PhysicsAndMathsTutor.com

OCR
Oxford Cambridge and RSA

Accredited

AS Level Further Mathematics B (MEI)
Y411 Mechanics a
Sample Question Paper

Model Answers.

Date – Morning/Afternoon
Time allowed: 1 hour 15 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)
- Scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer all the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet.
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is 60.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of 12 pages. The Question Paper consists of 8 pages.

Answer all the questions

1 A clock is driven by a 5kg sphere falling once through a vertical distance of 120cm over 2 days.

Calculate, in watts, the average power developed by the falling sphere. [4]

$$\text{Power} = \frac{mgh}{t} = \frac{5 \times 9.8 \times 1.2}{2 \times 24 \times 3600} = \underline{0.000340 \text{ W}} \ (3\text{sf.})$$

2 A triangular lamina, ABC, is cut from a piece of thin uniform plane sheet metal. The dimensions of ABC are shown in Fig. 2.

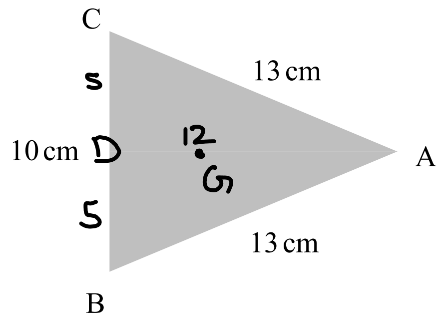


Fig. 2

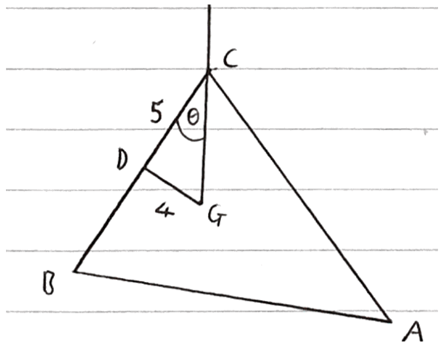
This piece of metal is freely suspended from a string attached to C and hangs in equilibrium.

Calculate the angle of BC with the downward vertical, giving your answer in degrees. [5]

By symmetry, the centre of mass is on AD.

Let C.O.M = G

$$\therefore DG = \frac{0 + 0 + 12}{3} = \underline{\underline{4}}$$



$\tan\theta = 4/5$

$\Rightarrow \theta = \arctan(4/5)$

$= \underline{\underline{38.7°}}$ (3sf)

3 Solid toy aeroplane nose cones of various sizes are made in the shape shown in Fig. 3.1, where OA is its line of symmetry.

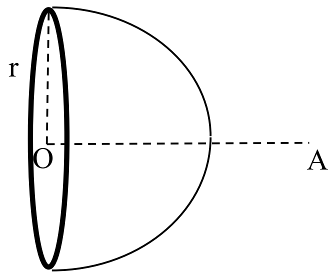


Fig. 3.1

The air resistance against the nose cone as the aeroplane flies through the air is initially modelled by $R = krv\eta$ where R is the air resistance, r is the radius of the circular flat end of the nose cone, v is the velocity of the nose cone, η is the viscosity of the air and k is a dimensionless constant.

(i) Use dimensional analysis to show that the dimensions of η are $ML^{-1}T^{-1}$. [3]

$R = krv\eta \Rightarrow [R] = [r][v][\eta]$

$\Rightarrow \frac{ML}{T^2} = L \times \frac{L}{T} \times [\eta] \Rightarrow [\eta] = \frac{MLT}{L^2T^2}$

$\Rightarrow [\eta] = MT^{-1}L^{-1}$ (shown)

In an experiment conducted on a particular nose cone, measurements of air resistance are taken for different velocities. The viscosity of the air does not vary during the experiment. The graph in Fig. 3.2 shows the results. Measurements are given using the appropriate S.I. units.

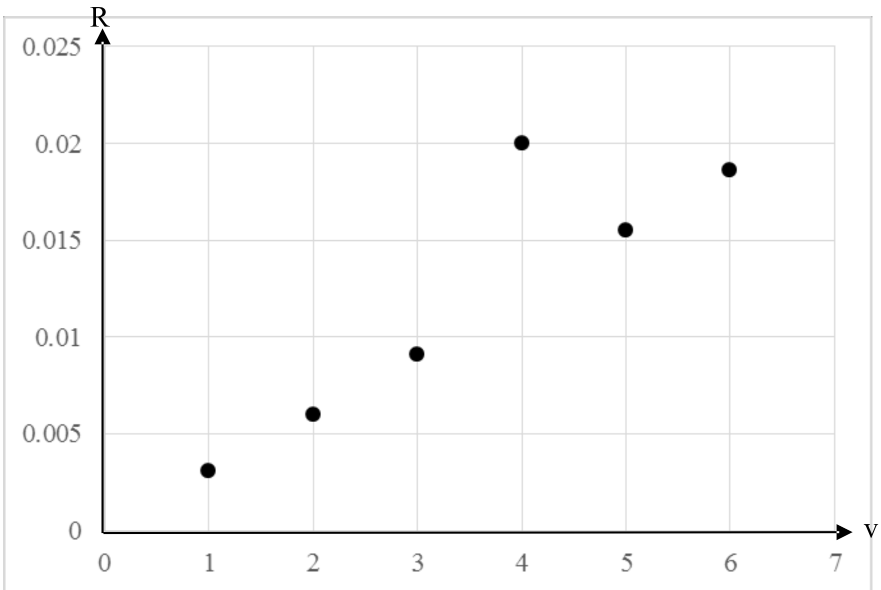


Fig. 3.2

(ii) Comment on whether the results of this experiment are consistent with the initial model. [3]

The points seem to lie on a straight line suggesting that relationship b/w v and R is linear.
The result at v = 4 is likely due to an experimental error, as it does not fit the pattern It can therefore be discarded.
The experiment is consistent with the model as they both suggest a proportional relationship between v and R.

It is now suggested that a better model for the air resistance is $R = Krv\eta\left(\frac{\rho rv}{\eta}\right)^{\alpha}$, where ρ is the density of the air, K is a dimensionless constant and R, r, v and η are as before.

(iii) (A) Find the dimensions of $\frac{\rho rv}{\eta}$. [2]

$$R = krv\eta\left(\frac{\rho rv}{\eta}\right)^{\alpha}$$

$$\Rightarrow \frac{\cancel{ML}}{\cancel{T^2}} = L \times \frac{\cancel{L}}{\cancel{T}} \times \frac{\cancel{M}}{\cancel{LT}} \times \left[\frac{\rho rv}{\eta}\right]$$

$$\Rightarrow \left[\frac{\rho rv}{\eta}\right] = 1$$

(B) Explain why you cannot use dimensional analysis to find the value of α [1]

It is dimensionless, therefore raising it to a power of α will have no effect.

4 Fig. 4 shows a thin rigid non-uniform rod PQ of length 0.5m. End P rests on a rough circular peg. A force of T N acts at the end Q at 60° to QP. The weight of the rod is 40N and its centre of mass is 0.3m from P.

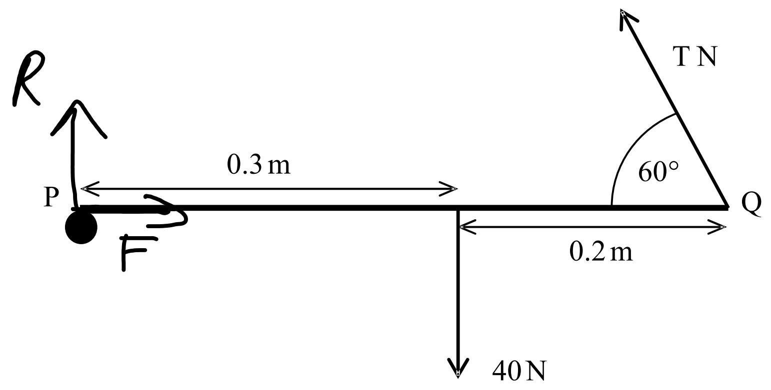


Fig. 4

The rod does not slip on the peg and is in equilibrium with PQ horizontal.

(i) Show that the vertical component of T is 24N. [2]

$\circlearrowleft P$: $40(0.3) = T\sin 60\,(0.5)$

$\Rightarrow T\sin 60 = 80 \times 0.3$

$\Rightarrow T\sin 60 = \underline{24N}$ shown

(ii) F is the contact force at P between the rod and the peg.
Find

- the vertical component of F,
- the horizontal component of F. [4]

From (i). $T\sin 60 = 24 \Rightarrow T = 24 \times \frac{2}{\sqrt{3}} = \underline{16\sqrt{3}\ N}$

$R(\rightarrow)$. $F = 16\sqrt{3}\cos 60$

$= \underline{\underline{8\sqrt{3}\ N}}$

$R(\uparrow)$: $R + T\sin 60 = 40$

$R = \underline{\underline{16\ N}}$

(iii) Given that the rod is about to slip on the peg, find the coefficient of friction between the rod and the peg. [2]

@ point of slipping $F = \mu R$

$\Rightarrow 8\sqrt{3} = 16\mu$

$\Rightarrow \mu = \frac{\sqrt{3}}{2} = \underline{\underline{0.866}}$ (3sf.)

5 In this question, all coordinates refer to the axes shown in Fig. 5.1.

Fig. 5.1 shows a system of four particles with masses 4m, 3m, m and 2m at the points A, B, C and D. These points have coordinates (–3, 4), (0, 0), (2, 0) and (5, 4).

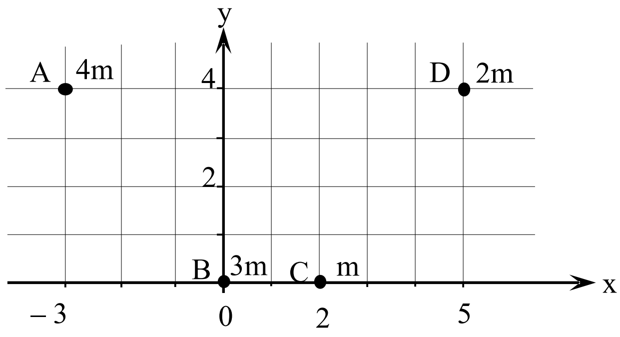


Fig. 5.1

(i) Calculate the coordinates of the centre of mass of the system of particles. [4]

$$4m\begin{pmatrix}-3\\4\end{pmatrix} + 3m\begin{pmatrix}0\\0\end{pmatrix} + 2m\begin{pmatrix}5\\4\end{pmatrix} + m\begin{pmatrix}2\\0\end{pmatrix} = 10m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\Rightarrow 10m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix} = \begin{pmatrix}0\\24m\end{pmatrix}$$

$$\Rightarrow (\bar{x}, \bar{y}) = \underline{\underline{(0, 2.4)}}$$

A thin uniform rigid wire of mass 12m connects the points A, B, C and D with straight line sections, as shown in Fig. 5.2.

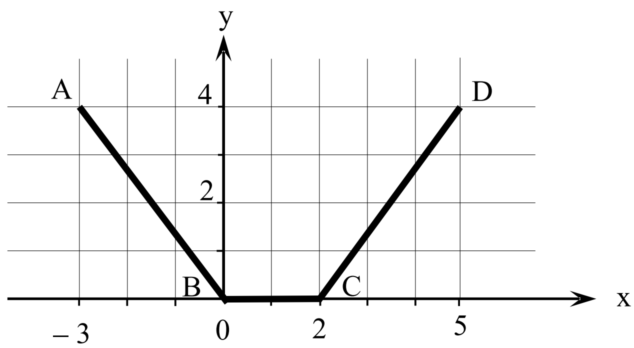


Fig. 5.2

(ii) Calculate the coordinates of the centre of mass of the wire. [4]

By symmetry, $\bar{x} = 1$

C.o.M of AB = C o M of CD = $\frac{4}{2} = 2$

PhysicsAndMathsTutor.com

Mass of AB = Mass of CD = 5m

Mass of BC = 2m

$\therefore \quad 12m\bar{y} = 5m \times 2 + 5m \times 2 + 2m \times 0$

$12\bar{y} = 20$

$\bar{y} = 5/3 \quad \Rightarrow (\bar{x}, \bar{y}) = (1, 5/3)$

The particles at A, B, C and D are now fixed to the wire to form a rigid object, R.

(iii) Calculate the x-coordinate of the centre of mass of R. [2]

$(12m + 10m)\bar{x} = 10m(0) + 12m(1)$

$22m\bar{x} = 12m$

$\bar{x} = \frac{6}{11}$

6 A sack of beans of mass 40kg is pulled from rest at point A up a non-uniform slope onto and along a horizontal platform. Fig. 6 shows this slope AB and the platform BC, which is a vertical distance of 12 m above A.

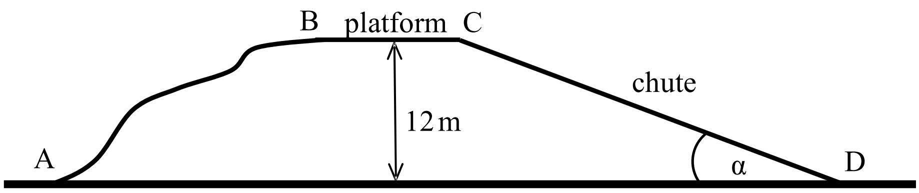


Fig. 6

(i) Calculate the gain in the gravitational potential energy of the sack when it is moved from A to the platform. [1]

Change in gpe = mgh

$= 40 \times 9.8 \times 12$

$= \underline{4704\text{ J}}$

The sack has a speed of $4\,\text{m s}^{-1}$ by the time it reaches C at the far end of the platform. The total work done against friction in moving the sack from A to C is 4841J. There are no other resistances to the sack's motion.

(ii) Calculate the total work done in moving the sack between the points A and C. [3]

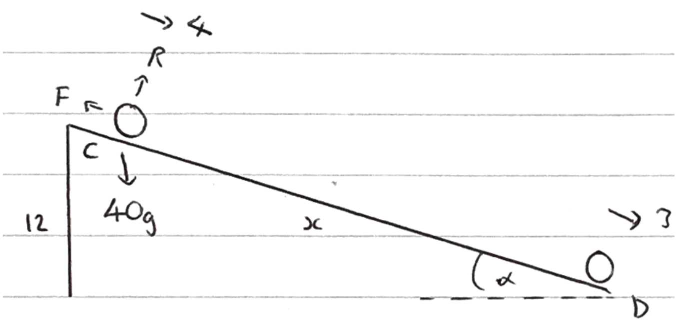
Total work = ↑KE + ↑GPE + W against friction

$= \frac{1}{2} \times 40 \times 4^2 + 4704 + 484$

$= \underline{\underline{5508J}}$

At point C, travelling at $4\,\text{m}\,\text{s}^{-1}$, the sack starts to slide down a straight chute inclined at α to the horizontal. Point D at the bottom of the chute is at the same vertical height as A, as shown in Fig. 6. The chute is rough and the coefficient of friction between the chute and the sack is 0.6. During this part of the motion, again the only resistance to the motion of the sack is friction.

(iii) Use an energy method to calculate the value of α given that the sack is travelling at $3\,\text{m}\,\text{s}^{-1}$ when it reaches D. [7]



$x = \frac{12}{\sin\alpha}$

$WD = Fx = \mu R x$

$= 0.6 \times 40g\cos\alpha \times \frac{12}{\sin\alpha}$

$= 2822.4\cot\alpha$

At C: $KE = \frac{1}{2} \times 40 \times 4^2 = 320J$

At D: $KE = \frac{1}{2} \times 40 \times 3^2 = 180J$

$\Delta KE \downarrow = \underline{140J}$

WD by friction $= \downarrow GPE + \downarrow KE$

$\Rightarrow 2822.4\cot\alpha = 4704 + 140$

$\Rightarrow \tan\alpha = 0.5827...$

$\Rightarrow \alpha = \underline{\underline{30.2^\circ}}$

For safety reasons the sack needs to arrive at D with a speed of less than $3\,\text{m}\,\text{s}^{-1}$. The value of α can be adjusted to try to achieve this.

(iv) (A) Find the range of values of α which achieve a safe speed at D. [1]

If speed at end = 0.

$2822.4\cot\alpha = 4704 + 320$

$\Rightarrow \tan\alpha = 0.5618$

$\Rightarrow \alpha = 29.3^\circ$

$\therefore 29.3^\circ < \alpha < 30.2^\circ$

(B) Comment on whether adjusting α is a practical way of achieving a safe speed at D. [1]

It is not practical because the range of values of α is very small.

7 Rose and Steve collide while sitting firmly on trays that are sliding on smooth horizontal ice. There are no external driving forces. Fig. 7 shows the masses of Rose and of Steve with their trays, their velocities just before their collision and the line of their motion and of their impact.

Immediately after the collision, Rose has a velocity of $0.28\,\mathrm{m\,s^{-1}}$ in the direction of her motion before the collision.

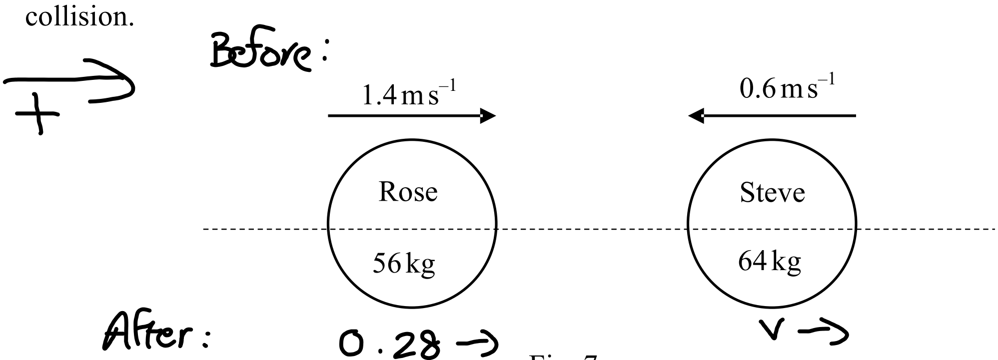


Fig. 7

(i) Find Steve's velocity after the collision. [3]

PCLM: $1.4(56) - 0.6(64) = 0.28(56) + v(64)$

$\Rightarrow 78.4 - 38.4 = 15.68 + 64v$

$\Rightarrow 24.32 = 64v$

$\Rightarrow v = \underline{0.38\ \mathrm{ms^{-1}}}$

$\therefore v = \underline{0.38\ \mathrm{ms^{-1}}}$ to the right

(ii) Find the coefficient of restitution between Rose and Steve on their trays. [2]

$e = \dfrac{\text{speed separation}}{\text{speed approach}} \Rightarrow \dfrac{0.38 - 0.28}{-0.6 - 1.4} = -e$

$\Rightarrow e = \underline{0.05}$

PhysicsAndMathsTutor.com

Shortly after the collision, Steve catches Rose's hand, pulls her towards him with a horizontal impulse of 4.48 N s and then lets go of her hand.

(iii) Calculate Rose's velocity after the pull. [2]

$$I = m(v-u)$$

$$4.48 = 56(v - 0\ 28)$$

$$v = \frac{4.48}{56} + 0.28 = \underline{\underline{0.36 \text{ ms}^{-1}}}$$

When they collide again they hold one another and move together with a common speed of V m s^{-1}.

(iv) Calculate V. [3]

$\xrightarrow{+}$

Start: $\text{R} \rightarrow 1.4$ (56g) $\quad 0.6 \leftarrow \text{S}$ (64g)

End $\quad \rightarrow V$ (120g)

PCLM: $56(1.4) - 64(0.6) = 120V$

$\Rightarrow 40 = 120V$

$\Rightarrow V = \frac{1}{3} \text{ ms}^{-1}$

(v) Why did you need to know that there are no driving forces and that the ice is smooth? [1]

So that linear momentum is conserved.

END OF QUESTION PAPER