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AS Level Further Mathematics B (MEI)
Y415 Mechanics b
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 15 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)
- Scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 A particle, P, has velocity $\mathbf{v}$ m s^{-1} at time t seconds given by $\mathbf{v} = \begin{pmatrix} 6(t^2-3t+2) \\ 2(1-t) \\ 3(t^2-1) \end{pmatrix}$, where $0 \le t \le 3$.

(i) Show that there is just one time at which P is instantaneously at rest and state this value of t. **[3]**

All the components of velocity need to be 0 for P to be at rest (i.e. v=0). This happens at t=1s.

$6(1^2-3(1)+2)=0$
$2(1-1)=0$
$3(1^2-1)=0$

shown $\therefore t = 1s$

P has a mass of 5 kg and is acted on by a single force **F** N.

(ii) Find **F** when $t = 2$. **[4]**

$$a = \frac{dv}{dt} = \begin{pmatrix} 12t-18 \\ -2 \\ 6t \end{pmatrix}$$

$$t = 2s \Rightarrow a = \begin{pmatrix} 6 \\ -2 \\ 12 \end{pmatrix} \text{ms}^{-2} \qquad F = ma$$

$$F = 5\begin{pmatrix} 6 \\ -2 \\ 12 \end{pmatrix} = \begin{pmatrix} 30 \\ -10 \\ 60 \end{pmatrix} N$$

(iii) Find an expression for the position, $\mathbf{r}$ m, of P at time t s, given that $\mathbf{r} = \begin{pmatrix} -5 \\ 2 \\ 6 \end{pmatrix}$ when $t = 0$. **[5]**

$$\int v \, dt = r \Rightarrow r = \int \begin{pmatrix} 6t^2-18t+12 \\ 2-2t \\ 3t^2-3 \end{pmatrix} dt$$

$$= \begin{pmatrix} 2t^3-9t^2+12t+c \\ 2t-t^2+d \\ t^3-3t+e \end{pmatrix} m \qquad \{c, d, e \text{ are constants}\}$$

$$t=0 \quad r = \begin{pmatrix} -5 \\ 2 \\ 6 \end{pmatrix} \Rightarrow \begin{matrix} c=-5 \\ d=2 \\ e=6 \end{matrix} \Rightarrow \therefore r = \begin{pmatrix} 2t^3-9t^2+12t-5 \\ 2t-t^2+2 \\ t^3-3t+6 \end{pmatrix} m$$

2 A smooth wire is bent to form a circle of radius 2.5m; the circle is in a horizontal plane. A small ring of mass 0.2 kg is travelling round the wire.

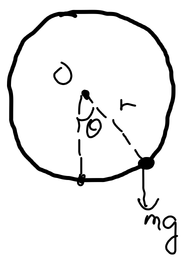
(i) At one instant the ring is travelling at an angular speed of 120 revolutions per minute.

(*A*) Calculate the angular speed in radians per second. **[1]**

$\omega = 120rpm = 2rps \Rightarrow f = 2Hz$

$\omega = 2\pi f = 2\pi \times 2 = 4\pi \, rad\,s^{-1}$

(*B*) Calculate the component towards the centre of the circle of the force exerted on the ring by the wire. r→radius **[4]**



Radial force by N2L $= m\omega^2 r$

$= 0.2 \times (4\pi)^2 \times 2.5$

$= 0.2 \times 16 \times 2.5\pi^2$

$= 78.96N$

$\sim 79.0N$ (3 s.f.)

(ii) Why must the contact between the wire and the ring be smooth if your answer to part **(i)** (*B*) is also the total horizontal component of the force exerted on the ring by the wire? **[1]**

If the contact was rough there would be a tangential force along with the force acting to the centre of circle.

3 A young woman wishes to make a bungee jump. One end of an elastic rope is attached to her safety harness. The other end is attached to the bridge from which she will jump.

She calculates that the stretched length of the rope at the bottom of her motion should be 20m, she knows that her weight is 576N and the stiffness of the elastic rope is 90 N m^{-1}. She has to calculate the unstretched length of rope required to perform the jump safely.

She models the situation by assuming the following.

- The rope is of negligible mass.
- Air resistance may be neglected.
- She is a particle.
- She moves vertically downwards from rest.
- Her starting point is level with the fixed end of the rope.
- The length she calculates for the rope does not include any extra for attaching the ends.

(i) (*A*) Show that the greatest extension of the rope, *X*, satisfies the equation $X^2 = 256$. **[3]**

$l_0 + \Delta x = 20\text{m}$
$W = 576\text{N}$
$K = 90 \text{ Nm}^{-1}$

GPE = Elastic PE at bottom

$\Rightarrow mg(20) = \frac{1}{2} k \Delta x^2$

$\Rightarrow \frac{2 \times W \times 20}{k} = \Delta x^2$

$\Rightarrow \frac{2 \times 20 \times 576}{90} = x^2 = 256$ (shown)

(*B*) Hence determine the natural length of rope she needs. **[2]**

$l_0 = ?$

$x^2 = 256 \Rightarrow x = \sqrt{256} = 16\text{m}$

$l_0 + x = 20$

$\Rightarrow l_0 = 4\text{m}$

(ii) To remain safe she wishes to be sure that, if air resistance is taken into account, the stretched length of the rope of natural length determined in part **(i)** will not be more than 20m. Advise her on this point. **[1]**

If energy is dissipated, EPE at bottom is less than the GPE at top. Therefore the extension would be lower than 16m. This means that if $l_0 = 4\text{m}$ then string won't go past 20m.

4 Two uniform circular discs with the same radius, A of mass 1 kg and B of mass 5.25 kg, slide on a smooth horizontal surface and collide obliquely with smooth contact.

Fig. 4 gives information about the velocities of the discs just before and just after the collision.

- The line XY passes through the centres of the discs at the moment of collision
- The components parallel and perpendicular to XY of the velocities of A are shown
- Before the collision, B is at rest and after it is moving at $2\,\text{m s}^{-1}$ in the direction XY

$\overset{+}{\rightarrow}$

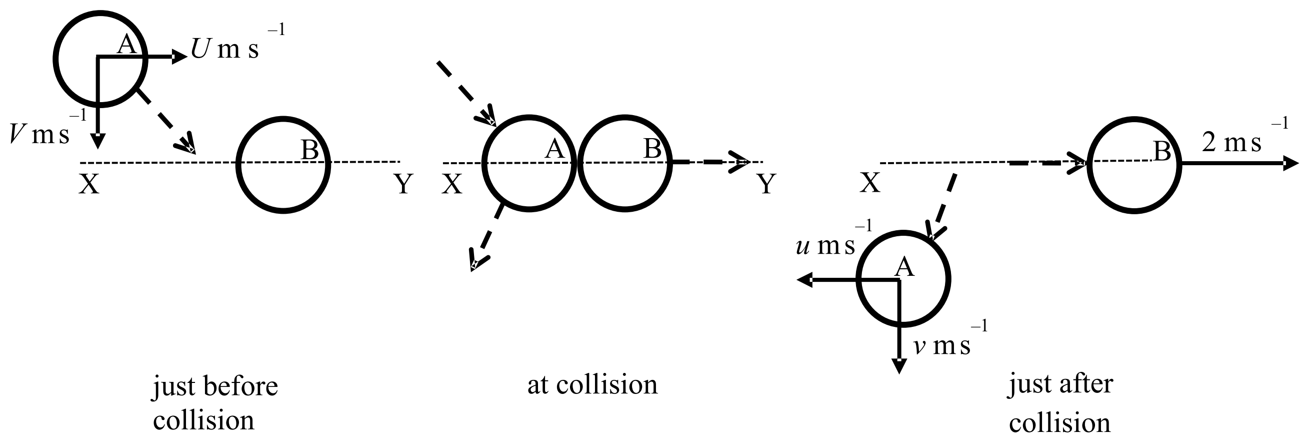


Fig. 4

The coefficient of restitution between the two discs is $\frac{2}{3}$.

(i) Find the values of U and u. **[5]**

PCLM $\overset{+}{\rightarrow}$: $U + 0 = 2\times5.25 - u$

$\Rightarrow U + u = 10.5$ — (i)

NEL: $-u - 2 = -\frac{2}{3}U$

$\Rightarrow u + 2 = \frac{2}{3}U \Rightarrow 3u + 6 = 2U$

$\Rightarrow 6 = 2U - 3u$ — (ii)

Simultaneous Eqns.:

$3U + \cancel{3u} = 31.5$

$2U - \cancel{3u} = 6$

$U = 7.5\,\text{ms}^{-1}$

$\Rightarrow u = 3\,\text{ms}^{-1}$

(ii) What information in the question tells you that $v = V$? **[1]**

The contact of discs is smooth.

The speed of disc A before the collision is $8.5 \mathrm{m\,s}^{-1}$.

(iii) Find the speed of disc A after the collision. **[2]**

$\Rightarrow \sqrt{U^2 + V^2} = 8.5$

$\Rightarrow 7.5^2 + V^2 = 8.5^2$

$\Rightarrow V^2 = 16$

$\therefore v^2 = V^2 = 16$

$\Rightarrow$ Speed after $= \sqrt{16 + (-3)^2} = \underline{\underline{5 \mathrm{ms}^{-1}}}$

5

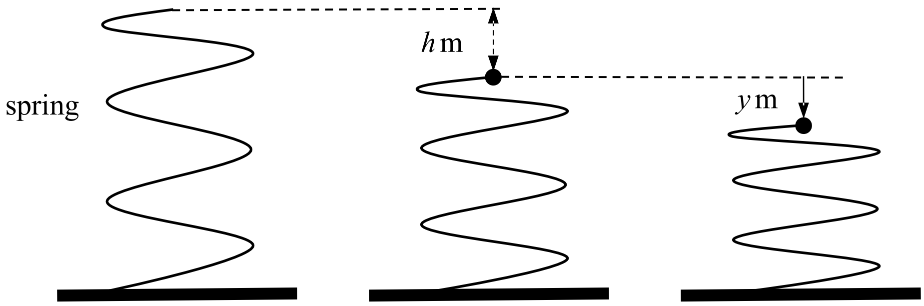


Fig. 5.1 **Fig. 5.2** **Fig. 5.3**

Fig. 5.1 shows a vertical light elastic spring. It is fixed to a horizontal table at one end. Fig 5.2 shows the spring with a particle of mass m kg attached to it at the other end. The system is in equilibrium when the spring is compressed by a distance h m.

(i) Find an expression for the stiffness of the spring, k N m^{-1}, in terms of m, h and g. **[1]**

$K = \frac{F}{\Delta x} = \frac{mg}{h}$

The particle is pushed down a further distance from the equilibrium position and released from rest. At time t seconds, the displacement of the particle from the equilibrium position of the system is y m in the downward direction, as shown in Fig. 5.3. You are given that $|y| \le h$.

$F = k\Delta x$

(ii) Show that the motion of the particle is modelled by the differential equation $\frac{d^2y}{dt^2} + \frac{gy}{h} = 0$. **[4]**

N2L on particle: $mg - \frac{mg}{h}(h+y) = m\ddot{y}$

$\Rightarrow \cancel{g} - \cancel{g} - \frac{gy}{h} = \ddot{y}$

$\Rightarrow 0 = \ddot{y} + \frac{gy}{h} \Rightarrow \frac{d^2y}{dt^2} + \frac{gy}{h} = 0$ shown

(iii) Find an expression for the period of the motion of the particle. **[1]**

$\frac{d^2y}{dt^2} = \frac{-gy}{h}$ from part (ii) $\ddot{y} = -\omega^2 y$

$\Rightarrow \omega^2 = \frac{g}{h} \Rightarrow \omega = \sqrt{\frac{g}{h}}$ $\omega = \frac{2\pi}{T}$

continued on next page

Turn over

$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{h}} \Rightarrow T = 2\pi\sqrt{\frac{h}{g}}$

(iv) Would the model for the motion of the particle be valid for large values of m? Justify your answer. **[1]**

No, if m is too large the spring will end up too compressed.

6 **In this question you must show detailed reasoning.**

As shown in Fig. 6.1, the region R is bounded by the lines $x = 1$, $x = 2$, $y = 0$ and the curve $y = 2x^2$ for $1 \le x \le 2$. A uniform solid of revolution, S, is formed when R is rotated through 360° about the x-axis.

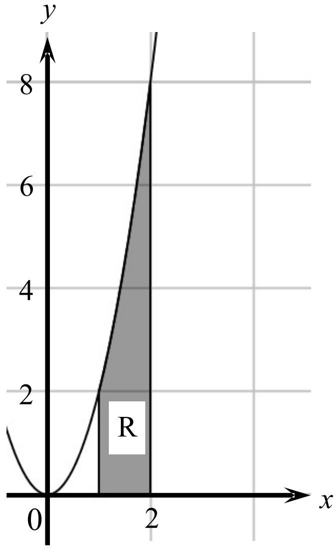


Fig. 6.1

(i) Show that the volume of S is $\frac{124\pi}{5}$. **[3]**

$V = \int_a^b \pi y^2 dx = \pi \int_1^2 y^2 dx = \pi \int_1^2 4x^4 dx = \pi \left[\frac{4}{5}x^5\right]_1^2$

$\Rightarrow \frac{4(2)^5\pi}{5} - \frac{4}{5}(1)^5\pi = \frac{124\pi}{5}$ cubic units (shown)

(ii) Show that the distance of the centre of mass of S from the centre of its smaller circular plane surface is $\frac{43}{62}$. **[5]**

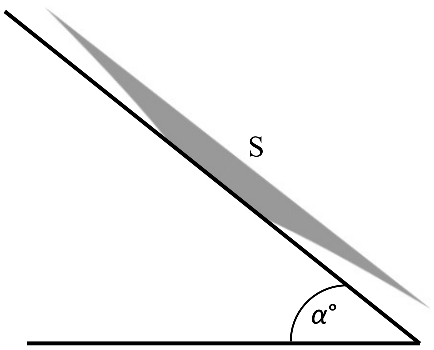
$V\bar{x} = \pi \int_a^b y^2 x \, dx$

$\Rightarrow \frac{124}{5}\cancel{\pi}\,\bar{x} = \pi\int_1^2 4x^5\,dx = \left[\frac{4}{6}x^6\right]_1^2 = \frac{256}{6} - \frac{4}{6}$

$\Rightarrow \frac{124}{5}\bar{x} = \frac{252}{6} \Rightarrow \bar{x} = \frac{252\times5}{124\times6} = \frac{105}{62}$ (this is from y axis)

$\therefore$ C.o.M from top of S $= \frac{105}{62} - 1 = \frac{43}{62}$ shown.

Fig. 6.2 shows S placed so that its smaller circular plane surface is in contact with a slope inclined at $\alpha°$ to the horizontal. S does not slip but is on the point of tipping.

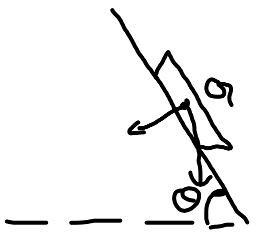


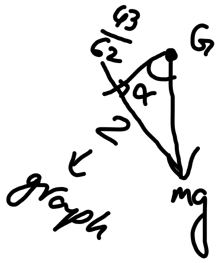
G – Centre of Mass

Fig. 6.2

(iii) Find the value of α, giving your answer in degrees correct to 3 significant figures. **[4]**

@ point of tipping C.o.M is directly above pivot.





$\tan\alpha = 2 \div \frac{43}{62}$

$\Rightarrow \alpha = \arctan\left(\frac{2\times62}{43}\right)$

$= 70.9°$ (3sf.)

7 A plane is inclined at 30° above the horizontal. A particle is projected up the plane from a point C on the plane with a velocity of $14\,\mathrm{m\,s^{-1}}$ at 40° above a line of greatest slope of the plane. The particle hits the plane at D. See Fig. 7.

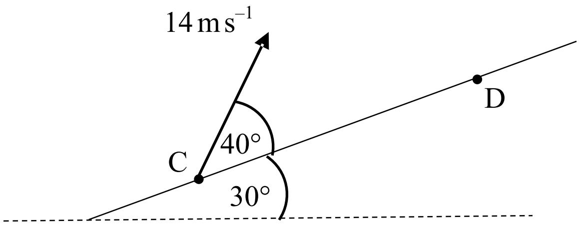


Fig. 7

(i) Using the standard model for projectile motion, show that the time of flight, T, is given by $T = \dfrac{28\sin 40°}{g\cos 30°}$. **[5]**

$\ddot{y} = -g\cos(30) \qquad \ddot{x} = -g\sin(30)$

$y: \; y = 14\sin 40\, t - \frac{1}{2} g\cos 30\, t^2$

When $y=0 \Rightarrow 14\sin(40)\,t = \frac{1}{2} g\cos 30\, t^2$

$\Rightarrow \quad t = \dfrac{28\sin(40)}{g\cos(30)}$ shown

(ii) Calculate the distance CD. **[4]**

$x = 14\cos 40\, T - \frac{1}{2} \times g\sin 30\, T^2$

$= 14\cos 40 \times \dfrac{28\sin 40}{g\cos 30} - \dfrac{g}{2}\sin 30\left(\dfrac{28\sin 40}{g\cos 30}\right)^2$

$= \sim$ 11.7m (3sf.)

END OF QUESTION PAPER