OCR
Oxford Cambridge and RSA

Thursday 08 October 2020 – Afternoon

AS Level Further Mathematics B (MEI)

Y411/01 Mechanics a

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

## ADVICE
- Read each question carefully before you start your answer.

OCR is an exempt Charity

Turn over

1   Brent is riding his bicycle along a straight horizontal road.

While riding along this road Brent can attain a maximum speed of $6.25\,\text{ms}^{-1}$ and the wind resistance acting on Brent and his bicycle is constant and equal to $19.2\,\text{N}$. Brent and his bicycle have a combined mass of $72\,\text{kg}$.

Brent later begins to ride up a hill which is inclined at an angle of $3°$ to the horizontal.

Given that the wind resistance and the maximum power developed by the bicycle is unchanged, determine Brent's maximum speed up the hill.   [4]

$6.25\,\text{ms}^{-1}$

$19.2\,\text{N}$

D_f

$72g\cos 3°$

$3°$

$3°$

$72g\sin 3°$

$72g$
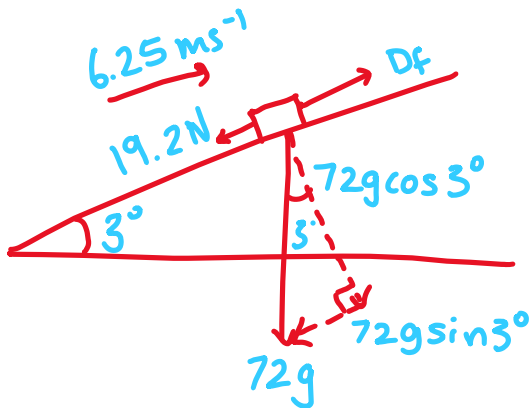
$P = fv$

$F = ma$

Normal Driving Force : $D_1 = 19.2\,\text{N}$

Maximum Power Output : $P = 19.2 \times 6.25 = 120\,\text{W}$

Driving Force on Slope : $D_2 - 19.2 = 72g\sin 3°$

$$\therefore D_2 = 72g\sin 3° + 19.2 = 56.128\ldots$$

$$\therefore \text{Maximum Speed} = \frac{P}{D_2} = \frac{120}{56.128\ldots} = 2.137\ldots \approx \boxed{2.14\,\text{ms}^{-1}}$$

(3sf)

2   George is investigating the time it takes for a ball to reach a certain height when projected vertically upwards. George believes that the time, t, for the ball to reach a certain height, h, depends on

- the ball's mass m,
- the projection speed u, and
- the height h.

George suggests the following formula to model this situation

$$t = km^{\alpha}u^{\beta}h^{\gamma},$$

where k is a dimensionless constant.

(a) Use dimensional analysis to show that $t = \dfrac{kh}{u}$. [3]

(b) Hence explain why George's formula is unrealistic. [1]

Mandy argues that any model of this situation must consider the acceleration due to gravity, g. She suggests the alternative formula

$$t = \frac{u - \sqrt{u^2 + gh}}{g}.$$

(c) Show that Mandy's formula is dimensionally consistent. [3]

(d) Explain why Mandy's formula is incorrect. [1]

(a.) $\text{RHS} = M^{\alpha}(LT^{-1})^{\beta}L^{\gamma} = M^{\alpha}L^{\beta+\gamma}T^{-\beta}$

$\therefore \alpha = 0, \ \beta+\gamma = 0, \ -\beta = 1$

$\therefore \beta = -1, \ \gamma = -\beta = 1$

$\boxed{\therefore t = \dfrac{kh}{u}}$

(b.) For a fixed projection speed, the model suggests it is possible for the particle to reach any height given enough time.

(c.) $u^2 = (LT^{-1})^2 = L^2 T^{-2}$

$gh = LT^{-2} \cdot L = L^2 T^{-2}$

$\sqrt{u^2 + gh} = (L^2 T^{-2})^{\frac{1}{2}} = LT^{-1}$

$u = LT^{-1}$

$\therefore \dfrac{u - \sqrt{u^2 + gh}}{g} = \dfrac{LT^{-1}}{LT^{-2}} = T$

$\boxed{\therefore t = T}$ $\therefore$ Dimensionally correct.

(d.) Since $\underline{\sqrt{u^2 + gh}} > u$, this model predicts a <u>negative</u>

<u>value for t.</u>

3   Fig. 3 shows a light square lamina ABCD, of side length 0.75 m, suspended vertically by wires attached to A and B so that AB is horizontal. A particle P of mass m kg is attached to the edge DC. The lamina hangs in equilibrium.
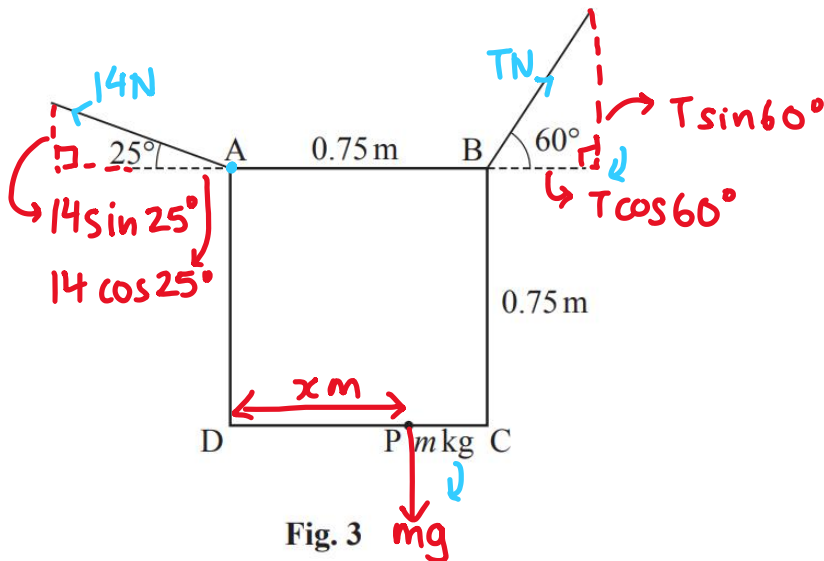

14N

☐ 25°

14sin25°

14cos25°

A   0.75 m   B   60°

TN

↗ Tsin60°

↳ Tcos60°

0.75 m

x m

D   P m kg C

mg

Fig. 3

The tension in the wire attached to A is 14 N and the tension in the wire attached to B is T N. The wire at A makes an angle of 25° with the horizontal and the wire at B makes an angle of 60° with the horizontal.

(a) Determine the value of T.   [2]

(b) Determine

   (i) the value of m,   [2]

   (ii) the distance of P from D.   [2]

P is moved to the midpoint of CD. A couple is applied to the lamina so that it remains in equilibrium with AB horizontal and the tension in both wires unchanged.

(c) Determine

   • the magnitude of the couple,
   • the direction of the couple.

    [3]

(a.) Horizontal Forces (⟷):

$$14\cos25° = T\cos60°$$

$$T = \frac{14\cos25°}{\cos60°} = 25.37... \approx 25.4 \text{ N (3sf)}$$

$$\therefore T = 25.4 \text{ N}$$

(b.) (i) Vertical Forces ($\updownarrow$):

$$14\sin 25° + T\sin 60° = mg$$

$$m = \frac{14\sin 25° + T\sin 60°}{g} = 2.846... \approx 2.85 \text{ kg} \quad (3sf$$

$$\boxed{\therefore m = 2.85 \text{ kg}}$$

(ii) M(A): $mgx = T\sin 60° \times 0.75$

$$(2.85)(9.8)x = (25.4)\left(\frac{\sqrt{3}}{2}\right)(0.75)$$

$$x = \frac{16.49...}{27.93} = 0.5906... \approx 0.591 \text{ m} \quad (3sf)$$

$$\boxed{\therefore x = 0.591 \text{ m}}$$

(c.) M(A): $mg\left(\frac{0.75}{2}\right) - T\sin 60° (0.75)$

$$= (2.85)(9.8)(0.375) - (25.4)\left(\frac{\sqrt{3}}{2}\right)(0.75)$$

$$= 10.47... - 16.49...$$

$$= -6.024...$$

$$\approx -6.02 \quad (3sf)$$

$\therefore$ Couple has magnitude <u>6.02 Nm</u> in the <u>clockwise</u> direction.

4   Fig. 4 shows a uniform beam of length $2a$ and weight W leaning against a block of weight $2W$ which is on a rough horizontal plane. The beam is freely hinged to the plane at O and makes an angle θ with the horizontal. The contact between the beam and the block is smooth. The beam and block are in equilibrium, and it may be assumed that the block does not topple.
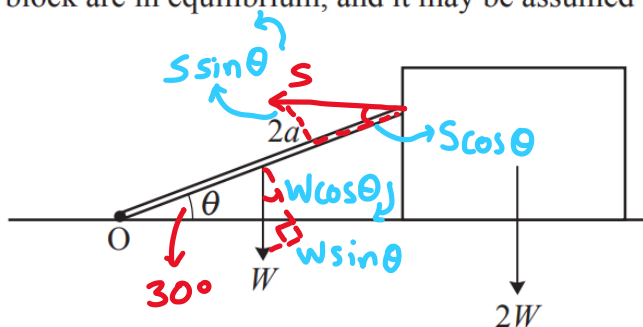

Fig. 4

Let

- S be the normal contact force between the beam and the block,
- R be the normal contact force between the plane and the block,
- F be the frictional force between the plane and the block.

Partially complete force diagrams showing the beam and the block separately are given in the Printed Answer Booklet.

(a) Add the forces listed above to these diagrams.   [2]

It is given that $\theta = 30°$.

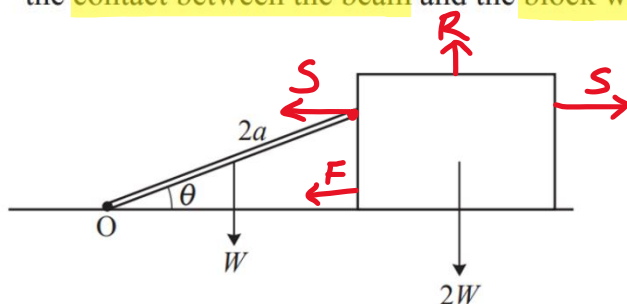**(b)** Determine the minimum possible value of the coefficient of friction between the block and the plane.   [5]

(c) In each case explain, with justification, how your answer to part **(b)** would change (assuming the rest of the system remained unchanged) if

    (i) $\theta < 30°$,   [1]

    (ii) the contact between the beam and the block were rough.   [1]

(b.) $M(0):$ $W\cos\theta\, \not{a} = S\sin\theta\,(2\not{a})$

$\qquad\qquad \theta = 30° \Rightarrow W\cos 30° = 2S\sin 30°$

$\qquad\qquad \therefore S = \dfrac{\sqrt{3}}{2}W$

Forces on the Block: $\quad F = S = \dfrac{\sqrt{3}}{2}W$

$\qquad\qquad\qquad\qquad\qquad R = 2W$

$F \le \mu R$

$\dfrac{\sqrt{3}}{2}\not{W} \le 2\mu\not{W}$

$\therefore \mu \ge \dfrac{\sqrt{3}}{4}$ $\qquad$ $\boxed{\therefore \text{Minimum Value of } \mu = \dfrac{\sqrt{3}}{4}}$

(c.) (i) If $\theta < 30°$, S would be greater, making the friction required greater, so minimum value of μ would be greater.

(ii) Minimum value of μ would be less as S would decrease and the beam would then exert a downwards force on the block, making R increase.

5   Throughout this question it may be assumed that there are no resistances to motion.

Model trucks A and B, with masses 5 kg and 3 kg respectively, rest on a set of straight, horizontal rails.

Truck A is given an impulse of 3.8 N s towards B.

(a) Calculate the initial speed of A.   [1]

Truck A collides directly with B. After the collision, B moves with a speed of $0.6 \, \text{m s}^{-1}$.

(b) Determine

(i)   the velocity of A after the collision,   [2]

(ii) the kinetic energy lost due to the collision.   [2]

(c) B continues to move with a speed of $0.6 \, \text{m s}^{-1}$ and collides with a model truck C, of mass 4 kg, which is travelling at a speed of $0.2 \, \text{m s}^{-1}$ towards B on the same set of rails. After the collision between B and C, the speeds of B and C are in the ratio 1 to 2.

Determine the two possible values of the coefficient of restitution between B and C.   [6]

(a.) $I = m(v-u)$

$3.8 = 5(v - 0)$

$\therefore v = \dfrac{3.8}{5} = \boxed{0.76 \ \text{m s}^{-1}}$

(b.) Before: $\xrightarrow{0.76}$ Rest $p = mv$
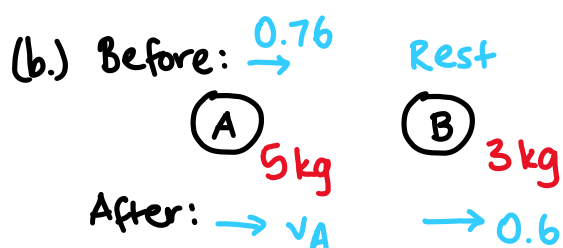
$\overset{A}{\textcircled{A}} \ 5kg \qquad \overset{B}{\textcircled{B}} \ 3kg$ Conservation of Momentum:

After: $\rightarrow v_A \qquad \rightarrow 0.6$ Before p = After p

i) $5(0.76) = 5v_A + 3(0.6)$

$3.8 = 5v_A + 1.8$

$v_A = \dfrac{3.8 - 1.8}{5} = 0.4 \ \text{m s}^{-1}$ $\boxed{\therefore \ v_A = 0.4 \ \text{m s}^{-1}}$

in the same direction as it was travelling before the collision

(ii) $KE = \frac{1}{2}mv^2$

$KE \ Start = \frac{1}{2}(5)(0.76)^2 = 1.444$

$KE \ End = \frac{1}{2}(5)(0.4)^2 + \frac{1}{2}(3)(0.6)^2 = 0.94$

$KE \ Lost = KE \ Start \ - \ KE \ End$

$= 1.444 - 0.94$

$= \boxed{0.504 \ J}$

(c.) B: $\rightarrow 0.6$ $\leftarrow -0.2$

Ⓑ $3kg$ Ⓒ $4kg$

A: x $2x$

B : C $e = \dfrac{u_B - u_A}{v_A - v_B}$

1 : 2

Case 1: Trucks travel in same direction after collision.

$3(0.6) + 4(-0.2) = 3x + 4(2x)$

$1 = 11x$

$\therefore x = \frac{1}{11}$

$e = \dfrac{\left(\frac{2}{11} - \frac{1}{11}\right)}{0.6 - -0.2} = \frac{5}{44} \approx 0.114 \ (3sf)$

Case 2: Trucks travel in opposite directions after collision.

$3(0.6) + 4(-0.2) = 3(-x) + 4(2x)$

$1 = 5x$

$\therefore x = \frac{1}{5}$

$e = \dfrac{\left(\frac{2}{5} - -\frac{1}{5}\right)}{0.6 - -0.2} = \frac{3}{4} = 0.75$

$\boxed{\therefore e = \frac{5}{44} \ OR \ e = \frac{3}{4}}$

6   Fig. 6.1 shows a solid uniform prism OABCDEFG. The Ox, Oy and Oz axes are also shown. The cross-section of the prism is a trapezium.

Fig. 6.2 shows the face OABC.

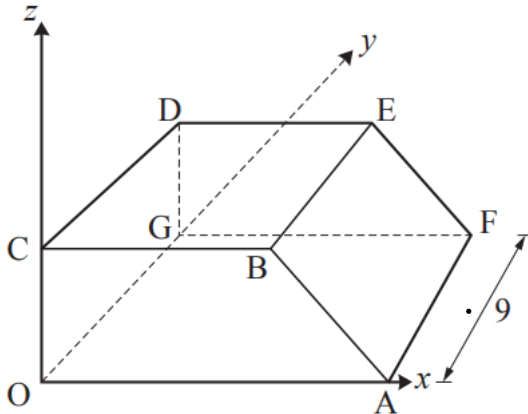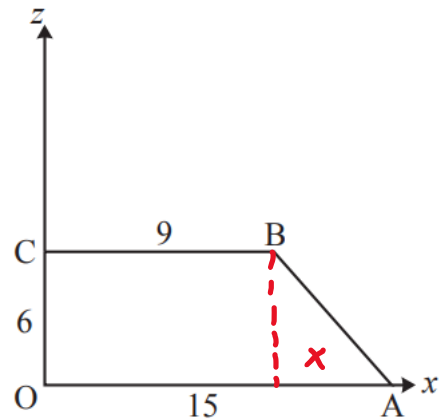The dimensions shown in the figures are in centimetres.

Fig. 6.1   Fig. 6.2

The centre of mass of the prism has coordinates $(\overline{x}, \overline{y}, \overline{z})$.

(a) Determine the values of $\overline{x}, \overline{y}$ and $\overline{z}$.   [7]

(b) By considering triangle PBA, where P has coordinates $(\overline{x}, 0, \overline{z})$, determine whether it is possible for the prism to rest with the face ABEF in contact with a horizontal plane without toppling.   [3]

(a.) $\overline{y} = \frac{9}{2} = 4.5$

Area $OABC = \frac{1}{2}(15+9)(6) = 72 \text{ cm}^2$

Coordinates of centre of mass of triangle $= (11, 2)$

$72\overline{x} = (9)(6)(4.5) + \frac{1}{2}(6)(6)(11)$

$\therefore \overline{x} = 6.125$

$72\overline{z} = (9)(6)(3) + \frac{1}{2}(6)(6)(2)$

$\therefore \overline{z} = 2.75$

$\boxed{\therefore \overline{x} = 6.125, \overline{y} = 4.5, \overline{z} = 2.75}$

(b.) $PB^2 + BA^2 = (9 - 6.125)^2 + (6 - 2.75)^2 + 6^2 + 6^2 = \dfrac{5813}{64}$

$PA^2 = (15 - 6.125)^2 + (0 - 2.75)^2 = \dfrac{5525}{64}$

$PA^2 < PB^2 + BA^2$

$\therefore \angle PBA < 90°$ so yes, the prism will stay in that position.

7   Fig. 7.1 shows one end of a light inextensible string attached to a block A of mass 4.4 kg. The other end of the string is attached to a block B of mass 5.2 kg.

Block A is in contact with a smooth horizontal plane. The string is taut and passes over a small smooth pulley at the end of the plane. Block B is inside a hollow vertical tube and the vertical sides of B are in contact with the tube. Initially B is 1.6 m above the horizontal base of the tube.
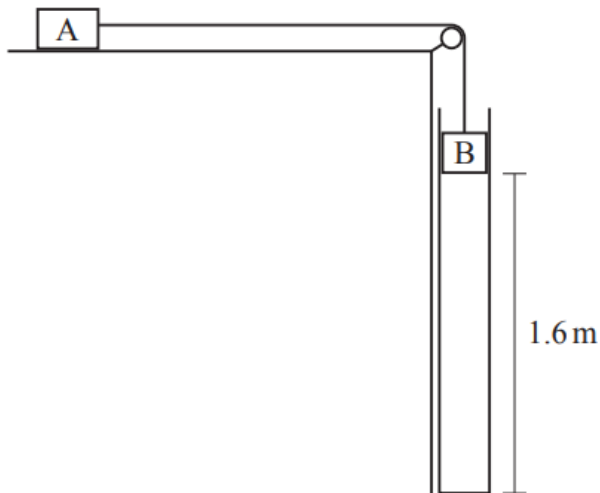

Fig. 7.1

The blocks are released from rest. It may be assumed that in the subsequent motion A does not reach the pulley and the string remains taut.

Block B reaches the base of the tube with speed 3.5 m s^{-1}.

(a) Given that the frictional force exerted by the tube on B is constant, use an energy method to show that the magnitude of this force is 14.21 N.   **[4]**

(a) Energy Start = Energy End

$$KE = \frac{1}{2}mv^2$$

$$GPE = mgh$$

$$WD = dF$$

$$GPE - WD = KE + KE$$

$$5.2g(1.6) - 1.6F = \frac{1}{2}(4.4)(3.5)^2 + \frac{1}{2}(5.2)(3.5)^2$$

$$F = \frac{81.536 - 58.8}{1.6} = 14.21 \text{ N} \qquad \boxed{\therefore F = 14.21 \text{ N}}$$

Blocks A and B remain attached to the opposite ends of a light inextensible string, but A is now in contact with a rough plane inclined at $\theta°$ to the horizontal, as shown in Fig. 7.2.

The string connecting A and B is taut and passes over a small smooth pulley at the top of the plane. Block B is inside the same hollow vertical tube as before with the vertical sides of B in contact with the tube. It may be assumed that the frictional force exerted by the tube on B remains unchanged.
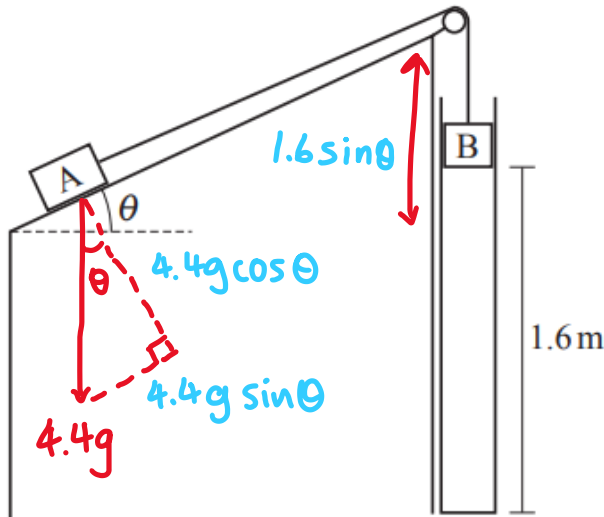

1.6sinθ

4.4g cosθ

4.4g sinθ

4.4g

B

1.6 m

Fig. 7.2

The coefficient of friction between block A and the plane is $\frac{3}{11}$.
The blocks are released from rest, with block B 1.6 m above the base of the tube. It may be assumed that in the subsequent motion A does not reach the pulley and the string remains taut.

(b) Given that block B reaches the base of the tube with speed $0.7\,\text{ms}^{-1}$, show that θ satisfies the equation

$$3\cos\theta + 11\sin\theta = k,$$

where k is a constant to be determined. [5]

(b.) GPE gain of block A = $4.4g\,(1.6\sin\theta) = 68.992\sin\theta$

Fr = μR → Friction on A = $\frac{3}{11}(4.4g\cos\theta)(1.6) = 18.816\cos\theta$

End KE of A & B = $\frac{1}{2}(4.4+5.2)(0.7)^2 = 2.352$

GPE of A + Total Friction = −KE of A & B + GPE of B

$68.992\sin\theta + 18.816\cos\theta + 14.21(1.6) = -2.352 + 5.2g(1.6)$

$68.992\sin\theta + 18.816\cos\theta = 56.448$ ÷ $\frac{784}{125}$

∴ $3\cos\theta + 11\sin\theta = 9$

where K = 9.