$\int_{0}^{\infty} x = \frac{\cos\theta \cos\alpha - \cos\theta \cos\alpha}{g\cos^2\alpha}$	$\sin\theta \sin\alpha$				
			`				
		$\Rightarrow OR = \frac{50\sin\theta \cos(\theta + \alpha)}{g\cos^2\alpha}$)				
		$g\cos^2\alpha$					
				[8]			
Ь		L L					<u> </u>

0	uestio	n Answer	Marks	AOs	Guidance	
9	(b)	$\sin\theta\cos(\theta + \alpha) = \frac{1}{2}(\sin(2\theta + \alpha) - \sin\alpha)$	M1	1.1	Use of given identity to re-write numerator from (a) as a difference of two sines	
		$OR = \frac{25}{g \cos^2 \alpha} \left(\sin(2\theta + \alpha) - \sin \alpha \right)$ $R_{\text{max}} = \frac{25}{8 \left(1 - \sin \alpha \right)} \left(1 - \sin \alpha \right)$	A1	1.1	Silies	
		$R_{\text{max}} = \frac{25}{8\left(1 - \sin \alpha\right)} \left(1 - \sin \alpha\right)$	A1	3.1a	Use of correct trig. identity and setting $\sin(2\theta + \alpha)$ equal to 1 – oe e.g. $R = \frac{25}{g(1 + \sin \alpha)}$	R_{max} occurs when $\sin(2\theta + \alpha) = 1$
			[3]			
9	(c)	$\frac{25}{g(1+\sin\alpha)} = 1.8 \text{ or } \frac{25(1-\sin\alpha)}{g(1-\sin^2\alpha)} = 1.8$	M1*	3.4	Setting their expression equal to 1.8	Expression must only contain $\sin \alpha$ terms
		$\frac{25}{g\left(1+\sin\alpha\right)} = 1.8 \Rightarrow \sin\alpha = \dots$	M1dep*	1.1	Attempting to solve for $\sin \alpha$ or α - for reference $\sin \alpha = \frac{184}{441}$ or $\alpha = 24.660053$ (or 0.430399 in radians)	If solving a 3TQ in sine then must solve using a correct method
		$\theta = 45 - 0.5\alpha$	M1	3.1a	Follow through their $lpha$	
		$\theta = 32.7$	A1	1.1		32.6699733 or 0.5701986 (in radians)
			[4]			

Question	Answer	Marks	AOs	Guidance	
10 (a)	•	B1	1.1	Guidance	Note that the reference
10 (11)	[At B,] KE = $\frac{1}{2}mu^2$, PE = 0				level for zero GPE
	-				might be taken at C
	[At θ ,] KE = $\frac{1}{2}mv^2$, PE = $mga(1-\cos\theta)$	B1	1.1		
	_	M1*	3.3	Use of conservation of energy – correct number of terms	
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mga\left(1 - \cos\theta\right)$	A1	1.1	cao	
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mga(1 - \cos\theta)$ $R - mg\cos\theta = \frac{mv^2}{a}$	M1*	3.3	N2L radially with correct number of terms and weight resolved	
	$R - mg \cos\theta = \frac{m}{a} \left(u^2 - 2ga \left(1 - \cos\theta \right) \right)$	M1dep*	3.4	Substitute an expression for v^2	
	$R - mg \cos\theta = \frac{m}{a} \left(u^2 - 2ga \left(1 - \cos\theta \right) \right)$ $R = m \left(3g \cos\theta - 2g + \frac{u^2}{a} \right)$	A1	1.1		
		[7]			

Q	uestio		Marks	AOs	Guidance	
10	(b)	Before collision at C, $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mga$	M1	3.4	Substituting $\theta = \frac{\pi}{2}$ into their	
			A 1	11	conservation of energy equation from (a)	
		After collision at C, speed of P is $e\sqrt{u^2 - 2ga}$	A1	1.1		
		After collision at C, speed of P is $e\sqrt{u^2 - 2ga}$ $\frac{1}{2}mv^{\frac{B}{2}} = mga + \frac{1}{2}m\left(e\sqrt{u - 2ga}\right)^2$	M1	3.1b	Conservation of energy to find an expression for the speed of P at B	Where v_B is the speed of P at B
		$v_{\rm B}^2 = 2ga + e^2\left(u^2 - 2ga\right)$				
		$\int_{B}^{1} mv^{2} - \int_{B}^{1} mv^{2} = Fb$	M1	3.1b	Work-energy principle for motion between B and A	
			M1	2.5	Set $v_A \ge 0$ and substitute for v_B^2	
		$m\left(2ga + e^2\left(u^2 - 2ga\right)\right) - 2bF \ge 0$	A1	2.2a	k need not be stated explicitly	
		$Fb \le mga + \frac{1}{2}me^2u^2 - me^2ga$				
		$\Rightarrow Fb \le \frac{1}{2}m \left[e^2u^2 + 2(1-e^2)ga\right], \text{ so } k=2$				
			[6]			
11	(a)		M1*	3.3	Conservation of linear momentum with correct number of terms	Where v_A is the speed
					correct number of terms	of A after 1 st impact and similarly for v_B
		$4V = 4v_{\rm A} + 3v_{\rm B}$	A1	1.1	cao	, D
			M1*	3.3	Newton's experimental law with correct number of terms	
		$v_{\rm A} - v_{\rm B} = -eV$	A1	1.1	Must be consistent with CLM	
			M1dep*	1.1	Solve the simultaneous equations to find both speeds	
		$v_{\rm A} = \frac{V(4-3e)}{7}$ and $v_{\rm B} = \frac{4V(1+e)}{7}$	A1	1.1		
			[6]			

	noctio	n	Answer	Marks	AOs	Guidance	
	Question 11 (b)		Let θ be the angle subtended by A in time t	Marks	AUS	Guidance	
11	(b)		For A, $t = \frac{r\theta}{\frac{V(4-3e)}{7}}$	M1	3.1b	Use of $s = ut$ with their v_A and $s = r\theta$	Where <i>r</i> is the radius of the circular groove
			For B, $t = \frac{2\pi r + r\theta}{\frac{4V(1+e)}{7}}$	M1	1.1	Use of $s = ut$ with their v_B and $s = 2\pi r + r\theta$	
			$\frac{2\pi + \theta}{4V(1+e)} = \frac{7}{V(4-3e)}$ $\theta = \frac{2\pi (4-3e)}{7e}$	M1	3.4	Equate expressions for t to form an equation in terms of θ , V and e	
			$2\pi(4-3e)$	A1	2.2a	AG	
			$\theta = \frac{2N(1-3\epsilon)}{7a}$				
			16	[4]			
	l	 -	Alternative method	r - 3			
			ALT: $v_B - v_A = \frac{4V(1+e)}{7} - \frac{V(4-3e)}{7} = eV$	M1*		Difference in speeds calculated	
			Time for B to catch up to A is $\frac{2\pi r}{eV}$	M1dep*		Using their eV	Where <i>r</i> is the radius of the circular groove
			$d_{A} = \frac{2\pi r}{eV} {V (4-3e) \choose 7} = \frac{2\pi r}{7e} (4-3e)$	M1		Where d_A is the distance travelled by A	
			$\theta = \frac{2\pi r \left(4 - 3e\right)}{7er} = \frac{2\pi \left(4 - 3e\right)}{7e}$	A1		AG	

O	uestio	n	Answer	Marks	AOs	Guidance	
_	(c) (i)		$3w_{\rm B} + 4w_{\rm A} = \frac{12}{7}V(1+e) + \frac{4}{7}V(4-3e)$	M1*	3.3	CLM correct number of terms using their	Where w_A is the
	(1)		7 7			expressions from (a)	speed of A after the
			$\begin{pmatrix} 4\mathbf{v}(\mathbf{t}_{1}) & 1\mathbf{v}(\mathbf{t}_{2}) \end{pmatrix}$	M1*	3.3	NEL correct number of terms	second collision
			$w - w = -e \left(\frac{4}{7} V (1+e) - \frac{1}{7} V (4-3e) \right)$	1411	3.3	NEE correct number of terms	
			$3w_{\rm B} + 4w_{\rm A} = 4V$ and $w_{\rm B} - w_{\rm A} = -e^2V$	A1	1.1	oe	
				M1dep*	1.1	Solve simultaneously for $w_{\rm B}$	
			$W_{\rm B} = \frac{4}{7}V\left(1 - e^2\right)$	A1	1.1	cao	For reference:
			В 7				$w_{A} = \frac{1}{7}V(4+3e^{2})$
				[5]			
11	(c)		If the collision is perfectly elastic ($e = 1$) B is brought to	B1	3.5a	oe correct statement	
	(ii)		rest by the second collision and A is moving with speed				
			V (which is the situation before the first collision)	[1]			
12	(a)	H	PE = -mg (l + e) (while P is at rest)	B1	1.1	Where e is the extension in the string	Taking the horizontal
	(4)		$1L - mg(t + \epsilon)$ (while 1 is at lest)	21	1,1	where e is the entension in the string	through O as the
							reference level for
							zero GPE
			$EPE = \frac{12mge^2}{2l}$	B1	1.1		
				B. Fed als	2.2	~	
			$\frac{6mge^2}{l} - mg\left(l + e\right) = 0$	M1*	3.3	Conservation of energy with correct number of terms	
			$6e^2 - el - l^2 = 0$	M1dep*	1.1a	Solving three-term quadratic in e	
			$6e^{2} - el - l^{2} = 0$ $(3e + l)(2e - l) = 0$				
			$e = \frac{l}{2} \Rightarrow \text{ length of string is } \frac{1}{2}l + l = \frac{3}{2}l$	A1	2.2a	AG	
				[5]			

Q	uestio	n	Answer	Marks	AOs	Guidance	
12	(b)		$mg - T = m\ddot{x}$	M1	3.3	N2L vertically with correct number of terms	
			$mg - \frac{12mgx}{l} = m\ddot{x}$	M1	3.4	Use of Hooke's law and substitute for <i>T</i> in N2L	
			$\ddot{x} + \frac{12g}{l}x = g$ so $\ddot{x} + \omega^2 x = g$ where $\omega^2 = \frac{12g}{l}$	A1	2.2a	AG	
				[3]			
12	(c)		$x = y + \frac{g}{\omega^2} \Rightarrow y + \omega^2 y = 0$	M1	1.1	Use given substitution to form differential equation in <i>y</i>	
			$y = A\cos\omega t + B\sin\omega t$	A1ft	1.2	Correctly solves their differential equation in <i>y</i>	
			$x = A\cos\omega t + B\sin\omega t + \frac{g}{\omega^2}$	A1	1.1	oe e.g. $x = A\cos\omega t + B\sin\omega t + \frac{l}{12}$	
			$t = 0, x = 0 \Rightarrow A = -\frac{g}{g^2}$	M1	3.4	Use correct initial conditions in their expression for <i>x</i>	
			$\frac{1}{2}mv^2 = mgl$	M1*	3.1b	Use conservation of energy to find speed v_P of P at time $t = 0$	
			$v_{\rm P} = \sqrt{2gl}$	A1	1.1		
			$t = 0, x = \sqrt{2gt} \Rightarrow B = \frac{\sqrt{2gt}}{\omega}$	M1dep*	3.4	Use initial speed in an expression for \dot{x}	
			$x = -\frac{g}{\omega^2}\cos\omega t + \frac{\sqrt{2gT}}{\omega}\sin\omega t + \frac{g}{\omega^2}$	A1	1.1	oe e.g. $x = \frac{l}{12} (1 - \cos \alpha t + 2\sqrt{\sin \alpha t})$	
			$\frac{l}{12} \left(1 - \cos \omega t + 2 \sqrt{\sin \omega t} \right) = 0$	M1	3.1b	Sets $x = 0$ and replaces $\omega^2 = \frac{12g}{l}$	Dependent on all previous M marks
			$\cos \omega t - \sqrt{24} \sin \omega t = 1$ so $k = 24$	A1	2.2a	k need not be stated explicitly	
				[10]			

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